Low-temperature resistance in metals without inversion center

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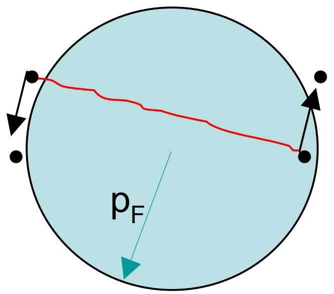
How? You don't know this?
We absorbed it with mother's milk.
I.M.Khalatnikov

OUTLINE

- **Fermi liquid kinetic properties**
- **Metals without inversion**
- **Band splitting**
- **Kinetic equation**
- **Electron-electron collision integral**
- **Dissipation in the ballistic regime**
- **Residual resistivity in impurityless metals**
- **Conclusion**

Fermi liquid $T \ll \varepsilon_F$



Scattering rate in Fermi liquid $W = \frac{1}{2\tau} \propto T^2$

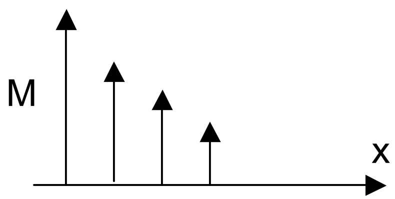
Resistivity $\rho = \rho_0 + AT^2$ Landau & Pomeranchuk 1936

Viscosity $\eta \propto \frac{1}{T^2}$

Pomeranchuk 1950

Abrikosov & Khalatnikov 1957

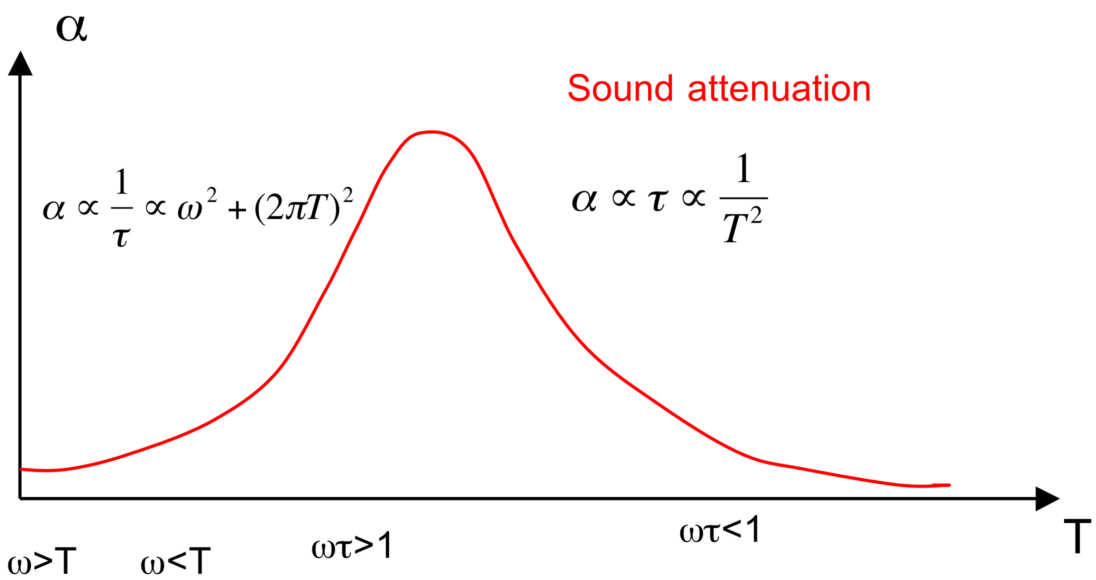
Thermalconductivity $\kappa \propto \frac{1}{T}$



Longitudinal spin diffusion $D_{\parallel} \propto \frac{1}{T^2}$ D. Hone 1961

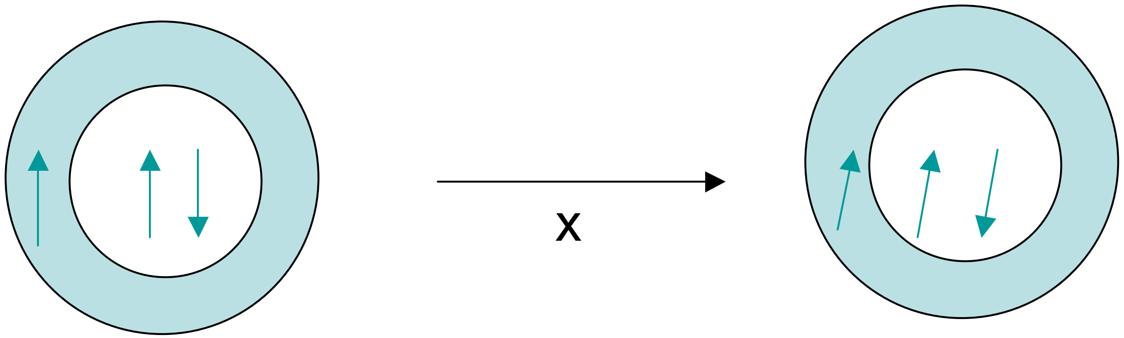
Sound attenuation

Landau 1957



M.Meizel
2004

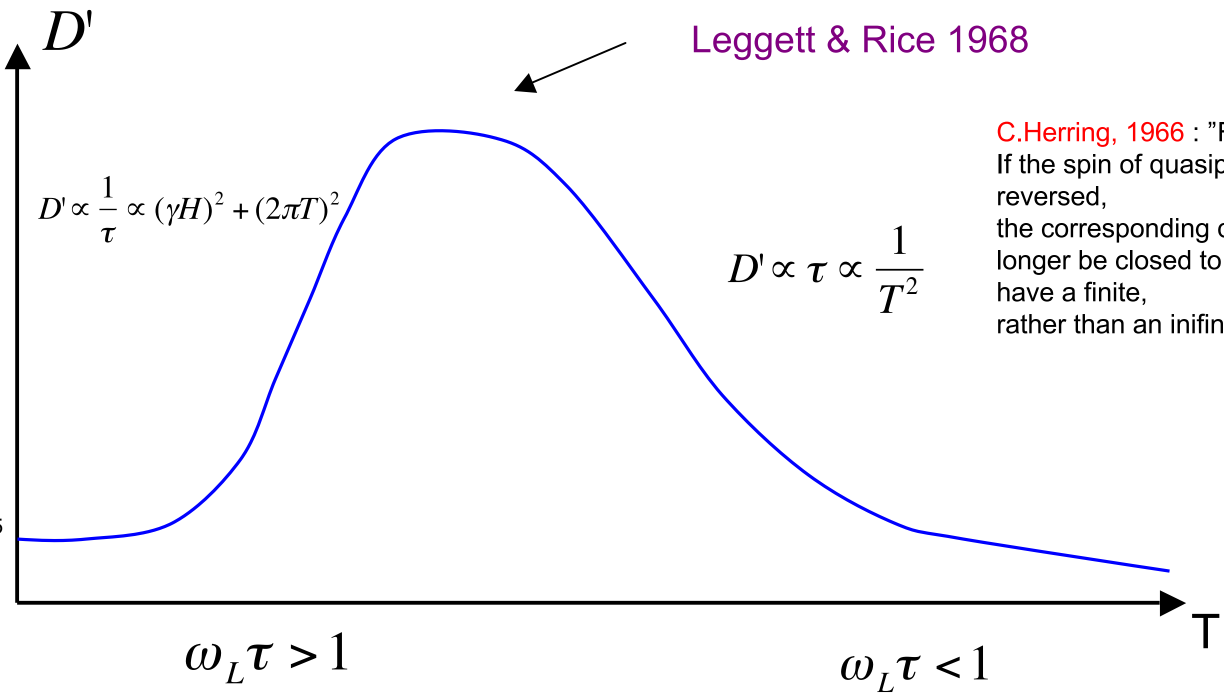
Transverse spin-diffusion in polarized Fermi liquid



$$\omega = \omega_L + (D'' - iD')k^2$$

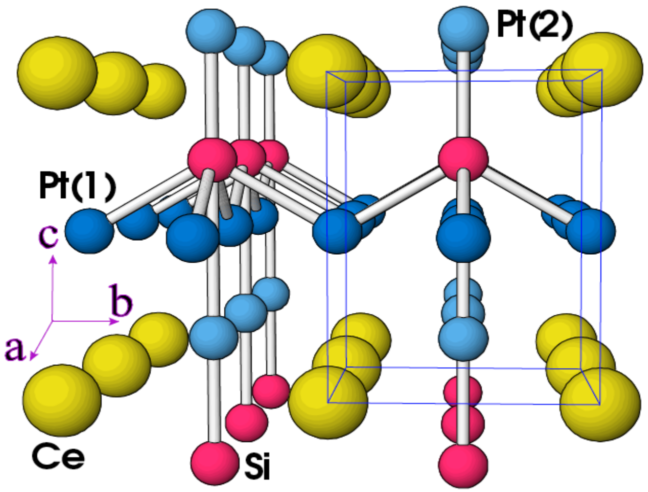
Jeon&Mullin 1989,1992
Meyerovich&Musaelyan 1994
Golosov&Ruckenstein 1995, 1998
Mineev 2004

O.Buu et all 2002
H.Akimoto et al 2003
Vermeulen&Roni 2003
Vermeulen&Perisanu 2005



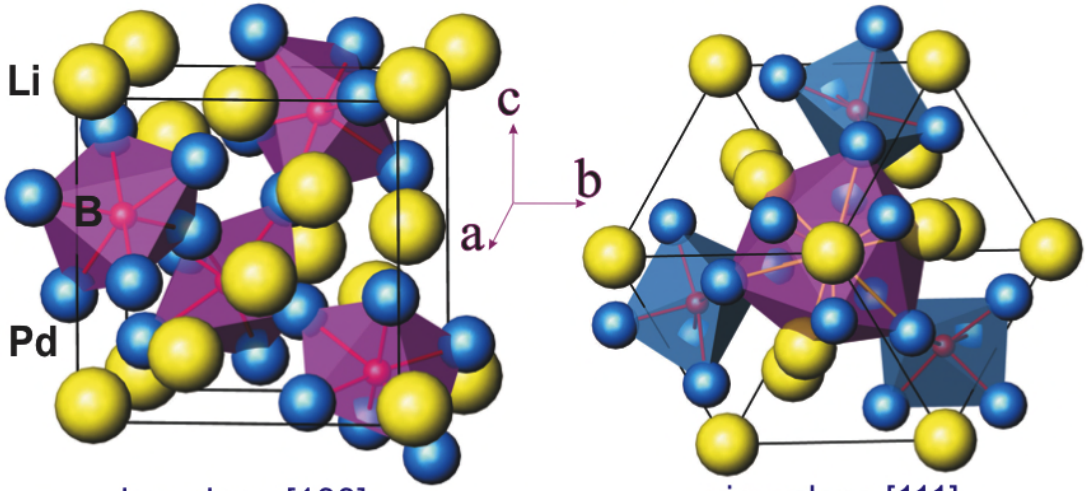
C.Herring, 1966 : "For a ferromagnetic metal….
If the spin of quasiparticle at Fermi surface is reversed,
the corresponding quasiparticle state will no longer be closed to the Fermi surface, and it will have a finite,
rather than an inifinitesimal, decay rate."

Metals without inversion



CePt$_3$Si - P4mm; CePt$_3$B-type

Li$_2$Pd$_3$B - P4$_1$32; filled β-Mn-type



view along [100]   view along [111]

E.Bauer et al, 2004   CePt$_3$Si

CeRhSi$_3$, CeIrSi$_3$ G=C_{4v}

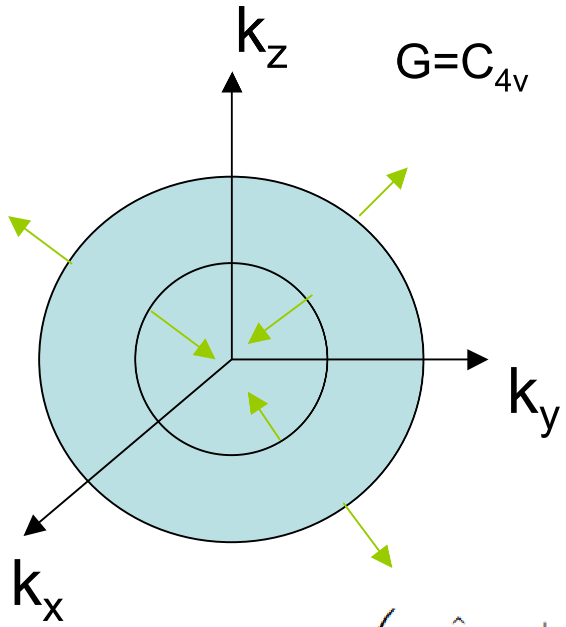
Li$_2$(Pd$_{1-x}$Pt$_x$)$_3$B   G=O

G=C_{2v}

UIr

Band splitting

$$\hat{\varepsilon}(\mathbf{k}) = \varepsilon(\mathbf{k})\hat{\delta} + \boldsymbol{\gamma}(\mathbf{k})\cdot\boldsymbol{\sigma}$$



G=C_{4v} $$\boldsymbol{\gamma}(\mathbf{k}) = \gamma(k_y\hat{x} - k_x\hat{y}) + \gamma_{||}k_xk_yk_z(k_x^2 - k_y^2)\hat{z}$$

G=O $$\boldsymbol{\gamma}(\mathbf{k}) = \gamma\mathbf{k}$$

$$\varepsilon_{\pm}(\mathbf{k}) = \varepsilon(\mathbf{k}) \pm |\boldsymbol{\gamma}(\mathbf{k})|$$

FS equation $$\varepsilon_{\pm}(\mathbf{k}) = \mu$$

$$k_{F\pm} = \mp m\gamma + \sqrt{2m\mu + (m\gamma)^2}$$

$$\Psi_\sigma^+(\mathbf{k}) = C_\mathbf{k}\begin{pmatrix} \hat{\gamma}_{\mathbf{k}z} + 1 \\ \hat{\gamma}_{\mathbf{k}x} + i\hat{\gamma}_{\mathbf{k}y} \end{pmatrix}$$

$$\hat{n}_0 = \frac{n_+ + n_-}{2}\hat{1} + \frac{n_+ - n_-}{2|\boldsymbol{\gamma}|}\boldsymbol{\gamma}\cdot\boldsymbol{\sigma}$$

$$\Psi_\sigma^-(\mathbf{k}) = C_\mathbf{k}\begin{pmatrix} -\hat{\gamma}_{\mathbf{k}x} + i\hat{\gamma}_{\mathbf{k}y} \\ \hat{\gamma}_{\mathbf{k}z} + 1 \end{pmatrix}$$

$$n_\pm = \frac{1}{e^{\xi_\pm} + 1} \qquad \xi_\pm \approx v_F(k - k_{F\pm}) = \varepsilon - \mu_\pm$$

? $$\tau_{ee} \propto \frac{1}{(2\pi T)^2 + (v_F\Delta k_F)^2}$$

$$\mathbf{v}_F = \frac{\partial(\varepsilon \pm \gamma k)}{\partial\mathbf{k}}|_{k=k_{F\pm}} = \hat{\mathbf{k}}\sqrt{\frac{2\mu}{m} + \gamma^2}$$

$$\mu_\pm = v_Fk_{F\pm}, \quad \mu_+ - \mu_- = -2mv_F\gamma$$

Kinetic equation
(spin representation)

V.P.Silin 1957

$$\frac{\partial \hat{n}_1}{\partial t} - i[\hat{\varepsilon}_1, \hat{n}_1] + \frac{1}{2}\left(\frac{\partial \hat{\varepsilon}_1}{\partial \mathbf{k}_1}\frac{\partial \hat{n}_1}{\partial \mathbf{r}} + \frac{\partial \hat{n}_1}{\partial \mathbf{r}}\frac{\partial \hat{\varepsilon}_1}{\partial \mathbf{k}_1}\right) - \frac{1}{2}\left(\frac{\partial \hat{\varepsilon}_1}{\partial \mathbf{r}}\frac{\partial \hat{n}_1}{\partial \mathbf{k}_1} + \frac{\partial \hat{n}_1}{\partial \mathbf{k}_1}\frac{\partial \hat{\varepsilon}_1}{\partial \mathbf{r}}\right) = \hat{I}$$

$$\hat{I} = 2\pi \int d^3\mathbf{k}' \frac{d^3\mathbf{k}''}{(2\pi)^3}\frac{d^3\mathbf{k}_2}{(2\pi)^3}\sum_{\mathbf{m}} \delta(\varepsilon_1 + \varepsilon_2 - \varepsilon' - \varepsilon'')\delta\left(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}' - \mathbf{k}'' - \frac{2\pi\mathbf{m}}{a}\right)\hat{F}$$

$$\hat{F} = \frac{1}{2}W_1\left\{[\hat{n}', (\hat{1} - \hat{n}_1)]_+ Tr((\hat{1} - \hat{n}_2)n'') - [(\hat{1} - \hat{n}'), \hat{n}_1]_+ Tr(\hat{n}_2(\hat{1} - \hat{n}''))\right\}$$
$$+\frac{1}{2}W_2\left\{[\hat{n}'(\hat{1} - \hat{n}_2)\hat{n}'', (\hat{1} - \hat{n}_1)]_+ - [(\hat{1} - \hat{n}')\hat{n}_2(\hat{1} - \hat{n}''), \hat{n}_1]_+\right\}$$

$$\hat{n} = \frac{1}{2}\left[f\ \hat{1} + \mathbf{g}\cdot\boldsymbol{\sigma}\right]$$

$$\left(e\mathbf{E}\cdot\frac{\partial}{\partial \mathbf{k}_1}\right) f_1 = Tr\hat{I}$$

$$\left(e\mathbf{E}\cdot\frac{\partial}{\partial \mathbf{k}_1}\right) \mathbf{g}_1 = Tr(\boldsymbol{\sigma}\hat{I})$$

Kinetic equation

(band representation)

$$f_{\alpha\beta}(\mathbf{k}) = \Psi^{\alpha\star}_{\sigma_1}(\mathbf{k}) n_{\sigma_1\sigma_2} \Psi^{\beta}_{\sigma_2}(\mathbf{k})$$

$$f^0_{\alpha\beta}(\mathbf{k}) = \Psi^{\alpha\star}_{\sigma_1}(\mathbf{k}) n^0_{\sigma_1\sigma_2} \Psi^{\beta}_{\sigma_2}(\mathbf{k}) = \begin{pmatrix} n_+ & 0 \\ 0 & n_- \end{pmatrix}_{\alpha\beta}$$

$$\frac{\partial \hat{g}}{\partial t} + e\mathbf{E}\frac{\partial \hat{n}^0}{\partial \mathbf{k}} - i[\hat{\varepsilon}, \hat{g}] = \hat{I}_{st} \qquad \hat{g} = \hat{n} - \hat{n}^0$$

$$-i\omega \begin{pmatrix} g_+ & g_\pm \\ g_\mp & g_- \end{pmatrix} + e \begin{pmatrix} (\mathbf{v}_+\mathbf{E})\frac{\partial n_+}{\partial \xi_+} & (\mathbf{v}_\pm\mathbf{E})(n_- - n_+) \\ (\mathbf{v}_\mp\mathbf{E})(n_+ - n_-) & (\mathbf{v}_-\mathbf{E})\frac{\partial n_-}{\partial \xi_-} \end{pmatrix} + \begin{pmatrix} 0 & ig_\pm(\varepsilon_- - \varepsilon_+) \\ ig_\mp(\varepsilon_+ - \varepsilon_-) & 0 \end{pmatrix} = I_{\alpha\beta}$$

$$\mathbf{v}_\alpha = \frac{\partial \varepsilon_\alpha}{\partial \mathbf{k}}, \qquad \mathbf{v}_\pm = \Psi^{+\star}_{\sigma}(\mathbf{k}) \frac{\partial \Psi^-_\sigma(\mathbf{k})}{\partial \mathbf{k}}$$

Electron - impurity collision integral

$$O_{\alpha\beta}(\mathbf{k}, \mathbf{k}') = \Psi^{\alpha\star}_{\sigma}(\mathbf{k}) \Psi^{\beta}_{\sigma}(\mathbf{k}')$$

$$I_{\alpha\beta}(\mathbf{k}) = 2\pi n_{imp} \int \frac{d^3k'}{(2\pi)^3} |V(\mathbf{k} - \mathbf{k}')|^2 \{O_{\alpha\nu}(\mathbf{k}, \mathbf{k}') [g_{\nu\mu}(\mathbf{k}') O_{\mu\beta}(\mathbf{k}', \mathbf{k}) - O_{\nu\mu}(\mathbf{k}', \mathbf{k}) g_{\mu\beta}(\mathbf{k})] \delta(\varepsilon'_\nu - \varepsilon_\beta)$$
$$+ [O_{\alpha\nu}(\mathbf{k}, \mathbf{k}') g_{\nu\mu}(\mathbf{k}') - g_{\alpha\nu}(\mathbf{k}) O_{\nu\mu}(\mathbf{k}, \mathbf{k}')] O_{\mu\beta}(\mathbf{k}', \mathbf{k}) \delta(\varepsilon'_\mu - \varepsilon_\alpha)\}$$

Solution

$$g_{\alpha\beta} = \begin{pmatrix} g_+ & g_\pm \\ g_\mp & g_- \end{pmatrix} = e \begin{pmatrix} (\mathbf{w}_+\mathbf{E}) & (\mathbf{w}_\pm\mathbf{E}) \\ (\mathbf{w}_\mp\mathbf{E}) & (\mathbf{w}_-\mathbf{E}) \end{pmatrix}$$

Electron-electron collision integral

$$\hat{I}(\mathbf{k}_1) = 2\pi \int d^3\mathbf{k}' \frac{d^3\mathbf{k}''}{(2\pi)^3} \frac{d^3\mathbf{k}_2}{(2\pi)^3} \sum_{\mathbf{m}} \delta\left(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}' - \mathbf{k}'' - \frac{2\pi \mathbf{m}}{a}\right) \hat{F}$$

$$\begin{aligned}
F_{\alpha\beta} = \frac{1}{2} W_1 \{ & [O_{\alpha\nu}(\mathbf{k}_1,\mathbf{k}') f_{\nu\mu}(\mathbf{k}') O_{\mu\lambda}(\mathbf{k}',\mathbf{k}_1)(\delta_{\lambda\beta} - f_{\lambda\beta}(\mathbf{k}_1))(\delta_{\xi\eta} - f_{\xi\eta}(\mathbf{k}_2)) O_{\eta\zeta}(\mathbf{k}_2,\mathbf{k}'') f_{\zeta\rho}(\mathbf{k}'') O_{\rho\xi}(\mathbf{k}'',\mathbf{k}_2) \\
& - O_{\alpha\nu}(\mathbf{k}_1,\mathbf{k}')(\delta_{\nu\mu} - f_{\nu\mu}(\mathbf{k}')) O_{\mu\lambda}(\mathbf{k}',\mathbf{k}_1) f_{\lambda\beta}(\mathbf{k}_1) f_{\xi\eta}(\mathbf{k}_2) O_{\eta\zeta}(\mathbf{k}_2,\mathbf{k}'')(\delta_{\zeta\rho} - f_{\zeta\rho}(\mathbf{k}'')) O_{\rho\xi}(\mathbf{k}'',\mathbf{k}_2)]\ \delta(\varepsilon'_\nu - \varepsilon_{1\beta} - \varepsilon_{2\xi} + \varepsilon''_\zeta) \\
& + [(\delta_{\alpha\nu} - f_{\alpha\nu}(\mathbf{k}_1)) O_{\nu\mu}(\mathbf{k}_1,\mathbf{k}')(\delta_{\mu\lambda} - f_{\mu\lambda}(\mathbf{k}')) O_{\lambda\beta}(\mathbf{k}',\mathbf{k}_1)(\delta_{\xi\eta} - f_{\xi\eta}(\mathbf{k}_2)) O_{\eta\zeta}(\mathbf{k}_2,\mathbf{k}'') f_{\zeta\rho}(\mathbf{k}'') O_{\rho\xi}(\mathbf{k}'',\mathbf{k}_2) \\
& - f_{\alpha\nu}(\mathbf{k}_1)) O_{\nu\mu}(\mathbf{k}_1,\mathbf{k}') f_{\mu\lambda}(\mathbf{k}') O_{\lambda\beta}(\mathbf{k}',\mathbf{k}_1) f_{\xi\eta}(\mathbf{k}_2) O_{\eta\zeta}(\mathbf{k}_2,\mathbf{k}'')(\delta_{\zeta\rho} - f_{\zeta\rho}(\mathbf{k}'')) O_{\rho\xi}(\mathbf{k}'',\mathbf{k}_2)]\ \delta(\varepsilon_{1\alpha} - \varepsilon'_\mu + \varepsilon_{2\xi} - \varepsilon''_\zeta) \} \\
+ \frac{1}{2} W_2 \{ & [O_{\alpha\nu}(\mathbf{k}_1,\mathbf{k}') f_{\nu\mu}(\mathbf{k}') O_{\mu\lambda}(\mathbf{k}',\mathbf{k}_2)(\delta_{\lambda\varphi} - f_{\lambda\varphi}(\mathbf{k}_2)) O_{\varphi\psi}(\mathbf{k}_2,\mathbf{k}'') f_{\psi\rho}(\mathbf{k}'')) O_{\rho\omega}(\mathbf{k}'',\mathbf{k}_1)(\delta_{\omega\beta} - f_{\omega\beta}(\mathbf{k}_1) \\
& - O_{\alpha\nu}(\mathbf{k}_1,\mathbf{k}')(\delta_{\nu\mu} - f_{\nu\mu}(\mathbf{k}')) O_{\mu\lambda}(\mathbf{k}',\mathbf{k}_2) f_{\lambda\varphi}(\mathbf{k}_2) O_{\varphi\psi}(\mathbf{k}_2,\mathbf{k}'')(\delta_{\psi\rho} - f_{\psi\rho}(\mathbf{k}'')) O_{\rho\omega}(\mathbf{k}'',\mathbf{k}_1) f_{\omega\beta}(\mathbf{k}_1]\ \delta(\varepsilon'_\nu - \varepsilon_{1\beta} - \varepsilon_{2\varphi} + \varepsilon''_\psi) \\
& + [(\delta_{\alpha\nu} - f_{\alpha\nu}(\mathbf{k}_1) O_{\nu\mu}(\mathbf{k}_1,\mathbf{k}') f_{\mu\lambda}(\mathbf{k}') O_{\lambda\varphi}(\mathbf{k}',\mathbf{k}_2)(\delta_{\varphi\psi} - f_{\varphi\psi}(\mathbf{k}_2)) O_{\psi\rho}(\mathbf{k}_2,\mathbf{k}'') f_{\rho\omega}(\mathbf{k}'')) O_{\omega\beta}(\mathbf{k}'',\mathbf{k}_1) \\
& - f_{\alpha\nu}(\mathbf{k}_1) O_{\nu\mu}(\mathbf{k}_1,\mathbf{k}')(\delta_{\mu\lambda} - f_{\mu\lambda}(\mathbf{k}')) O_{\lambda\varphi}(\mathbf{k}',\mathbf{k}_2) f_{\varphi\psi}(\mathbf{k}_2) O_{\psi\rho}(\mathbf{k}_2,\mathbf{k}'')(\delta_{\rho\omega} - f_{\rho\omega}(\mathbf{k}'')) O_{\omega\beta}(\mathbf{k}'',\mathbf{k}_1)]\ \delta(\varepsilon_{1\alpha} - \varepsilon'_\mu + \varepsilon_{2\psi} - \varepsilon''_\rho) \} .
\end{aligned}$$

$$f_{\alpha\beta}(\mathbf{k}) = \Psi^{\alpha\star}_{\sigma_1}(\mathbf{k}) n_{\sigma_1\sigma_2} \Psi^{\beta}_{\sigma_2}(\mathbf{k}).$$

$$O_{\alpha\beta}(\mathbf{k},\mathbf{k}') = \Psi^{\alpha\star}_{\sigma}(\mathbf{k}) \Psi^{\beta}_{\sigma}(\mathbf{k}')$$

Current

$$\mathbf{j} = e \int \frac{d^3k}{(2\pi)^3} \frac{\partial \varepsilon_{\sigma\sigma_1}(\mathbf{k})}{\partial \mathbf{k}} g_{\sigma_1\sigma}(\mathbf{k},\omega)$$

$$\mathbf{j} = e^2 \int \frac{d^3k}{(2\pi)^3} \{\mathbf{v}_+(\mathbf{w}_+\mathbf{E}) + \mathbf{v}_-(\mathbf{w}_-\mathbf{E}) + [\mathbf{v}_\pm(\mathbf{w}_\mp\mathbf{E}) - \mathbf{v}_\mp(\mathbf{w}_\pm\mathbf{E})]\,(\varepsilon_- - \varepsilon_+)\}$$

Ballistic regime $\omega > \tau^{-1}$

$$\mathbf{v}_\alpha = \frac{\partial \varepsilon_\alpha}{\partial \mathbf{k}}, \qquad \mathbf{v}_\pm = \Psi_\sigma^{+\star}(\mathbf{k}) \frac{\partial \Psi_\sigma^-(\mathbf{k})}{\partial \mathbf{k}}$$

$$g_+ = e(\mathbf{w}_+\mathbf{E}) = \frac{e}{i\omega}(\mathbf{v}_+\mathbf{E})\frac{\partial n_+}{\partial \xi_+},$$

$$g_- = e(\mathbf{w}_-\mathbf{E}) = \frac{e}{i\omega}(\mathbf{v}_-\mathbf{E})\frac{\partial n_-}{\partial \xi_-},$$

$$g_\pm = e(\mathbf{w}_\pm\mathbf{E}) = \frac{e(\mathbf{v}_\pm\mathbf{E})(n_- - n_+)}{i\omega - i(\varepsilon_- - \varepsilon_+)},$$

$$g_\mp = e(\mathbf{w}_\mp\mathbf{E}) = \frac{e(\mathbf{v}_\mp\mathbf{E})(n_+ - n_-)}{i\omega - i(\varepsilon_+ - \varepsilon_-)}.$$

$$\mathbf{j} = e^2 \int \frac{d^3k}{(2\pi)^3} \left\{ \frac{\mathbf{v}_+(\mathbf{v}_+\mathbf{E})}{i\omega}\frac{\partial n_+}{\partial \xi_+} + \frac{\mathbf{v}_-(\mathbf{v}_-\mathbf{E})}{i\omega}\frac{\partial n_-}{\partial \xi_-} \right.$$
$$+2\frac{(n_+ - n_-)(\varepsilon_- - \varepsilon_+)}{\omega^2 - (\varepsilon_+ - \varepsilon_-)^2}\left[i\omega \Re(\mathbf{v}_\pm(\mathbf{v}_\pm^\star\mathbf{E}))\right.$$
$$\left.\left. + (\varepsilon_- - \varepsilon_+)]\Im(\mathbf{v}_\pm(\mathbf{v}_\pm^\star\mathbf{E}))\right]\right\}$$

Dissipative current = 0 in 2D

≠ 0 in 3D

$\omega \ll \varepsilon_+ - \varepsilon_- \approx 2\gamma k_F$

$C_{4v} \quad \mathbf{j} \perp \mathbf{E}$

$$\mathbf{j} = e^2 \int \frac{d^3k}{(2\pi)^3} \left\{ \frac{\mathbf{v}_+(\mathbf{v}_+\mathbf{E})}{i\omega}\frac{\partial n_+}{\partial \xi_+} + \frac{\mathbf{v}_-(\mathbf{v}_-\mathbf{E})}{i\omega}\frac{\partial n_-}{\partial \xi_-} - 2(n_+ - n_-)\Im(\mathbf{v}_\pm(\mathbf{v}_\pm^\star\mathbf{E}))\right\}$$

Conductivity

$$\sigma = const \cdot \tau_{ee}$$

$$\tau_{ee} \propto \frac{1}{(2\pi T)^2 + (v_F \Delta k_F)^2}$$

$\gamma = 0$ $\qquad \sigma = \frac{4e^2 v_F^2 N_0}{3\pi^3 m^3 \tilde{W}_0} \frac{1}{T^2}$ Landau-Pomeranchuk $\qquad \frac{1}{\tau_{ee}} \sim \frac{V^2}{\varepsilon_F^2} \frac{T^2}{\varepsilon_F}$

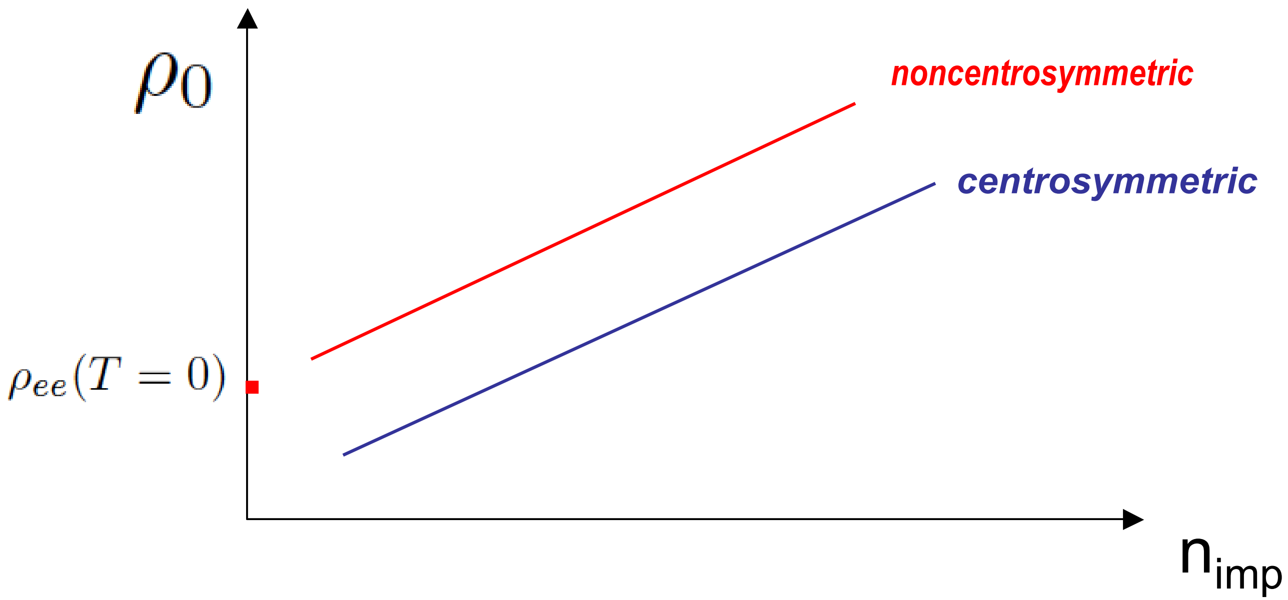
$\gamma \neq 0$ $\qquad \mu_+ - \mu_- = -2mv_F\gamma \gg T$ $\qquad \frac{1}{\tau_{ee}} \sim \frac{V^2}{\varepsilon_F^2} \frac{(\mu_+ - \mu_-)^2}{\varepsilon_F}$

How to measure

$$\rho = \rho_0 + AT^2 + BT^5$$

$$\rho_0 = \rho_{ee}(T=0) + \rho_{imp}$$

$$\rho_0 = \rho_{ee}(T=0) \qquad n_{imp} \to 0$$



***T**hermal resistivity*

$$\frac{T}{\kappa} = d_{imp} + d_{ee}(T=0) + f_{ee}T^2 + gT^3$$

Non Fermi-liquid ?

$$\frac{1}{\tau_{ee}} \sim \frac{V^2}{\varepsilon_F^2}\frac{(\mu_+ - \mu_-)^2}{\varepsilon_F} \ll \xi < |\mu_+ - \mu_-|$$

Conclusion

- *The kinetic properties in noncentrosymmetri metals are described by four coupled kinetic equations for diagonal (intraband) and off-diagonal (interband) matrix elements of distribution function.*
- *In contrast with space parity symmetric materials in 3D media without mirror symmetry the external forces in collision-less regime not only accelerate electrons but create specific currents directed perpendicular to external electric field (examle - tetragonal crystals with symmetry C_{4v}).*
- *Along with scattering on impurities the electron-electron scatterings in a medium without inversion symmetry are also responsible for the finite zero-temperature residual resistivity and residual thermal resistivity.*

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