Strange metal state near quantum metal-superconductor transition in thin films

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Outline of the talk

1. Motivation from experimental data: “anomalous metal” behavior near SMT and the issue of para-conductivity at very low T
2. Theoretical model: 2D metal with potential disorder and spatial fluctuations of the Cooper attraction constant $\lambda(\mathbf{r})$
3. Anderson localization of superconducting modes: emergent SC grains
4. Interaction between emergent grains: strong-disorder RG
5. Strange metal as a Griffiths phase: large T-independent conductivity

Classical results (since 1980's)

- No metal state in 2D due to Anderson localization:

  Abrahams, Anderson, Liccardello & Thouless

  Gor'kov,Larkin & Khmelnitskii

- Direct transition between superconducting

  and insulating states, with a metal as a critical point only

  M.P.A.Fisher

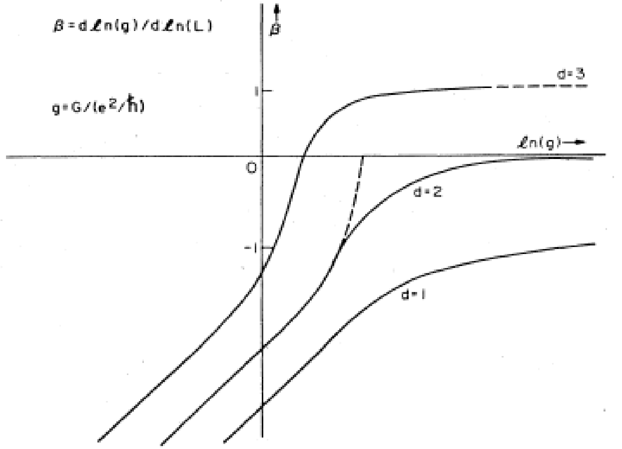
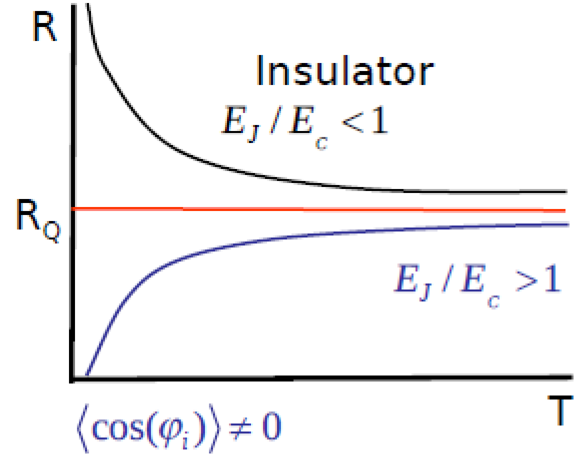


FIG. 3. The scaling function $\beta(g)$ vs the dimensionless conductance g for different dimensions. If $\sigma_{\min}$ exists in 2D, the behavior of β is shown by the dashed lines.

However, experimental data of quite different kind were collected in quite a large amount



REVIEWS OF MODERN PHYSICS, VOLUME 91, JANUARY–MARCH 2019

Colloquium: Anomalous metals: Failed superconductors

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Quite a number of various data for large, but T-independent conductivity at lowest temperatures

With magnetic field and without magnetic field

Few examples:

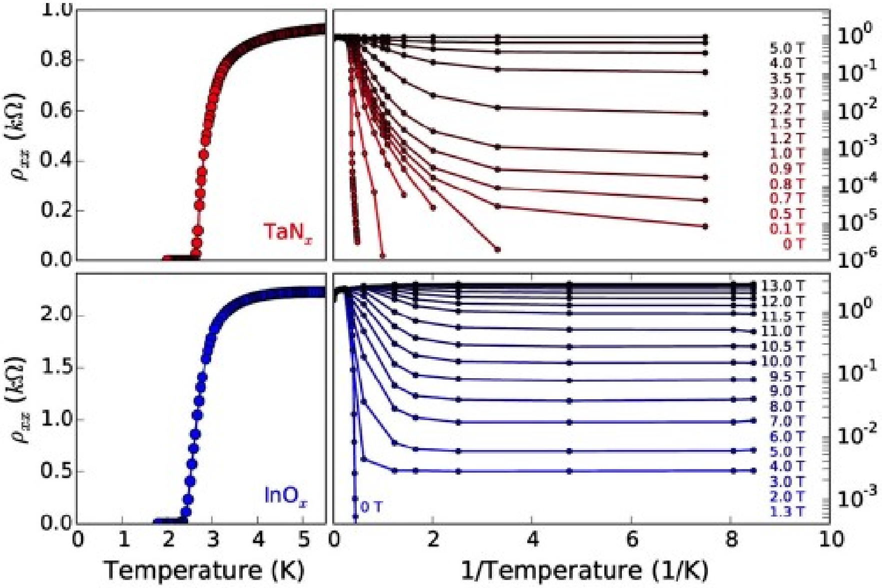


FIG. 8. Resistivity as a function of T for TaN_x and InO_x films. The left-hand panels show the superconducting transition in resistance vs T for $H = 0$. The right-hand panels show the resistance on a logarithmic scale as a function of $1/T$ for various values of the applied magnetic field. From Breznay and Kapitulnik, 2017.

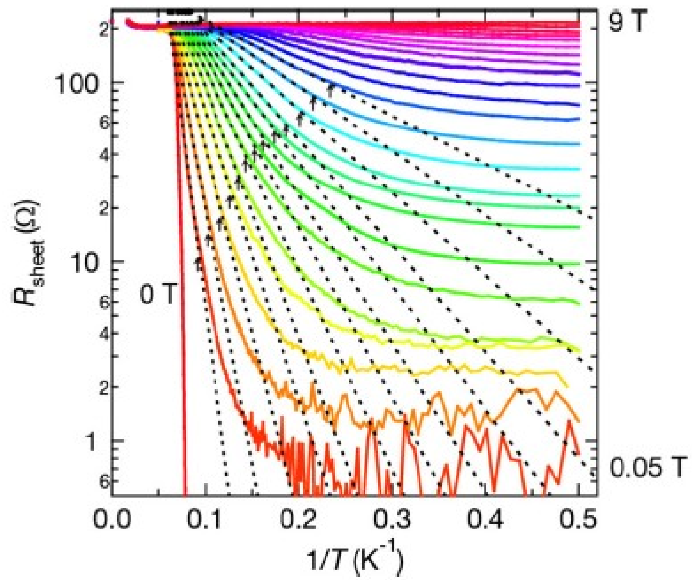


FIG. 9. Arrhenius plot of the sheet resistance of an electric double layer transistor of ZrNCl at gate voltage $V_G = 6.5$ V for different magnetic fields perpendicular to the surface. The black dashed lines demonstrate the activated behavior with activation energy $U(H) \propto \ln(H_0/H)$, similar to Ephron *et al.* (1996). The

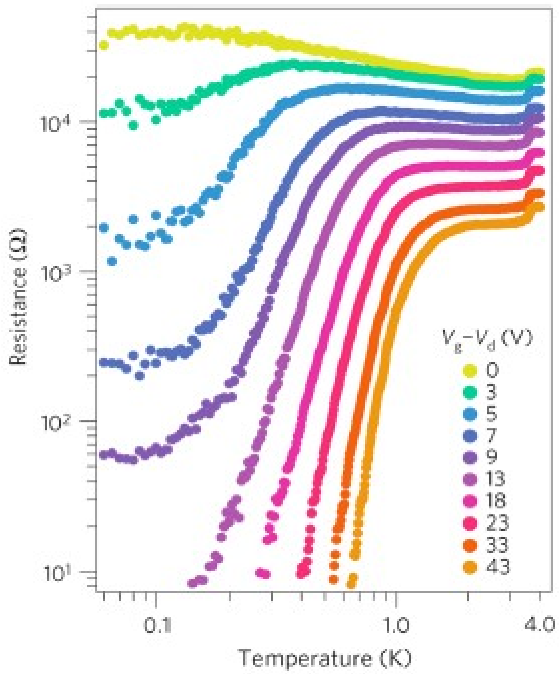


FIG. 11. The resistance vs T on a log-log scale of an ordered array of Sn disks on a graphene substrate; the density of electrons in the graphene is controlled by adjusting the voltage with a back gate. For the largest gate voltages (highest electron densities) there is a clear finite temperature transition to a superconducting state. However, for a broad range of lower gate voltages, we see the familiar several orders of magnitude drop in the resistance that terminates in a temperature independent plateau. From Han *et al.*, 2014.

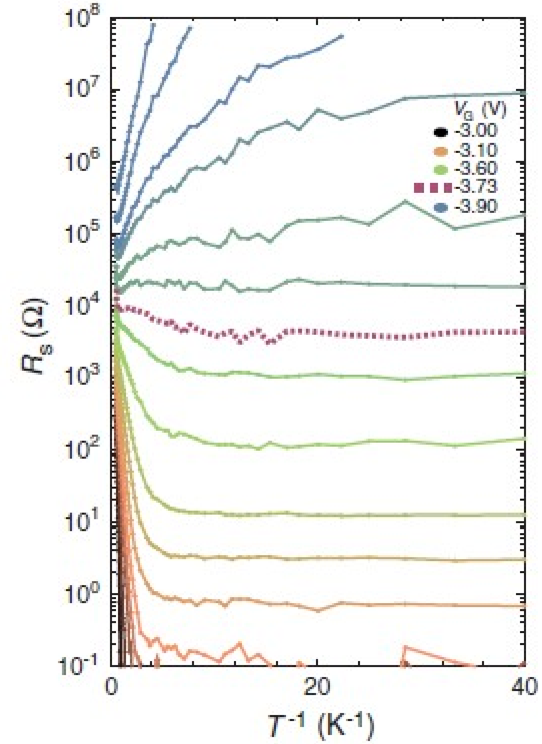


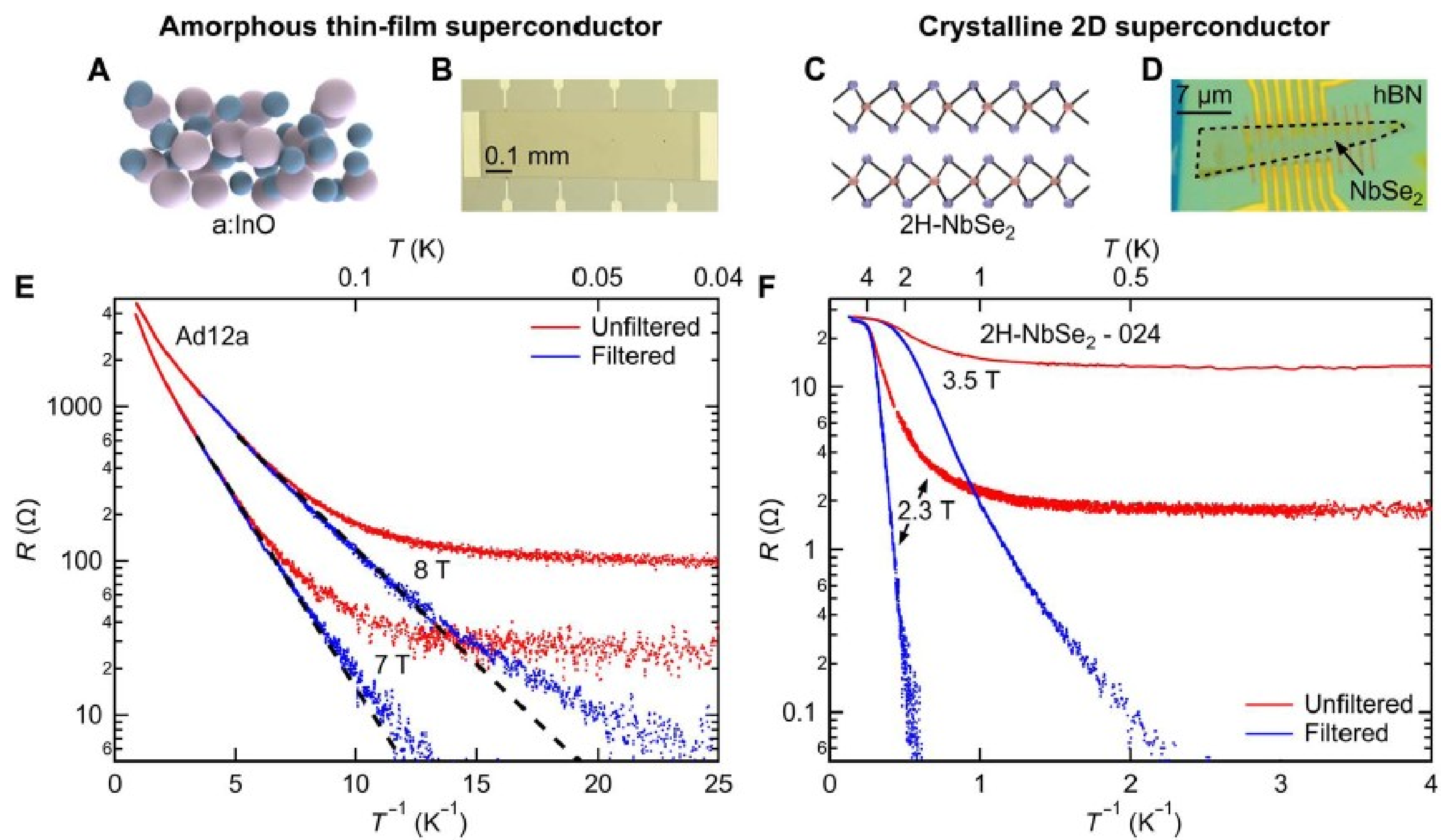
FIG. 12. Log of the sheet resistance R_s as a function of the inverse temperature T^{-1} in a gated InAs heterostructure with epitaxial Al patterned to form a regular array of superconducting islands. Data are shown for a range of gate voltages V_G from -3.0 to -3.9 V. The dashed curve corresponds to $V_G = -3.73$ V; the tendency of the curves with $V_G \geq -3.73$ V to saturate at low T is indicative of the occurrence of a metallic phase. From Bøttcher *et al.*, 2018.

Important counter example

CONDENSED MATTER PHYSICS

Sensitivity of the superconducting state in thin films

I. Tamir[1,2]*, A. Benyamini[3], E. J. Telford[4], F. Gorniaczyk[1], A. Doron[1], T. Levinson[1], D. Wang[4], F. Gay[5], B. Sacépé[5], J. Hone[3], K. Watanabe[6], T. Taniguchi[6], C. R. Dean[4], A. N. Pasupathy[4], D. Shahar[1,4]



What is known on the theory side ?

Standard scaling scenario for SMT at T=0

T. Kirkpatrick, D. Belitz (1997)

Metal-superconductor transition at $T = 0$: a case of unusual scaling

$$\Gamma^{(2)}(\omega, q) = t + \frac{1}{\ln 1/(q^2 - i\omega)}. \quad \xi \sim e^{1/2|t|}, \quad \Psi \sim \frac{\Theta(-t)}{|t|} e^{-1/|t|}$$

Gaussian fixed point is stable (phase volume of fluctuation modes is tiny)

⇓

no noticeable fluctuation effects

No conductivity enhancement near SMT

Can this scenario be broken?

2. Theoretical model:

frozen disorder in the Cooper constant

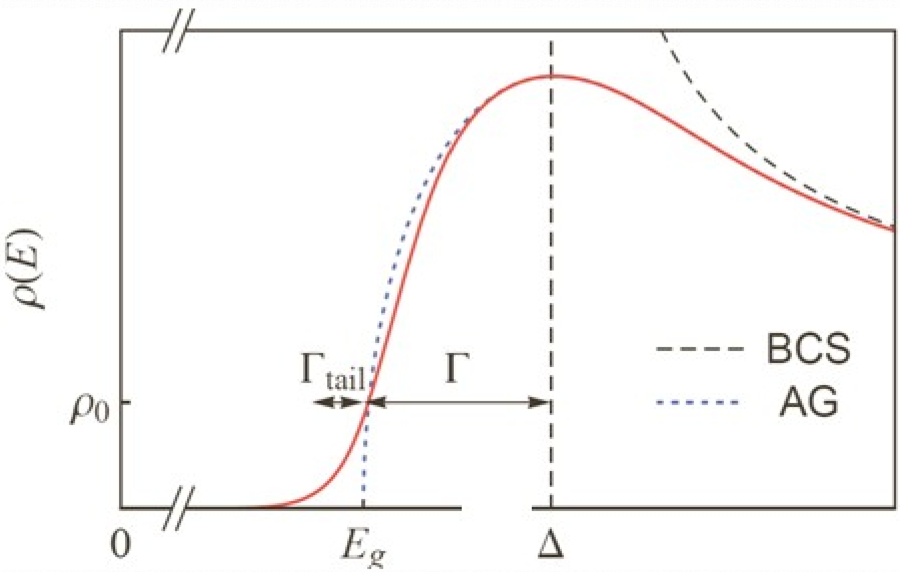
Cooper constant inhomogeneity

- Appears in any scenario of SM transition at $T = 0$
- Generates effective depairing in the superconducting phase

Larkin, Ovchinnikov (1972)

$$(D/2)\nabla^2\theta + iE\sin\theta + \Delta\cos\theta - \Delta\,\eta\,\cos\theta\sin\theta = 0$$

Feigel'man, Skvortsov (2012)



- What are its effects in the M phase?

Coulomb suppression of T_c

Critical temperature

$$\frac{1}{|\lambda_0|} = \frac{1}{\lambda_g}\tanh(\lambda_g \zeta_c) \quad \Rightarrow \quad T_c\tau = \left(\frac{1 - \frac{1}{\sqrt{2\pi g}} \ln \frac{1}{T_{c0}\tau}}{1 + \frac{1}{\sqrt{2\pi g}} \ln \frac{1}{T_{c0}\tau}} \right)^{\sqrt{\pi g/2}}$$

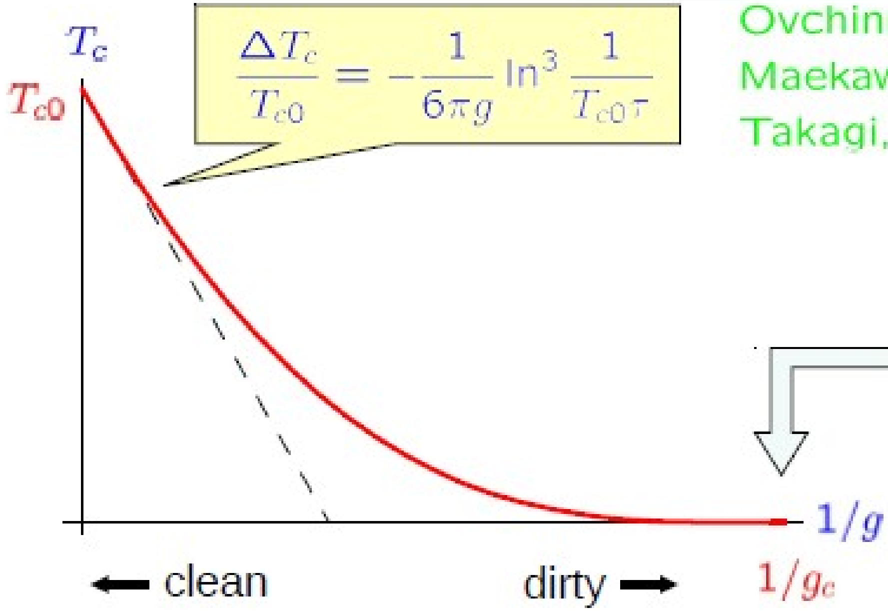
Finkelstein (1987)

$$\frac{\Delta T_c}{T_{c0}} = -\frac{1}{6\pi g} \ln^3 \frac{1}{T_{c0}\tau}$$

Ovchinnikov (1973) **(wrong sign)**

Maekawa, Fukuyama (1982)

Takagi, Kuroda (1982)



$$g_c = \frac{1}{2\pi} \ln^2 \frac{1}{T_{c0}\tau}$$

At $\ln(1/T_{c0}\tau) > 5$

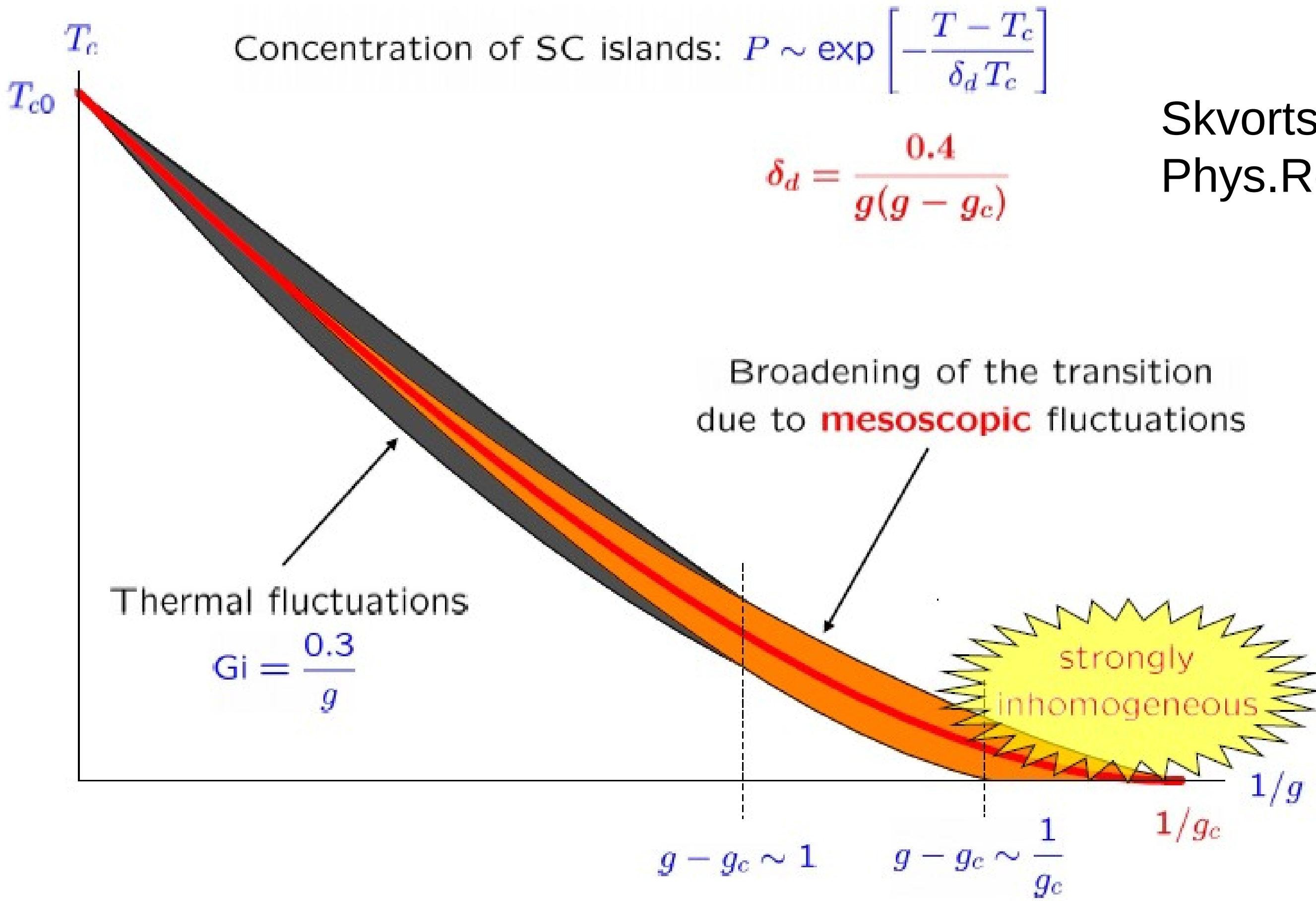
$g_c > 4$ i.e. $R_c < R_Q$ ⇒

Conclusion:

Superconductor – Metal

Quantum phase transition

Mesoscopic vs. thermal fluctuations



Skvortsov,Feigelman
Phys.Rev.Lett. 2005

Major conclusion: near SMT emergent granularity is a strong effect

Suppression of Cooper pairing

Consider superconducting propagator $\mathcal{L}$ on top of metallic state

electrons are integrated out (gaussian approximation)

$$S_2 = \int \frac{d\omega}{2\pi} \int d\mathbf{r}_1 d\mathbf{r}_2 \Delta_\omega(\mathbf{r}_1)\alpha(\omega, \mathbf{r}_1 - \mathbf{r}_2)\Delta_\omega(\mathbf{r}_2).$$

$$\alpha(0,q) = \frac{\nu}{\lambda_*} - \int d\epsilon \frac{\nu}{Dq^2 + 2|\epsilon|} w_q(\epsilon)$$

Cooperon suppression factor due to fluctuating electric field

w_q obeys RG equation

$$w_q(\epsilon) = 1 - \lambda_g^2 \int_0^{\zeta_q} d\zeta_1 \min(\zeta, \zeta_1) w_q(\zeta_1)$$

$$w_q(\epsilon) \equiv w_q(\zeta), \text{ with } \zeta = \ln(1/\epsilon\tau).$$

with $\lambda_g^2 = 1/2\pi g$ and $\zeta_q = -2\ln(ql)$

Its solution is

$$w_q(\zeta \le \zeta_q) = \cosh(\lambda_g \zeta) - \tanh(\lambda_g \zeta_q)\sinh(\lambda_g \zeta)$$

$$w_q(\zeta \ge \zeta_q) = \frac{1}{\cosh(\lambda_g \zeta_q)}$$

$$\alpha(0,q) = \frac{\nu}{\lambda_*} - \frac{\nu}{\lambda_g} + \frac{2}{\lambda_g} \frac{1}{1 + (ql)^{-4\lambda_g}}$$

Randomness of the Cooper pairing

$$S_2 = \int \frac{d\omega}{2\pi} \int d\mathbf{r}_1 d\mathbf{r}_2 \Delta_\omega(\mathbf{r}_1)\alpha(\omega, \mathbf{r}_1 - \mathbf{r}_2)\Delta_\omega(\mathbf{r}_2).$$

$$\alpha(0,q) = \frac{\nu}{\lambda_*} - \frac{\nu}{\lambda_g} + \frac{2}{\lambda_g}\frac{1}{1+(ql)^{-4\lambda_g}}$$

In presence of disorder and non-zero frequency:

$$\hat{\alpha}(\omega, q; \mathbf{r}) = \frac{\nu}{\lambda_g}\left[\delta_0 + u(\mathbf{r}) + C_q(\omega)\right]$$

$$C_q(\omega) \approx 2\left[(ql)^2 + 2\omega\tau\right]^{2\lambda_g}$$

$$\delta_0 = \left\langle \frac{\lambda_g}{\lambda_*(r)} \right\rangle - 1$$

$$\overline{u(\mathbf{r})u(\mathbf{r}')} = \Lambda f(|\mathbf{r}|/b)$$

$$(\delta_0 + u(\mathbf{r}))\,\mathcal{L}(\omega; \mathbf{r}, \mathbf{r}') + \int d^2\mathbf{r}_1 C(\omega; \mathbf{r} - \mathbf{r}_1)\mathcal{L}(\omega; \mathbf{r}_1, \mathbf{r}') = \frac{\lambda_g}{\nu}\delta(\mathbf{r} - \mathbf{r}')$$

$$L_\omega(\mathbf{r} - \mathbf{r}') = \overline{\mathcal{L}(\omega, \mathbf{r}, \mathbf{r}')} = \frac{\lambda_g}{\nu}\overline{\sum_n \frac{\psi_n(\mathbf{r})\psi_n^*(\mathbf{r}')}{E_n + \delta_0 - i0}}.$$

Superconducting propagator

$$L_\omega(\mathbf{r}-\mathbf{r}') = \overline{\mathcal{L}(\omega,\mathbf{r},\mathbf{r}')} = \frac{\lambda_g}{\nu}\overline{\sum_n \frac{\psi_n(\mathbf{r})\psi_n^*(\mathbf{r}')}{E_n+\delta_0-i0}};$$

The spectrum E_n depends on disorder $\overline{u(\mathbf{r})u(\mathbf{r}')} = \Lambda f(|\mathbf{r}|/b)$

$$\delta_0 = \left\langle \frac{\lambda_g}{\lambda_*(r)} \right\rangle - 1$$

without disorder $u \equiv 0,\ \delta_c = 0$

Clean DOS $\nu(E) \propto E^{-1+1/(2\lambda_g)}\Theta(E)$

Very small DoS at low E and small λ_g

Weak disorder spectral edge shifts to negative E (shift of δ_c)

Randomness in λ enhances SC!

No localized Lifshits tails at small disorder

Similar problem treated earlier

Key features: long-range, non-random hopping

Malyshev, Malyshev, Dominguez-Adame (2004)
Rodriguez, Malyshev, Sierra et al (2003)

$$\mathcal{H} = \sum_{n=1}^{N} \varepsilon_n |n\rangle\langle n| + \sum_{m,n=1}^{N} J_{mn} |m\rangle\langle n|$$

$$J_{mn} = J/|m-n|^{\mu}$$

$$\epsilon(q) \sim q^{\alpha}$$

$\alpha = \mu - d$

ϵ_n from fox dist. of width W

Under the condition $0 < \alpha < d/2$ weak disorder is irrelevant near the band edge

localization transition at the band bottom as a function of disorder

Transition happens at $d < \mu < 3d/2$

3. Anderson localization near the band edge: emergent superconducting islands

Randomness of the Cooper pairing

$$S_2 = \int \frac{d\omega}{2\pi} \int d\mathbf{r}_1 d\mathbf{r}_2 \Delta_\omega(\mathbf{r}_1)\alpha(\omega, \mathbf{r}_1 - \mathbf{r}_2)\Delta_\omega(\mathbf{r}_2).$$

$$\alpha(0,q) = \frac{\nu}{\lambda_*} - \frac{\nu}{\lambda_g} + \frac{2}{\lambda_g}\frac{1}{1+(ql)^{-4\lambda_g}}$$

In presence of disorder and non-zero frequency:

$$\hat{\alpha}(\omega, q; \mathbf{r}) = \frac{\nu}{\lambda_g}\left[\delta_0 + u(\mathbf{r}) + C_q(\omega)\right]$$

$$C_q(\omega) \approx 2\left[(ql)^2 + 2\omega\tau\right]^{2\lambda_g}$$

$$\delta_0 = \left\langle \frac{\lambda_g}{\lambda_*(r)} \right\rangle - 1$$

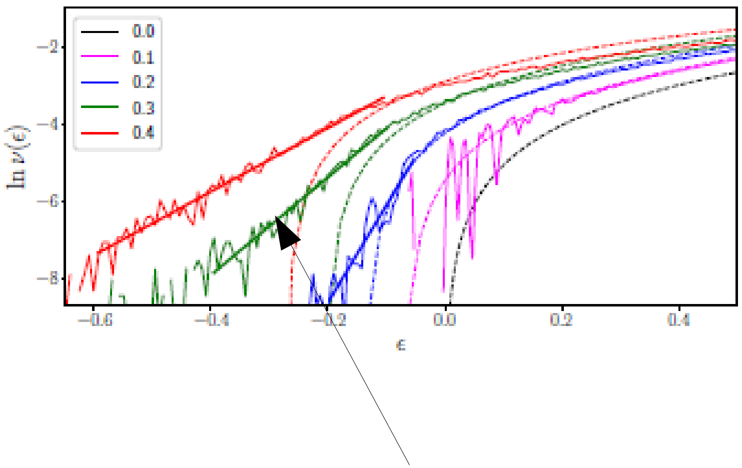
$$\overline{u(\mathbf{r})u(\mathbf{r}')} = \Lambda f(|\mathbf{r}|/b)$$

$$(\delta_0 + u(\mathbf{r}))\,\mathcal{L}(\omega; \mathbf{r}, \mathbf{r}') + \int d^2\mathbf{r}_1 C(\omega; \mathbf{r} - \mathbf{r}_1)\mathcal{L}(\omega; \mathbf{r}_1, \mathbf{r}') = \frac{\lambda_g}{\nu}\delta(\mathbf{r} - \mathbf{r}')$$

$$L_\omega(\mathbf{r} - \mathbf{r}') = \overline{\mathcal{L}(\omega, \mathbf{r}, \mathbf{r}')} = \frac{\lambda_g}{\nu}\overline{\sum_n \frac{\psi_n(\mathbf{r})\psi_n^*(\mathbf{r}')}{E_n + \delta_0 - i0}}.$$

Superconducting propagator: DoS

$\lambda_g = 0.2$ and several values of disorder.

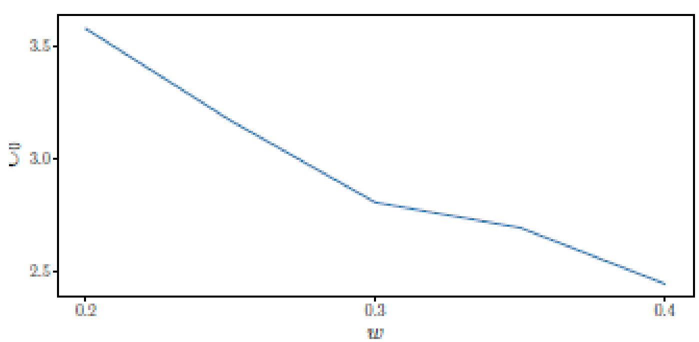


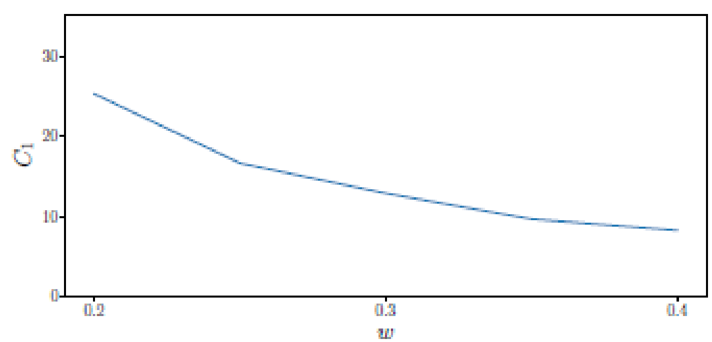
Dashed lines: self-consistent Born appr.

$$\text{SCBA: } \nu(\epsilon) = \frac{1}{4\pi^2} \text{Im}\Sigma(\epsilon + \sigma(\epsilon))$$

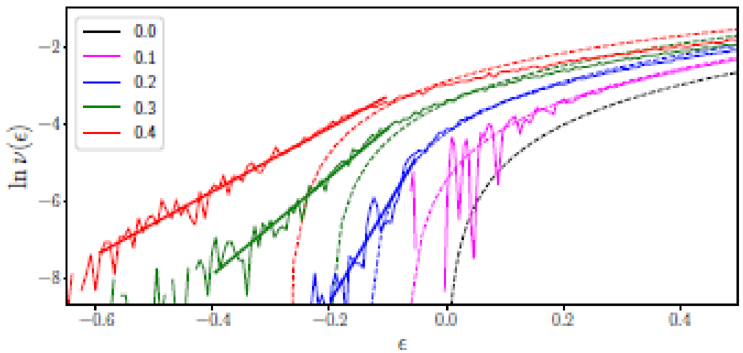
$$\sigma(\epsilon) = w^2 \Sigma(\epsilon + \sigma(\epsilon)), \quad \Sigma(\epsilon) = \frac{1}{4\pi^2} \int \frac{d^2 q}{-\epsilon + C_q}$$

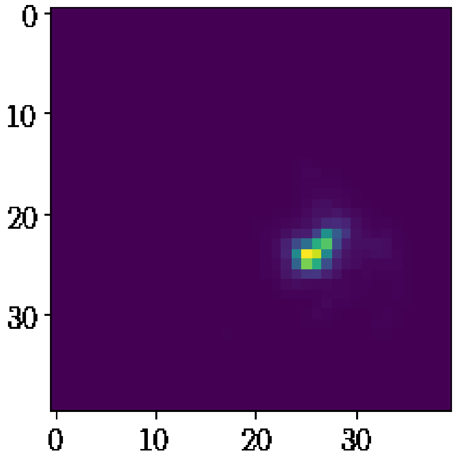
DoS in the tail: $\ln \nu(\epsilon) = C_1(w)\epsilon - C_0(w)$

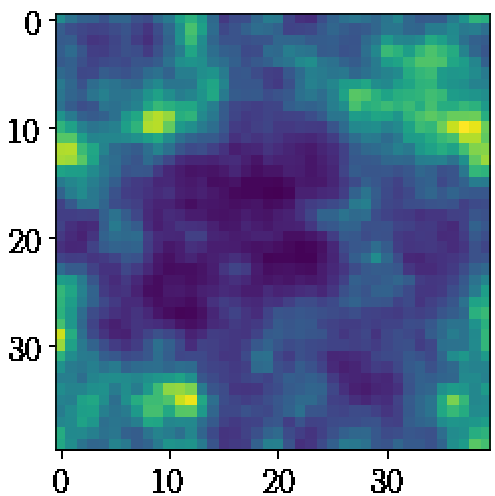




Superconducting propagator: eigenfunctions







$$P_2 = \langle \sum_r \psi^4(r) \rangle$$

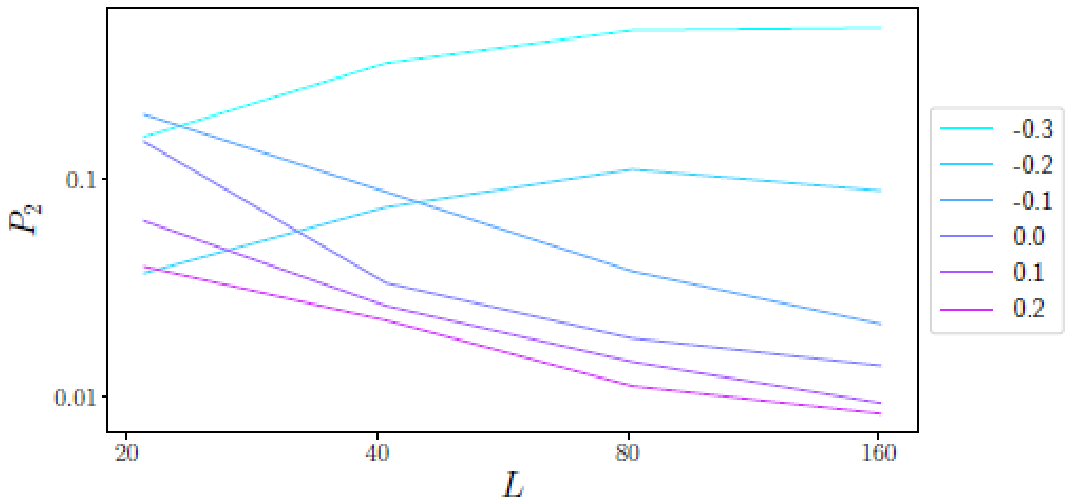


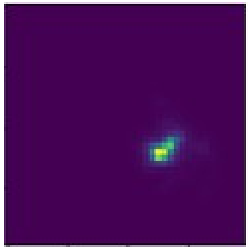
FIG. 2: Inverse participation ratio P_2 at $\lambda_g = 0.2$ and $w = 0.3$ (compare with Fig. 1), for several energies ϵ (see the legend). At larger energy, the eigenstates are delocalized and in the tail they are localized.

Localized emergent superconducting islands:
breakdown of the T=0 scaling theory for SMT

A spatial length scale comes in
as island's localization length

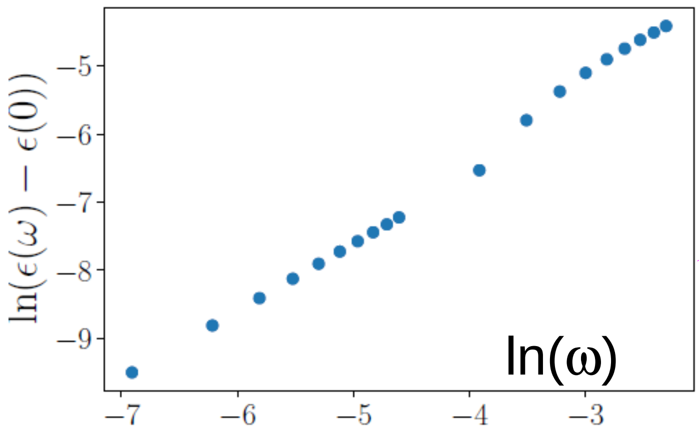
Localized SC islands

$$\Delta_i(\mathbf{r}, t) = a_i(t)\psi_i(\mathbf{r})$$



$\psi_i(r)$: normalized eigenmode with energy E_i, localized on the scale L_i

$$S = \frac{\nu}{\lambda_g}\int dt\left[\alpha_i|a_i|^2 + \frac{B_i}{2}|a_i|^4\right] + \nu\sum_{\omega}\Gamma_i|\omega||a_i(\omega)|^2$$



Quadratic action (diagonalized)

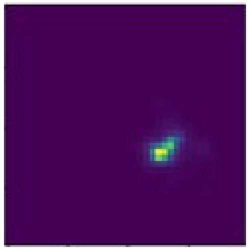
Nonlinear GL action

$$F_4(\Delta) = \int \Pi_{i=1}^4 d^2 r_i \Delta(r_1)\Delta^*(r_2)\Delta(r_3)\Delta^*(r_4)B(\{r_i\})$$

$$B_4(\{r_i\}) = \frac{\pi\nu}{2}T\sum_\epsilon \Pi_{k=1}^4 \frac{1}{|\epsilon| + \frac{1}{2}D(-i\partial_k)^2} \times$$
$$\delta(r_1 - r_2)\delta(r_1 - r_3)\delta(r_1 - r_4) \times \left[|\epsilon| + \frac{1}{8}D\left([-i\partial_1 + i\partial_3]^2 + [-i\partial_2 + i\partial_4]^2\right)\right]$$

Localized SC islands: dynamics

$$\Delta_i(\mathbf{r},t) = a_i(t)\psi_i(\mathbf{r})$$



$\psi_i(r)$: normalized eigenmode with energy E_i, localized on the scale L_i

$$S = \frac{\nu}{\lambda_g}\int dt\left[\alpha_i|a_i|^2 + \frac{B_i}{2}|a_i|^4\right] + \nu\sum_\omega \Gamma_i|\omega||a_i(\omega)|^2$$

$$\alpha_i = E_i + \delta_0$$

$$\delta_0 = \left\langle \frac{\lambda_g}{\lambda_*(r)} \right\rangle - 1$$

$B_i \sim L_i^2/D^2$ $\qquad$ $\Gamma_i \sim L_i^2/D$

diverge at $T \to 0$ without disorder in λ

In the unstable domain, $\alpha_i < 0$: $|a_i|^2 = -\alpha_i/B_i \sim |\alpha_i| D^2/L_i^2$

At $|\alpha_i| \gg 1/g$ only phase degrees of freedom survive, $S = \sum_i S_0[\phi_i(t)]$

$$S[\varphi(t)] = \frac{G_i}{2\pi^2}\int dt_1 dt_2 \frac{\sin^2[(\varphi(t_1)-\varphi(t_2))/2]}{(t_1-t_2)^2}$$

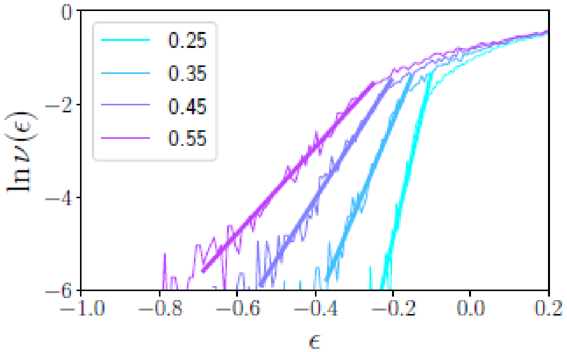
$G_i = g(E_i+\delta_0)\frac{\tilde{c}}{\lambda_g}$ >> 1

Andreev conductance

Correlation time $\quad t_i \approx \omega_i^{-1}\exp(G_i/2)$

$\omega_i \sim \omega_0 = D/L_{\mathrm{loc}}^2$

Localized SC islands: distribution of relaxation rates



$\gamma_i = 1/t_i.$ Relaxation rate

$$t_i \approx \omega_i^{-1} \exp(G_i/2) \qquad G_i = g(E_i + \delta_0)\frac{\tilde{c}}{\lambda_g} \qquad \ln\nu(\epsilon) = C_1(w)\epsilon - C_0(w).$$

Andreev conductance

$$P_0(\gamma)d\gamma \approx p_0 \left(\frac{\gamma}{\omega_0}\right)^{\eta_0} \frac{d\gamma}{\omega_0}, \qquad \eta_0 = \frac{2C_1(w)}{\tilde{c}g} - 1$$

??

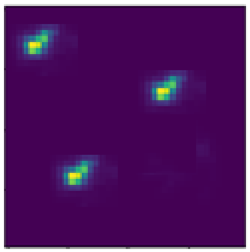
$0 < \eta_0 < 1$ ⇒ Average correlation time $\langle 1/\gamma \rangle$ is finite

but its dispersion diverges

Power-law distribution is indicative for possible Griffiths phase

4. Interaction between phases of different islands and strong-disorder renormalization group

Interaction of the modes

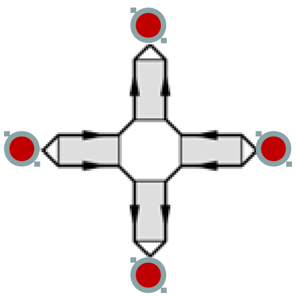


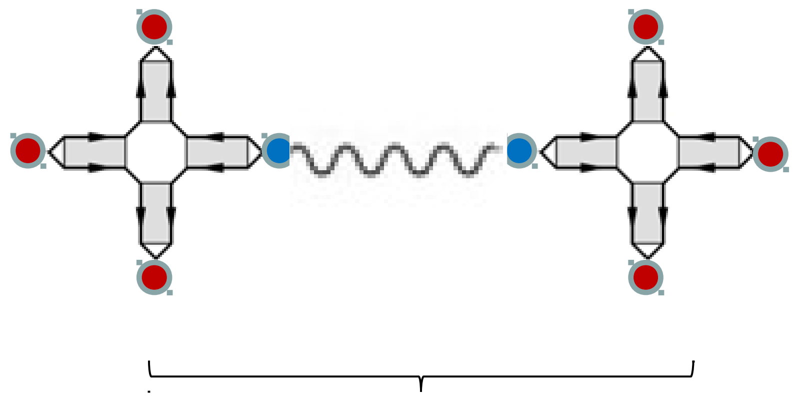
$$S = \frac{\nu}{\lambda_g} \int dt \left[\alpha_i |a_i|^2 + \frac{B_i}{2} |a_i|^4 \right] + \nu \sum_{\omega} \Gamma_i |\omega| |a_i(\omega)|^2$$

Other contributions of non-linear part of GL

localized mode

delocalized mode





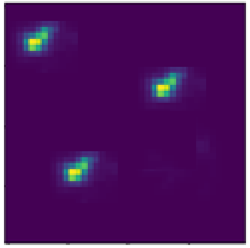
Interaction between localized modes via propagator of delocalized modes

 $= \dfrac{A}{\delta_0 + (Ql)^{4\lambda_g}}$

$$\delta_0 = \left\langle \frac{\lambda_g}{\lambda_*(r)} \right\rangle - 1$$

Interaction of localized modes

Fourrier – space interaction $$J(Q) = \frac{A}{\delta_0 + (Ql)^{4\lambda_g}}$$



Localized islands with pair interaction in the real space

$$J(\mathrm{r}) \approx \frac{A}{2\pi} \frac{4\lambda_g (r/l)^{4\lambda_g}}{r^2(1+\delta_0 (r/l)^{4\lambda_g})^2} \propto r^{-\beta} \quad \text{where} \quad 0 < \beta - 2 < 1$$

Islands are distributed at random over the plane,
thus $J_{nm} = J(r_{nm})$ fluctuate strongly

Single-island relaxation rates also fluctuate strongly between the islands:

$$P_0(\gamma) d\gamma \approx p_0 \left(\frac{\gamma}{\omega_0}\right)^{\eta_0} \frac{d\gamma}{\omega_0},$$

➡ Strong-disorder RG

Strong-disorder RG approach

Initially developed for disordered quantum Ising chain in transverse field

D. S. Fisher, Physical Review Letters **69**, 534 (1992).
D. S. Fisher, Physical Review B **51**, 6411 (1995).

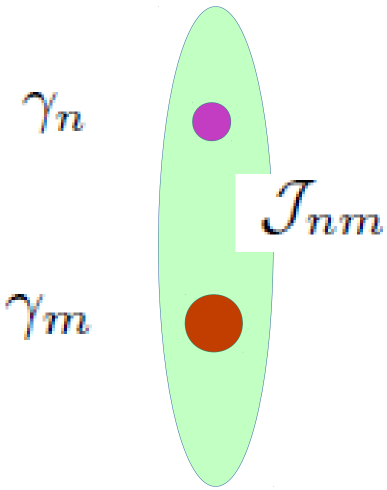
Recent reviews:

F. Igloi and C. Monthus, Physics Reports **412**, 277 (2005).
F. Igloi and C. Monthus, Eur. Phys. J. B **91**, 290 (2014).
G. Refael and E. Altman, Comptes Rendus Physique **14**, 725 (2013).

Most relevant previous publication:

R. Juhasz, I. A. Kovacs, and F. Igloi, Europhysics Letters **107**, 47008 (2014).

Strong-disorder RG approach: major idea



joint two-island decay rate γ_{nm}

becomes much smaller than the individual rates γ_n and γ_m:

$$\gamma_{nm} = \frac{\gamma_n \gamma_m}{\mathcal{J}_{nm}} \quad \text{for} \quad \min(\ln(\gamma_n), \ln(\gamma_m)) < \ln(\mathcal{J}_{nm})$$

Multiplicative renormalization is the key starting point of SDRG

lowest-frequency Andreev conductance of the two-island system $G_{nm} = G_n + G_m$

SDRG is the method to exclude (step by step) strongest interaction terms in the Hamiltonian, and keep track of the statistics of remaining lower-energy terms

Minimal energy scale of this RG is $\Omega_{min} = T > 0$

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$$\mathcal{J}(r) \propto r^{-\beta}$$

Strong-disorder RG equations

$$x_n = (2/\beta)\ln(\Omega/\gamma_n) \qquad y = (\Omega/\mathcal{J})^{2/\beta} - 1$$

Convenient representation of the energy variables

$$\tau = (2/\beta)\ln(\Omega_0/\Omega)$$

"RG time" evolution variable

Ω is the current upper energy scale

$$x_{nm} = x_n + x_m$$

$$y_{nm} = y_{ni} + y_{im} + 1$$

→ Anzats for the PDFs

$$P(x,\tau) = p(\tau)e^{-p(\tau)\,x}$$

$$Q(y,\tau) = q(\tau)e^{-q(\tau)\,y}$$

$$\frac{dp}{d\tau} = -p\cdot q$$

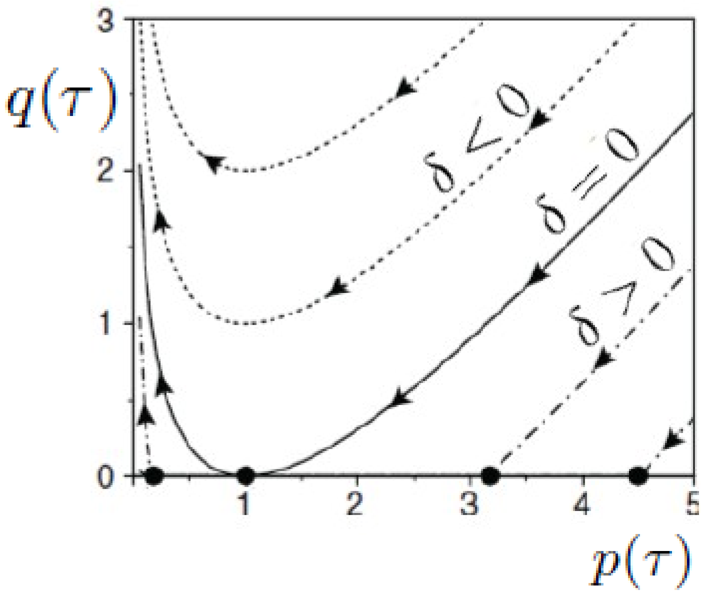
$$\frac{dq}{d\tau} = -p\cdot q + q$$

These two equations are formally equivalent to the Berezinsky-Kosterlitz-Thouless RG for 2D XY model

Strong-disorder RG: the solution

$$\frac{dp}{d\tau} = -p \cdot q$$
$$\frac{dq}{d\tau} = -p \cdot q + q$$

$$P(x,\tau) = p(\tau) e^{-p(\tau)\,x}$$
$$Q(y,\tau) = q(\tau) e^{-q(\tau)\,y}$$



1st integral: $q(\tau) - p(\tau) + \ln(p(\tau)) = \text{Const}$

$p(\tau) = 1 + \xi(\tau)$ $\xi(\tau) \ll 1$

near the critical point

Single RG equation:

$$\frac{d\xi}{d\tau} = -\frac{\xi^2}{2} + \delta$$

At small $\delta > 0$

$$p(\tau) = 1 + \sqrt{2\delta}\coth\left(\sqrt{\frac{\delta}{2}}(\tau + \tau_+)\right)$$

$$q(\tau) = \frac{\delta}{\sinh^2\left(\sqrt{\frac{\delta}{2}}(\tau + \tau_+)\right)}$$

Line of fixed points -
Griffiths metal phase

$$L_\delta = L_0\, e^{\frac{1}{2\sqrt{2\delta}}}$$

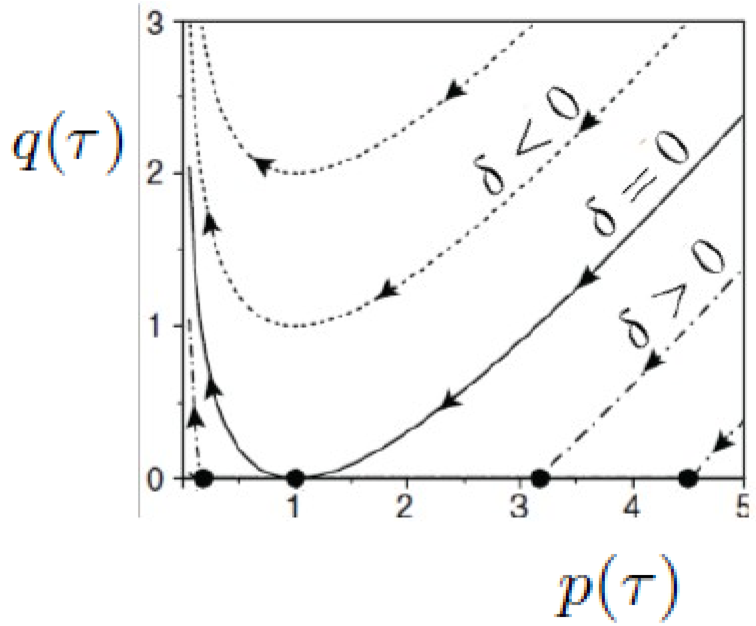
Largest strongly coupled clusters

Strong-disorder RG: solution for the superconducting state

At negative values of δ, the RG equation has qualitatively different solution

$$\xi(\tau) = \sqrt{2|\delta|}\tan\left(\sqrt{\frac{|\delta|}{2}}(\tau+\tau_-)\right)$$

singularity at $\tau+\tau_- = \pi/\sqrt{2|\delta|}$.



This solution describes strongly inhomogeneous superconducting state with a small energy gap

$$\Delta_\delta(0) = \Omega_0\, e^{-\frac{\beta\pi}{2\sqrt{2|\delta|}}}$$

$$T_c(\delta) \sim \Delta_\delta$$

5. Low-T conductivity of the Griffiths phase: "strange metal"

SDRG evolution stops at the lowest energy scale ~ T
The resulting distribution functions are:

$\beta > 2$

$$\mathcal{P}(\gamma;T)d\gamma = \frac{2p_T}{\beta}\left(\frac{\gamma}{T}\right)^{\frac{2p_T}{\beta}-1}\frac{d\gamma}{T} \qquad \text{where } (\gamma,\mathcal{J}) \leq T$$

$$\mathcal{Q}(\mathcal{J};T)d\mathcal{J} = \frac{2q_T}{\beta}\left(\frac{T}{\mathcal{J}}\right)^{\frac{2}{\beta}} e^{-q_T\left(\frac{T}{\mathcal{J}}\right)^{2/\beta}}\frac{d\mathcal{J}}{\mathcal{J}}$$

Large fractal clusters of SC islands

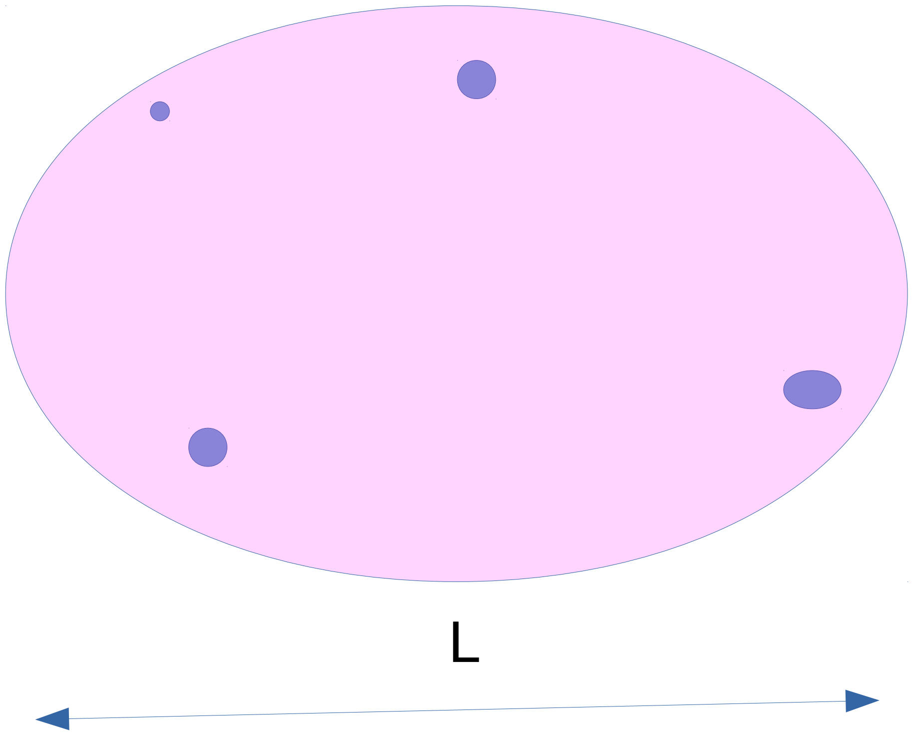
Close to the critical point $p_T \approx 1 + \sqrt{2\delta}$ and $q_T \ll 1$

the average correlation time $\langle 1/\gamma \rangle = \int_0^T \mathcal{P}(\gamma;T)d\gamma/\gamma$ will diverge, at some small $\delta > 0$

Thermal contribution to the relaxation rate should be added:

$\gamma_{tot} = \gamma + 2\pi T/G_a$ ⇨ $t_a^{cl} = \dfrac{G_a \hbar}{2\pi T}$

Effective Andreev conductance



Typical size of largest clusters

$$L_\delta = L_0 \, e^{\frac{1}{2\sqrt{2\delta}}}$$

Number of islands
within cluster of size L

$n(L) \sim \log^2(L)$

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Number of islands in largest clusters $n_\delta \sim 1/\delta$

Effective Andreev conductance of largest clusters
(in units of $4e^2/h$) $G_\delta \sim G_0 \, n_\delta \sim 1/\delta$

Aslamazov-Larkin paraconductivity near T=0 in 2D system

$$\sigma_{AL}^{cl} \sim \frac{e^2}{\hbar^2} T\, t_c(T) \sim \frac{e^2}{\hbar} \cdot \frac{1}{\delta}$$

No T-dependence

Divergence upon approach to SMT at $\delta \to 0$

Typical energy of superconducting fluctuations

Longest correlation time

$t_\delta(T) \sim \hbar\, G_\delta / T$

Conclusions

1. Usual scaling theory of the quantum SMT breaks down when spatial disorder in the Cooper coupling $\lambda(\mathbf{r})$ exceeds some critical magnitude

2. Proximity coupling between spontaneously formed SC islands can be described by Strong-Disorder RG

3. “Strange metal” phase in the vicinity of the SMT can be represented as a line of the SDRG fixed points

4. Conductivity of this “strange metal” is T – independent but diverges upon approach to the critical point of SM transition