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EXCITON BOSE-EINSTEIN CONDENSATION IN 2D HETEROSTRUCTURES WITH DISORDER

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MOTIVATION

3D EXCITON BE CONDENSATION

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2D EXCITON BE CONDENSATION

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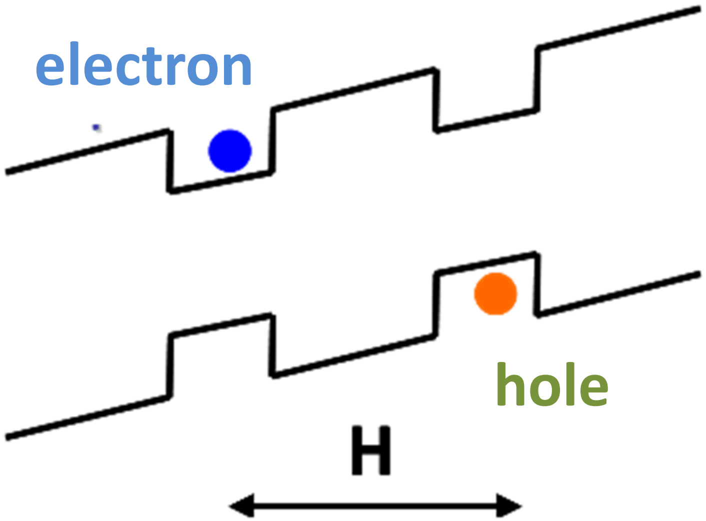
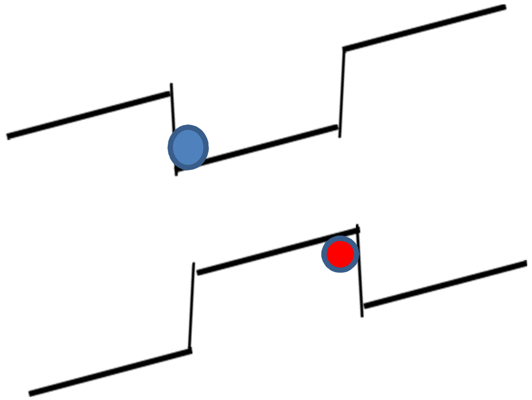
NEW HOPE -
SEMICONDUCTOR HETEROSTRUCTURES



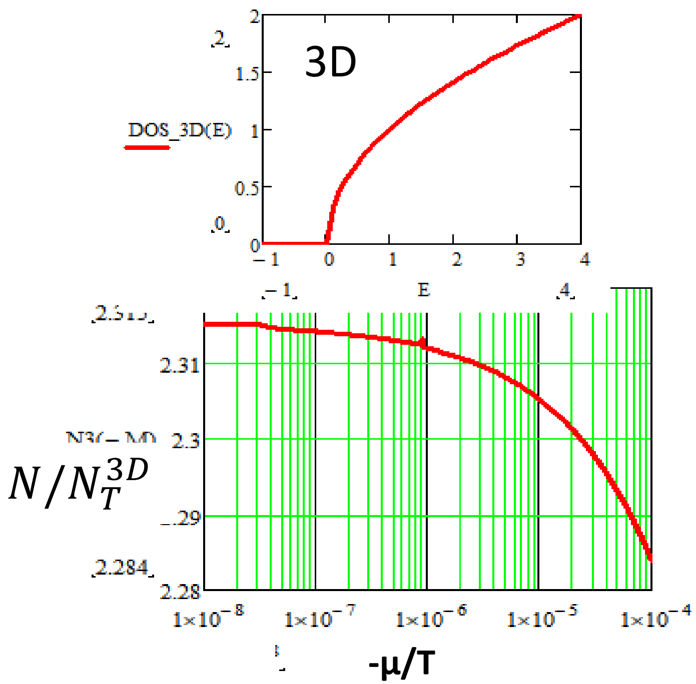


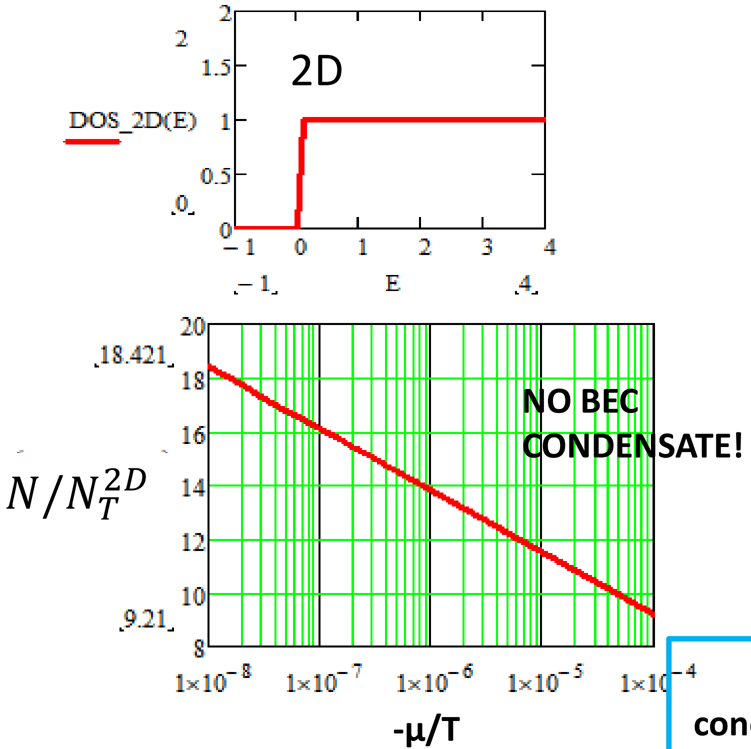
DIPOLAR EXCITON

WHAT IS BOSE-EINSTEIN CONDENSATION ? *(REMINDER)*

CONCENTRATION

$$N\left(\frac{\mu}{T}\right) = \int dE \frac{DOS(E)}{e^{\frac{E-\mu}{T}} - 1}$$





BEC concentration

BEC TEMPERATURE $T_C \approx 3.315 \frac{\hbar^2 N^{\frac{2}{3}}}{m}$, $T < T_C$ $N = \varsigma\left({}^3/_2\right)\left(\frac{mT}{2\pi\hbar^2}\right)^{\frac{3}{2}} + N_c$

THE CONDENSATE WITH PARTICLES FILLING A STATE WITH MOMENTUM *P = 0* IS *MACROSCOPICALLY COHERENT STATE* !

PROBLEMS & CHALLENGES

❑ There is no Bose-Einstein condensation in an ideal 2D system. Does a disorder make this statement wrong?

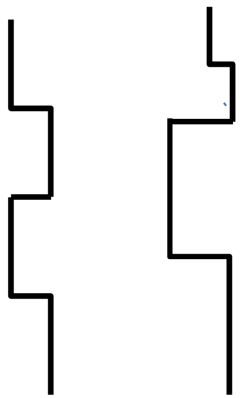
❑ Does the Bose-Einstein condensation take place if the boson life time is finite?

❑ What is the role of the boson-boson repulsion in forming of the system state?

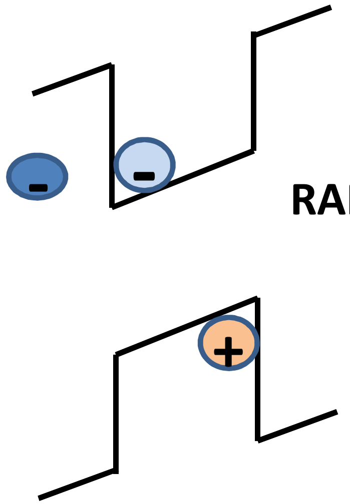
❑ What is the role of pumping intensity fluctuations if we deal with the pumped exciton system?

❑ How does a local perturbation of condensate density propagate in space?

DISORDER ORIGINS & DENSITY OF STATES



QW WIDTH FLUCTUATIONS



RANDOMLY DISTRIBUTED CHARGES IN BARRIER LAYERS

We consider the situation when the energy spectrum is bounded from below.

We set the bottom boundary of spectrum *E=0*

$$\mathrm{DOS}(E) = \begin{cases} \dfrac{N_p E_\nu}{E^2} \exp\left(-\dfrac{E_\nu}{E}\right), & E > 0, \\ 0, & E < 0. \end{cases}$$

I. M. LIFSHITZ, THEORY OF FLUCTUATION LEVELS IN DISORDERED SYSTEMS, Zh. Eksp. Teor. Fiz. 53, 743-758 (August, 1967)

$$E_\nu = \pi N_p / m$$

In the model of lateral potential wells, N_p is the well concentration

BOSE-EINSTEIN CONDENSATION IN 2D

EQUILIBRIUM DISTRIBUTION FUNCTION

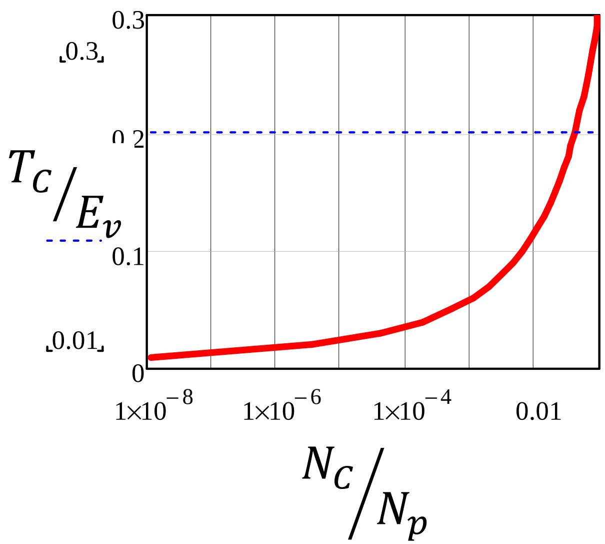
$$f(E;T,\mu) = \frac{1}{\exp\left(\frac{E-\mu}{T}\right) - 1}$$

BEC OCCURES AT ZERO CHEMICAL POTENTIAL

$\mu = 0$

THE INTERRELATION OF THE CRITICAL TEMPERATURE, T_c, AND CRITICAL CONCENTRATION, N_c, IS GIVEN BY THE EQUATION:

$$N_c = \int_0^\infty f(E;T_c,0)\, DOS(E)dE = \int_0^\infty \frac{N_p e^{-x} dx}{\exp\left(E_\nu / xT_c\right) - 1}$$

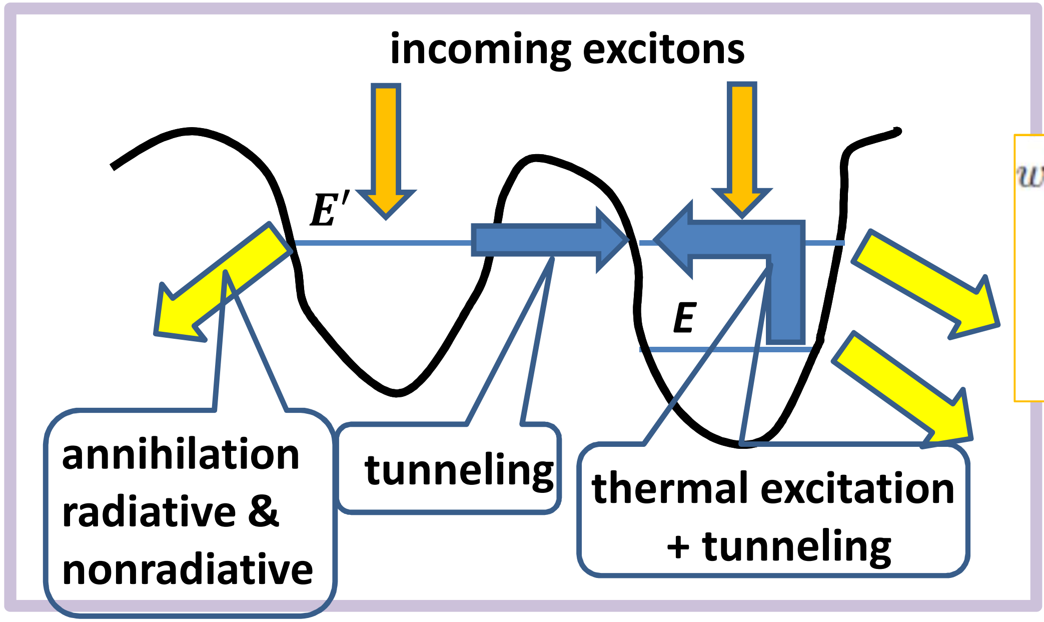


$$\frac{T_c}{T_d} \sim \frac{N_p}{N_c} \frac{1}{\ln^2\left(\sqrt{\pi}\frac{N_p}{N_c}\right)} \gg 1 \qquad T_d \sim N_c/m$$

In a disordered 2D system,
the BEC critical temperature T_C profoundly exceeds the temperature T_d corresponding to the de Broglie wave length close to the average distance between particles!
This is due to filling in of deep fluctuation potential wells.

DOES A CONDENSATE ARISE IF THE PARTICLES HAVE A FINITE LIFETIME?

MAIN PROCESSES



Probability of hopping between levels *E* and E' of 2 wells on distance r per unit of time ($\varkappa$ is inverse tunnel length):

$$w(E \to E', r) = w_0 \exp\left(-2\varkappa r\right) \times \times \begin{cases} 1, & E' \leqslant E, \\ \exp\left(\dfrac{E-E'}{T}\right), & E' > E. \end{cases}$$

We call the wells as TRAPS, if the time of annihilation of particles in them, τ_0, is less than the time of their hopping into neighboring wells

THE ENERGY DISTRIBUTION FUNCTION OF EXCITONS IN TRAPS (*G* is generation rate):

$$f_{tr}(E) = G\tau_0 P_{tr}(E).$$

$$P_{tr}(E) = exp\left(-\int_{w(E\to E',r)\tau_0>1} dE'dr2\pi rDOS(E')\right)$$

THE PROBABILITY OF A WELL BEING A TRAP

CONDITIONS OF BEC EXISTENCE

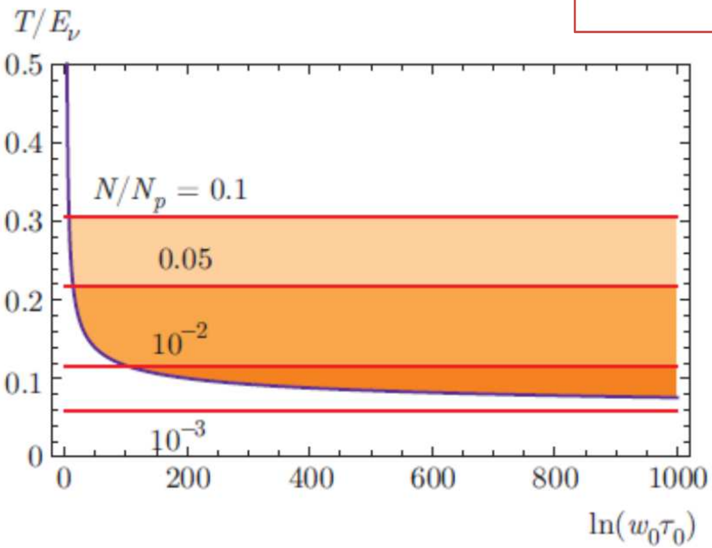
THRESHOLD ENERGY LEVEL, $\boldsymbol{E_C}$: THE WELLS WITH ENERGY $\boldsymbol{E} > \boldsymbol{E_C}$ ARE NOT THE TRAPS

$$P_{tr}(\boldsymbol{E_C}) = exp\left(-\int_{w(\boldsymbol{E_C}\to E',r)\boldsymbol{\tau_0}>1} dE'dr2\pi rDOS(E')\right) < 1$$

For long enough lifetimes where the exciton hopping between the wells is effective, i.e. $\frac{\pi N_p}{4\varkappa^2}\ln^2(w_0\tau_0) \gg 1$

$$E_c = \frac{E_\nu}{\ln\left[\frac{\pi N_p}{4\varkappa^2}\ln^2(w_0\tau_0)\right]}.$$

condencation is feasible under the condition $\boldsymbol{T} > \boldsymbol{E_C}$



State diagram

Horizontal lines are BEC temperature at $\tau_0 = \infty$ for the few values of N/N$_P$. The solid line is E_C at $\varkappa^{-2}N_p = 0.3$. The shadow areas are the regions of the BEC existence, $E_c < T < T_c(N)$

THE ROLE OF THE BOSON-BOSON REPULSION

In a lateral potential well of 2D system, the particle energy shift due to pair collisions in low density approximation is

$$\delta E \equiv \mathcal{E}_R - E_R = \mathcal{T} N, \quad \mathcal{T} = \frac{2\pi}{m} \frac{1}{\ln(E_0/\mathcal{E}_R)}.$$

$\mathcal{T}$ - the scattering amplitude

$E_R = 1/mR^2$

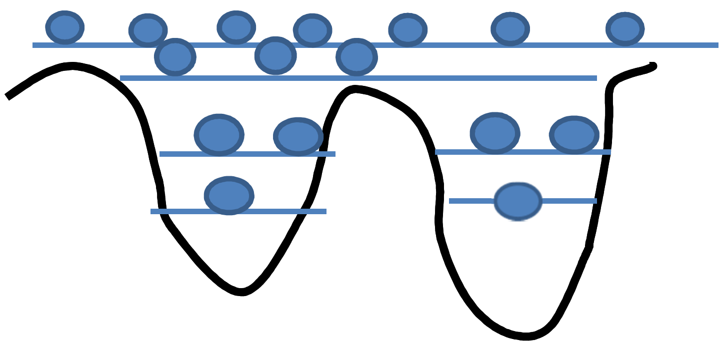
$E_0 \sim (1/ma^2)\exp(2\pi\Gamma/m)$

a – characteristic radius of the interaction potential

Γ - the bare interaction constant

With logarithmic accuracy

$$\mathcal{E}_R = \frac{2\pi N}{m} \frac{1}{\ln(mE_0/2\pi N)}.$$



Repulsion of particles results in a limitation of their number in each of the wells and screening of potential fluctuations

ELEMENTARY EXCITATIONS OF BEC

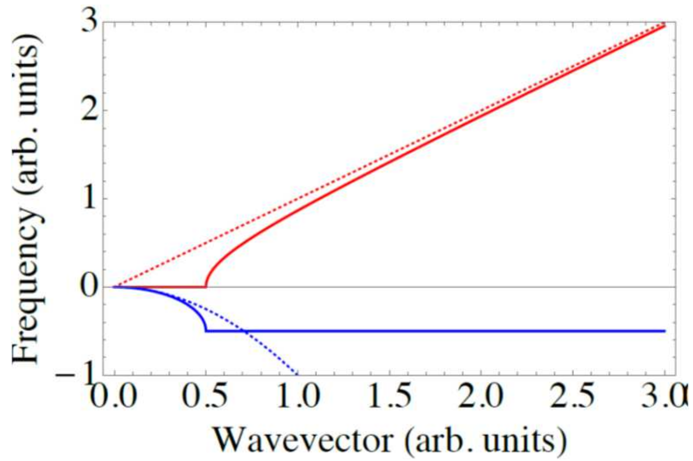
Elementary excitation spectrum and the correlation function might be obtained using ideal liquid hydrodynamic equations* supplemented with the terms describing particle generation and annihilation

*An introduction to the theory of superfluidity by **I.M. Khalatnikov ;** Cambridge, Mass., 2000

$$\frac{\partial \delta n}{\partial t} + \nabla \cdot (N\mathbf{v}) + \frac{\delta n}{\tau_0} = \delta g,$$

$$\frac{\partial \mathbf{v}}{\partial t} + \frac{1}{m}\nabla \delta p = 0,$$

$$\boxed{\omega\left(\omega + \frac{i}{\tau_0}\right) = \frac{TN}{m}k^2}$$



$$\omega = sk, \quad k \gg {}^{1}/_{s\tau_0} \quad s = \sqrt{TN/m}$$ Sound velocity

$$\omega = -iDk^2 \quad k \ll {}^{1}/_{s\tau_0} \quad D = s^2\tau_0$$ Diffusion coefficient

CORRELATION FUNCTION OF BEC

$$\varrho(\mathbf{r}_1,\mathbf{r}_2) = \langle\psi^\dagger(\mathbf{r}_1)\psi(\mathbf{r}_2)\rangle = N\exp\left[-\Phi(\mathbf{r}_1-\mathbf{r}_2,0)/2\right]$$

The damping of the correlation is due to the particle generation fluctuations

$$\langle g_{\mathbf{k},\omega}g_{\mathbf{k}',\omega'}\rangle = g_0\delta_{\mathbf{k},-\mathbf{k}'}\delta(\omega+\omega')$$

At $r = |\boldsymbol{r_1} - \boldsymbol{r_2}| \to \infty$

3D GAS

$$\Phi^{(3D)}(\mathbf{r},t) = \frac{\mathcal{T}m}{8\pi r}\operatorname{erf}\left(\frac{r}{2\sqrt{Dt}}\right) \to 0$$

LONG-RANGE ORDER

$\varrho(\mathbf{r}_1,\mathbf{r}_2) \neq 0$

2D GAS

$$\tilde{\Phi}(\mathbf{r},t) = -\frac{\mathcal{T}m}{8\pi}\left(4\gamma + \ln\frac{Dt}{r^2}\right)$$

$\varrho(\mathbf{r}_1,\mathbf{r}_2) \to 0$ as $(Dt/r^2)^{\frac{\mathcal{T}m}{8\pi}}$

NO LONG-RANGE ORDER

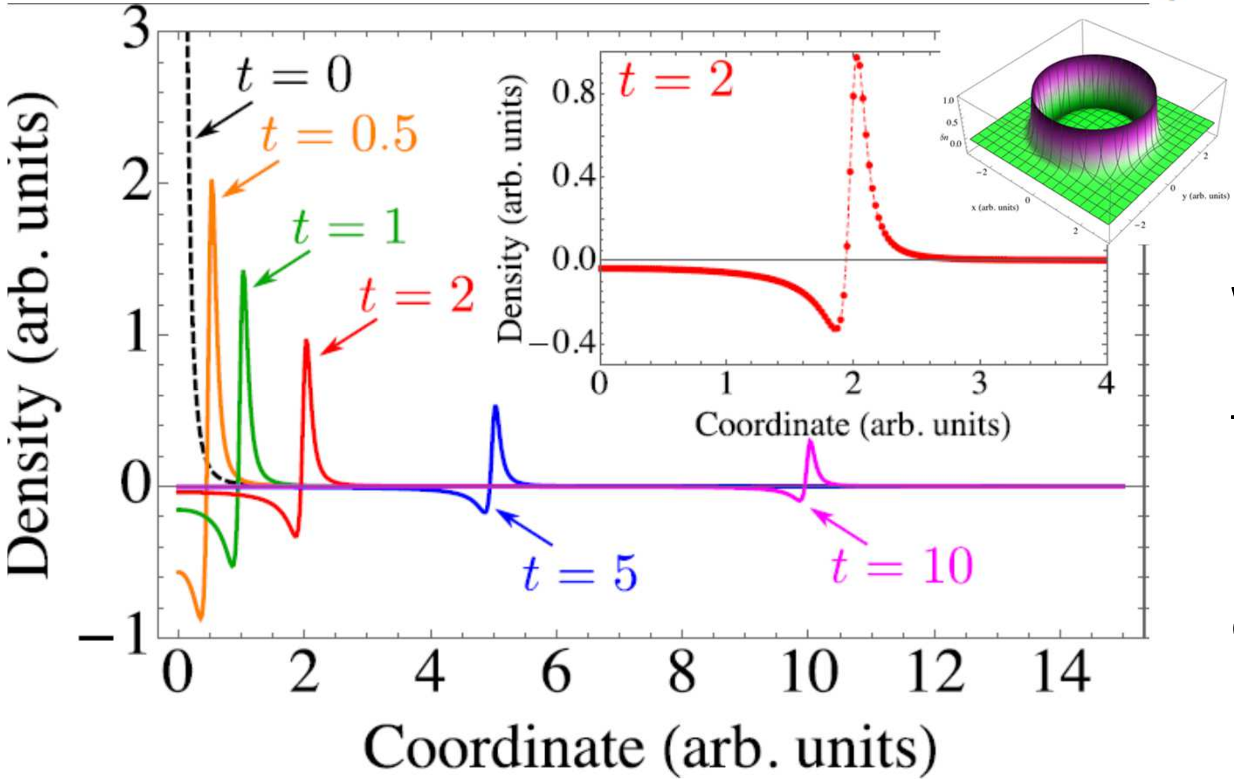
Naturally, thermal fluctuations also result in destruction of the long-range order

PROPAGATION OF A LOCAL PERTURBATION OF BEC DENSITY IN SPACE

$$\delta n(r,t) = \delta g \exp\left(-\frac{t}{2\tau_0}\right) \int \frac{dk\, k}{2\pi} J_0(kr) \exp(-kr_0) \left[\cos(\Omega_k t) - \frac{\sin(\Omega_k t)}{2\tau_0 \Omega_k}\right]$$

Simple approximation neglecting diffusion

$$\delta n(r,t) = \frac{\delta g_0}{2\pi} \exp\left(-\frac{t}{2\tau_0}\right) \mathrm{Re}\left\{\frac{r_0 - ist}{[r^2 + (r_0 - ist)^2]^{3/2}}\right\}$$



Development of the concentration perturbation profile in 2D system

We use the system of units: s = 1, τ_0= 10, k_0= 10.

The profiles calculated with the “exact” equation (dot line) and the analytical equation (solid line) at t = 2 are presented in the insertion.

CONCLUSIONS

- ❑ **In contrast to ideal 2D system, in the disordered system Bose-Einstein condensation is feasible**

- ❑ **The finite life time of bosons results in emergence of low-temperature boundary of the BEC region on the phase diagram**

- ❑ **The boson-boson repulsion results in a limitation of their number in each of the the lateral potential minima and in screening of the potential fluctuations**

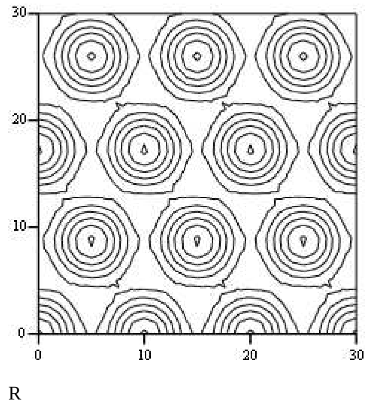
- ❑ **The pumping intensity fluctuations as well as thermal fluctuations destruct the long-range order and the correlation function falls off with distance r as 1/ $\boldsymbol{r}^{\alpha}$**

- ❑ **A local perturbation of condensate density propagate with the condensate sound velocity and its amplitude decays exponentially with time due to the finit particle life time**

M.M. Glazov, R.A. Suris, Exciton Condensation in a Two-Dimensional System with Disorder JETP, 2018, Vol. 126, No. 6, pp. 833–841.
M.M. Glazov, R.A. Suris “Collective states of excitons in semiconductors”
DOI: 10.3367/UFNe.2019.10.038663

2D CRYSTAL vs BE CONDENSATION

R. A. Suris, Gas–crystal phase transition in a 2D dipolar exciton system, 2016 JETP 122(3):602-607, DOI: 10.1134/S1063776116030110



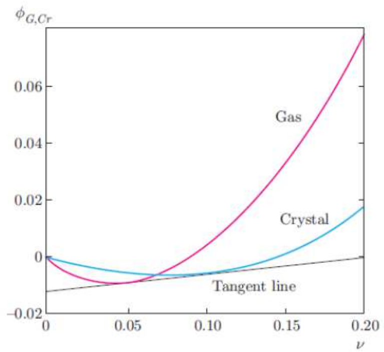


Рис. 1. Иллюстрация построения кривых фазового равновесия

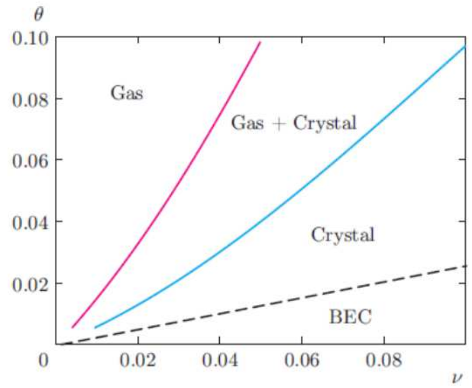
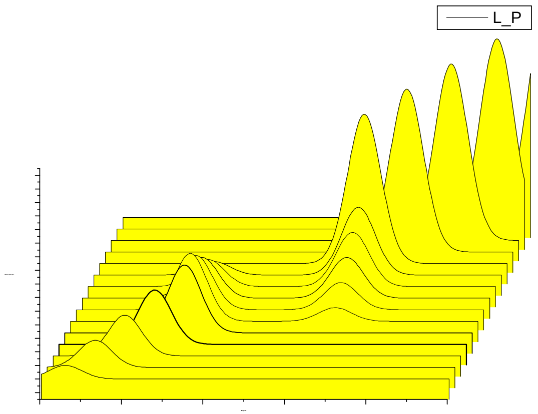


Рис. 2. Фазовая диаграмма «газ–кристалл» диполярных экситонов

$\nu = \mathbf{n}\ \mathbf{H}^2$

$\Theta = \mathbf{T}\ \varepsilon\ H/e^2$



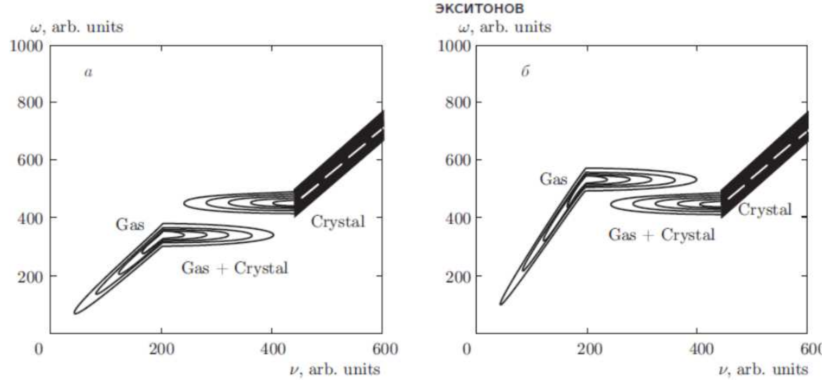


Рис. 3. Изменение спектра люминесценции в зависимости от концентрации для двух различных безразмерных температур: $a - \theta = 0.02$; $б - \theta = 0.08$