Russian Academy of Sciences

Landau Institute for Theoretical Physics

Mesoscopic Stoner instability in open quantum dots:

suppression of Coleman-Weinberg mechanism by electron tunneling

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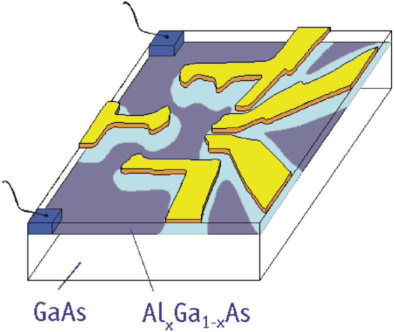
International conference dedicated to the 100th anniversary of I. M. Khalatnikov

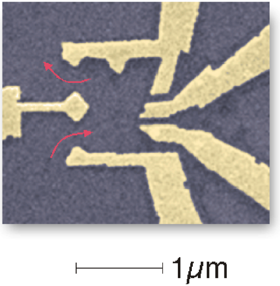
"Quantum Fluids, Quantum Field Theory, and Gravity"

Introduction: quantum dots

electrons are confined in all three dimensions, e.g.

- regions of confinement for 2D electrons of size $\sim 1\,\mu$m





[adopted from Kouwenhoven, Marcus (1998)]

- metallic grains of Al, Au, Pd of size $1 \div 10$ nm
- large molecules
- short nanowires

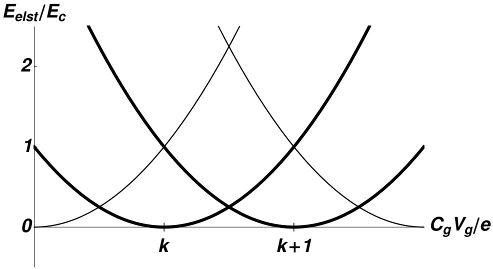
- charging energy of single electron

$$E_c = \frac{e^2}{2C} \approx 10\,\mathrm{K} \qquad \text{for} \qquad C = L = 1\,\mu m$$

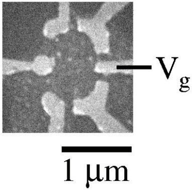
- from classical electrostatics to quantum mechanics

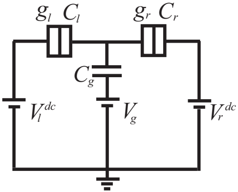
$$\frac{(Q - C_g V_g)^2}{2C} \qquad \Longrightarrow \qquad \widehat{H} = E_c(\hat{n} - N_0)^2,$$

$$\hat{n}|k\rangle = k|k\rangle, \qquad N_0 = C_g V_g / e$$

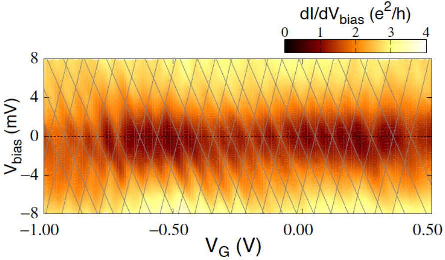


[Kulik, Shekhter (1975)]





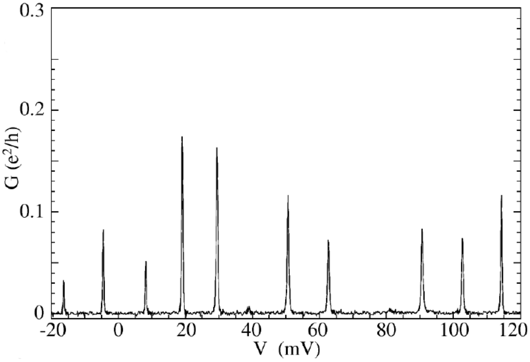
- Kirchhoff's laws and energy conservation ⇒ Coulomb diamonds



[adopted from Andresen et al. (2008)]

- differential conductance dI/dV as a function of bias and gate voltages

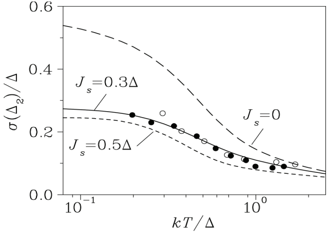
- gate voltage dependence of conductance



[adopted from Lüsher et al. (2001)]

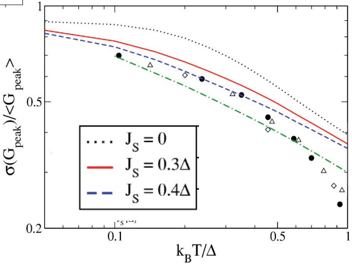
  - temperature $T = 120$ mK
  - single-particle mean level spacing $\Delta = 2.3$ K
  - charging energy $E_c = 14.5$ K
- no periodicity in gate voltage (peak heights and spacings are different) due to
  - randomness of single-particle levels
  - exchange interaction

Variation of conductance-peak spacing



Alhassid, Rupp (2003)

Variation of conductance-peak height



Usaj, Baranger (2003)

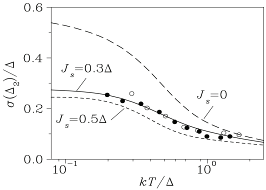
- data points from Patel et al.
- theory is applicable at relatively low temperatures $T \leqslant \Delta$
- typical exchange interaction is $J \sim \Delta/2$

- electrons with Heisenberg exchange interaction $H = H_0 + H_J$

$$H_0 = \sum_{\alpha,\sigma} \epsilon_\alpha a^\dagger_{\alpha,\sigma} a_{\alpha,\sigma}$$

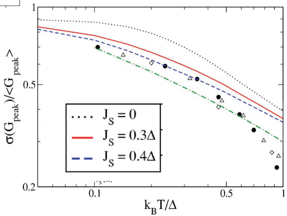
$$H_J = -J\hat{\boldsymbol{S}}^2$$

Variation of conductance-peak spacing



Alhassid, Rupp (2003)

Variation of conductance-peak height

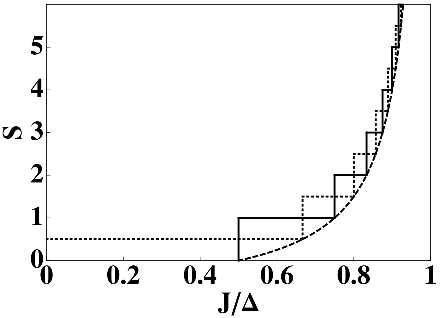


Usaj, Baranger (2003)

- equidistant spectrum is assumed: $\epsilon_{\alpha+1} - \epsilon_\alpha = \Delta$
- if the energy diff. between the GSs with the total spin S and $S+1$

$$E_{S+1} - E_S = (2S+1)\Delta - J(2S+2) < 0,$$

the GS has nonzero total spin S



$$S = \frac{2J - \Delta}{2(\Delta - J)} \qquad \text{at} \quad \Delta - J \ll J$$

in thermodynamic limit $N \to \infty$,
spin per electron

$$S/N \to \begin{cases} 0, & J < \Delta \\ 1/2, & \Delta \leqslant J \end{cases}$$

- large energy scale: renormalized exchange interaction

$$J_* = 2JS = \frac{J\Delta}{\Delta - J} \gg J \qquad \text{at} \qquad \Delta - J \ll \Delta$$

- Curie law for spin susceptibility

$$\chi = \frac{1}{3}\frac{S(S+1)}{T} = \frac{1}{12T}\left(\frac{J_*}{J}\right)^2, \qquad T \ll J_*$$

[Kurland, Aleiner, Altshuler (2000)]

how the mesoscopic Stoner instability and the spin susceptibility
are affected by tunneling of electrons to/from a reservoir?

- universal Hamiltonian for an isolated metallic QD

[Kurland, Aleiner, Altshuler (2000)]

$$H_{\rm qd} = \sum_{\alpha,\sigma} \varepsilon_\alpha a^\dagger_{\alpha,\sigma} a_{\alpha,\sigma} + E_c \left(\sum_{\alpha,\sigma} a^\dagger_{\alpha,\sigma} a_{\alpha,\sigma} - N_0 \right)^2 - \frac{J}{4} \left(\sum_{\alpha,\sigma} a^\dagger_{\alpha,\sigma} \boldsymbol{\sigma}_{\sigma\sigma'} a_{\alpha,\sigma'} \right)^2$$

- the tunneling Hamiltonian $H_{\rm tun} = \sum_{k,\alpha,\sigma} t_{k,\alpha} d^\dagger_{k,\sigma} a_{\alpha,\sigma} + \text{h.c.}$

- electrons in the reservoir $H_{\rm res} = \sum_{k,\sigma} \epsilon_k d^\dagger_{k,\sigma} d_{k,\sigma}$

- tunneling conductance $g = 4\pi^2 \sum_{k,\alpha} |t_{k,\alpha}|^2 \delta(\epsilon_k) \delta(\varepsilon_\alpha)$

- assumption: $g \gg 1$ (allows us to neglect the charging energy)
- assumption: $0 < \Delta - J \ll \Delta$ (deep in the mesoStoner phase)

- Hubbard-Stratonovich transformation with the vector field $\mathbf{\Phi}(\tau)$:

$$\exp\left[\frac{J}{4}\int_0^\beta d\tau \left(\sum_{\alpha,\sigma} a^\dagger_{\alpha,\sigma}\boldsymbol{\sigma}_{\sigma\sigma'}a_{\alpha,\sigma'}\right)^2\right]$$

$$\to \exp\left[-\int_0^\beta d\tau \frac{\mathbf{\Phi}^2}{4J} - \frac{1}{2}\int_0^\beta d\tau\ \mathbf{\Phi}(\tau)\sum_{\alpha,\sigma,\sigma'} a^\dagger_{\alpha,\sigma}\boldsymbol{\sigma}_{\sigma\sigma'}a_{\alpha,\sigma'}\right]$$

- effective action in the imaginary time ($\hat{\epsilon}_{\alpha\alpha'} = \epsilon_\alpha\delta_{\alpha\alpha'}$):

$$S = \frac{1}{4J}\int_0^\beta d\tau\ \mathbf{\Phi}^2 - \mathrm{Tr}\ln\left(-\partial_\tau - \hat{\epsilon} + \frac{1}{2}\boldsymbol{\sigma}\mathbf{\Phi} - \hat{\Sigma}\right)$$

- self-energy due to tunneling to the reservoir ($\varepsilon_n = \pi T(2n+1)$):

$$\hat{\Sigma}_{\alpha\alpha'} = \sum_k t^*_{\alpha k}(-\partial_\tau - \epsilon_k)^{-1}t_{k\alpha'} \quad \to \quad \hat{\Sigma}_{\alpha\alpha'}(i\varepsilon_n) = -\frac{i\gamma}{\pi}\,\mathrm{sgn}\,\varepsilon_n\delta_{\alpha\alpha'}$$

NB $\gamma = g\Delta/4$

Effective action: integrating out fermions

- effective action in the imaginary time ($\hat{\epsilon}_{\alpha\alpha'} = \epsilon_\alpha \delta_{\alpha\alpha'}$):

$$S = \frac{1}{4J} \int_0^\beta d\tau \; \mathbf{\Phi}^2 - \mathrm{Tr}\ln\Big(-\partial_\tau - \hat{\epsilon} + \frac{1}{2}\boldsymbol{\sigma}\mathbf{\Phi} - \hat{\Sigma}\Big)$$

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- spin susceptibility

$$\chi = \frac{T}{12J^2}\Big\langle\Big|\int_0^\beta d\tau\, \mathbf{\Phi}(\tau)\Big|^2\Big\rangle - \frac{1}{2J},$$

NB $\gamma = g\Delta/4$

- how to compute $\mathrm{Tr}\ln\Big(-\partial_\tau - \hat{\epsilon} + \frac{1}{2}\boldsymbol{\sigma}\boldsymbol{\Phi}\Big)$?

- gauge transformation

$$a_{\alpha,\sigma} \to U_{\sigma,\sigma'} a_{\alpha,\sigma'}, \qquad U(\tau) = \mathcal{T}\exp\left(\frac{1}{2}\int_0^\beta d\tau\, \boldsymbol{\sigma}\boldsymbol{\Phi}(\tau)\right)$$

NB $U(\beta) \neq U(0)$, i.e. rotated Grassmanian fields are not fermions

- explicit expression for the determinant

$$\exp\mathrm{Tr}\ln\Big(-\partial_\tau - \hat{\epsilon} + \frac{1}{2}\boldsymbol{\sigma}\boldsymbol{\Phi}\Big) = \prod_\alpha \det\Big[1 + U(\beta)e^{-\beta\epsilon_\alpha}\Big]$$

- gauge transformation

$$a_{\alpha,\sigma} \to U_{\sigma,\sigma'} a_{\alpha,\sigma'}, \qquad U(\tau) = \mathcal{T} \exp\left(\frac{1}{2} \int_0^\beta d\tau \, \boldsymbol{\sigma}\boldsymbol{\Phi}(\tau) \right)$$

- Wei-Norman representation of time-ordered $SU(2)$ exponent

$$U(\tau) = e^{\frac{1}{2}\int_0^\tau d\tau' \rho(\tau')} \begin{pmatrix} 1 & \int_0^\tau d\tau' \tilde{\kappa}(\tau') e^{-\int_0^{\tau'} dt \rho(t)} \\ \kappa(\tau) & e^{-\int_0^\tau d\tau' \rho(\tau')} + \kappa(\beta) \int_0^\tau d\tau' \tilde{\kappa}(\tau') e^{-\int_0^{\tau'} dt \rho(t)} \end{pmatrix}$$

where

$$\frac{\Phi_x - i\Phi_y}{2} = \tilde{\kappa}, \quad \frac{\Phi_x + i\Phi_y}{2} = \partial_\tau \kappa + \rho\kappa - \kappa^2 \tilde{\kappa}, \quad \Phi_z = \rho - 2\kappa\tilde{\kappa}$$

- Kolokolov's trick: $\displaystyle\int D\boldsymbol{\Phi} = \int D\rho D\tilde{\kappa} D\kappa \, \exp\Big(\int_0^\beta d\tau \rho/2 \Big)$

[Kolokolov (1986)]

Method: separation of slow and fast variables

- splitting $\rho(\tau) = 2h + \delta\rho(\tau)$, $\qquad \int\limits_0^\beta d\tau\,\delta\rho(\tau) = 0$

- integration over $\delta\rho$, κ, and $\tilde{\kappa} = \kappa^*$ within Gaussian approximation

- effective free energy (Coleman-Weinberg potential):

$$F(h) = \frac{h^2}{J_*} - h + 2T\,\mathrm{Re}\ln\frac{\Gamma\left(1 + \frac{ih}{\pi T} + \frac{\gamma}{\pi^2 T}\right)}{\Gamma\left(1 + \frac{ih}{\pi T}\right)\Gamma\left(1 + \frac{\gamma}{\pi^2 T}\right)}$$

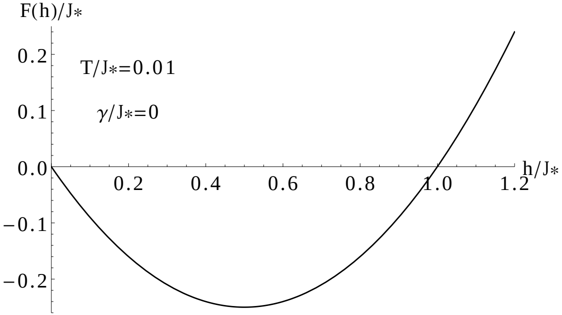
- spin susceptibility

$$\chi \approx \frac{1}{3TJ^2}\int\limits_{-\infty}^{\infty} dh\,h^2 e^{-\beta F(h)} \Big/ \int\limits_{-\infty}^{\infty} dh\,e^{-\beta F(h)}$$

NB Gaussian approximation is justified at

$$|h|, T \gg \max\left\{J, \min\{J_*, \sqrt{J\gamma}\}\right\}$$

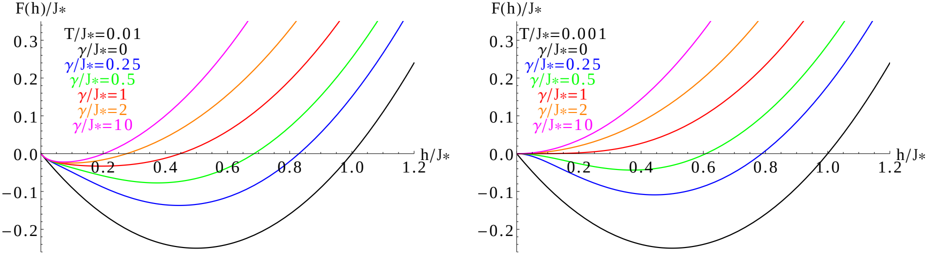
- effective free energy at $\gamma = 0$: $F_0(h) = \frac{h^2}{J_*} - h$
- spin susceptibility: $\chi \propto \langle h^2 \rangle / (T J^2)$
- Curie law at $T \ll J_*$: $\chi \sim J_*^2/(T J^2)$
- Pauli-type susceptibility at $T \gg J_*$: $\chi \sim J_*/J^2$



mesoscopic Stoner in isolated QD is a symmetry breaking phenomenon

- effective free energy (Coleman-Weinberg potential):

$$F(h) = \frac{h^2}{J_*} - h + 2T \,\mathrm{Re}\ln \frac{\Gamma\left(1 + \frac{ih}{\pi T} + \frac{\gamma}{\pi^2 T}\right)}{\Gamma\left(1 + \frac{ih}{\pi T}\right)\Gamma\left(1 + \frac{\gamma}{\pi^2 T}\right)}$$



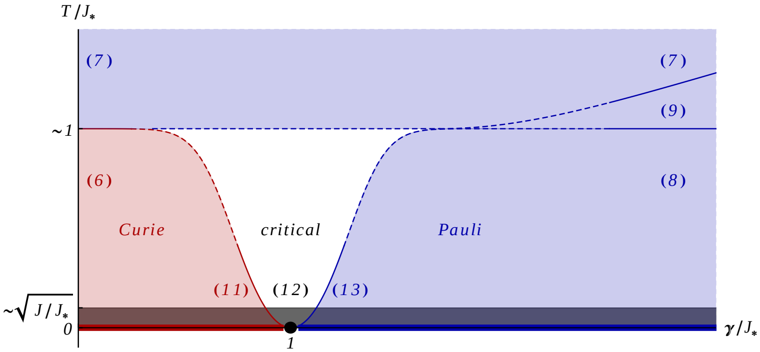
the minimum disappears at $\gamma \sim J_*$ for $T \to 0$

- Curie law at $T \ll J_*$, region (6)

$$\chi \sim (J_*/J)^2 \left[1 - 8\gamma/(\pi^2 J_*)\right]/T$$

- Pauli-type susceptibility at $T \gg J_*$, region (7)

$$\chi \sim J_* \left[1 + \psi''(1)\gamma J_*/(\pi^4 T^2)\right]/J^2$$



Results: strong tunneling regime, $\gamma \gg J_*$

- Pauli-type susceptibility at $T \ll J_*$, region (8)

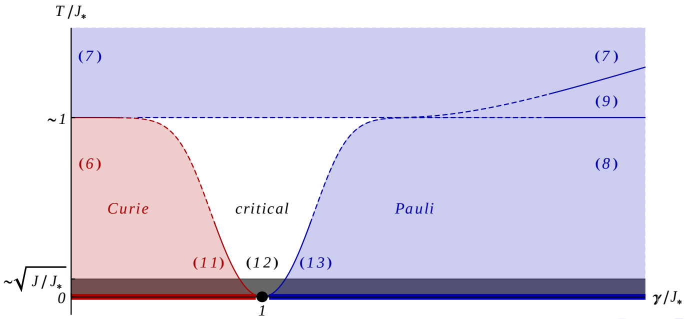
$$\chi \sim J_*\big(1 + J_*/\gamma\big)/J^2.$$

- Pauli-type susceptibility at $J_* \ll T \ll \sqrt{J_*\gamma}$, region (9)

$$\chi \sim J_*\big[1 - J_*/(6T)\big]/J^2$$

- Pauli-type susceptibility at $T \gg J_*$, region (7)

$$\chi \sim J_*\big[1 + \psi''(1)\gamma J_*/(\pi^4 T^2)\big]/J^2$$



- the free energy functional at $\sqrt{JJ_*} \ll T \ll h \ll \gamma_c \sim J_*$

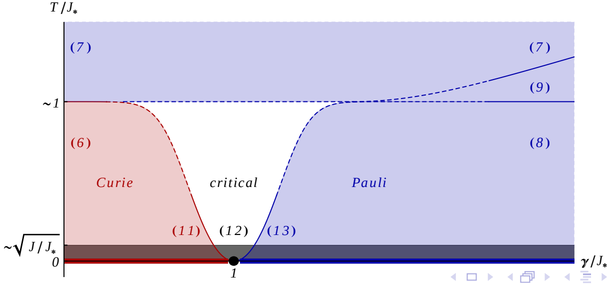
$$F(h) = (1/J_* - 1/\gamma)h^2 + \pi^2 h^4/(6\gamma_c^3)$$

- the scaling form of the spin susceptibility

$$\chi = \sqrt{J_*^3/T} f(T_X/T)/J^2, \qquad T_X = J_*\alpha^2, \quad \alpha = \gamma_c/\gamma - 1$$

- ordered phase, $\gamma < \gamma_c$ at $T \ll T_X$, region (11):

$$\chi \sim J_*^2\alpha/(TJ^2)$$



- the free energy functional at $\sqrt{JJ_*} \ll T \ll h \ll \gamma_c \sim J_*$

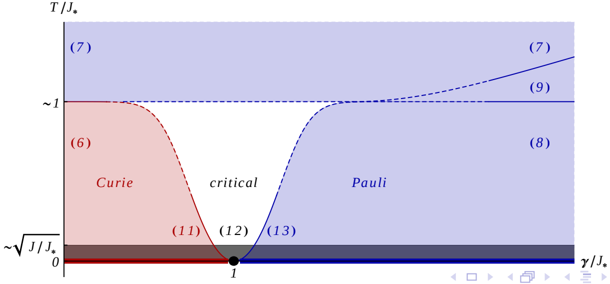
$$F(h) = (1/J_* - 1/\gamma)h^2 + \pi^2 h^4/(6\gamma_c^3)$$

- the scaling form of the spin susceptibility

$$\chi = \sqrt{J_*^3/T} f(T_X/T)/J^2, \qquad T_X = J_*\alpha^2, \quad \alpha = \gamma_c/\gamma - 1$$

- disordered phase, $\gamma > \gamma_c$ at $T \ll T_X$, region (13):

$$\chi \sim J_*/(J^2|\alpha|)$$



- the free energy functional at $\sqrt{JJ_*} \ll T \ll h \ll \gamma_c \sim J_*$

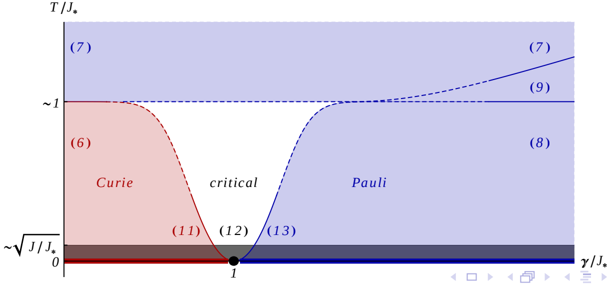
$$F(h) = (1/J_* - 1/\gamma)h^2 + \pi^2 h^4/(6\gamma_c^3)$$

- the scaling form of the spin susceptibility

$$\chi = \sqrt{J_*^3/T} f(T_X/T)/J^2, \qquad T_X = J_*\alpha^2, \quad \alpha = \gamma_c/\gamma - 1$$

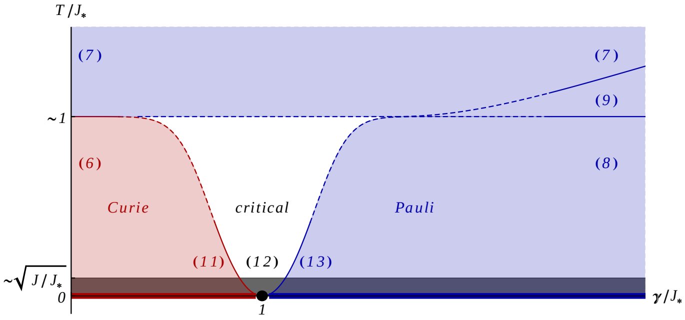
- critical region, $\gamma \sim \gamma_c$ at $T_X \ll T$, region (12):

$$\chi \sim J_*^{3/2}/(J^2\sqrt{T})$$



- our results for the spin susceptibility suggests the existence of quantum critical point at

$$\gamma_c = J_* \left[1 + O\left((J/J_*)^{1/4} \right) \right]$$



Conclusions:

- we studied the mesoscopic Stoner instability in open quantum dots at $0 < \Delta - J \ll \Delta$
- we computed the temperature dependence of the spin susceptibility
- our results suggest existence of the quantum phase transition at $\gamma_c = J_*$
- this quantum phase transition occurs due to suppression of Coleman-Weinberg mechanism

- the partion function of gauged fermions:

$$\mathcal{Z} = \prod_{\alpha} \lim_{M\to\infty} \int \prod_{k=1}^{M} d\bar{\tilde{\Psi}}_{\alpha,k} d\tilde{\Psi}_{\alpha,k} \exp\Big(\sum_{k,j=1}^{M} \bar{\tilde{\Psi}}_{\alpha k} S^{(\alpha)}_{kj} \tilde{\Psi}_{\alpha j} \Big).$$

where

$$S^{(\alpha)}_{kj} = \begin{bmatrix} 1 & 0 & & \dots & 0 & e^{-i\beta\phi_0} U(\beta) a_\alpha \\ -a_\alpha & 1 & 0 & & & 0 \\ 0 & -a_\alpha & 1 & \ddots & & \vdots \\ & 0 & -a_\alpha & \ddots & 0 & \\ \vdots & & 0 & \ddots & 1 & 0 \\ 0 & & & \dots & -a_\alpha & 1 \end{bmatrix}, \quad a_\alpha = 1 - \beta\,\epsilon_\alpha / M.$$

- integration over the Grassmanian variables yields

$$\mathcal{Z} = \prod_{\alpha} \lim_{\Delta\to 0} \det S^{(\alpha)} = \prod_{\alpha} \det \left[1 + U(\beta) e^{-\beta(\epsilon_\alpha + i\phi_0)} \right].$$