Spacetime TRINITY

Lavinia Heisenberg
(ETH-ITP)
Institute for Theoretical Physics, Zürich
Lavinia.Heisenberg@phys.ethz.ch

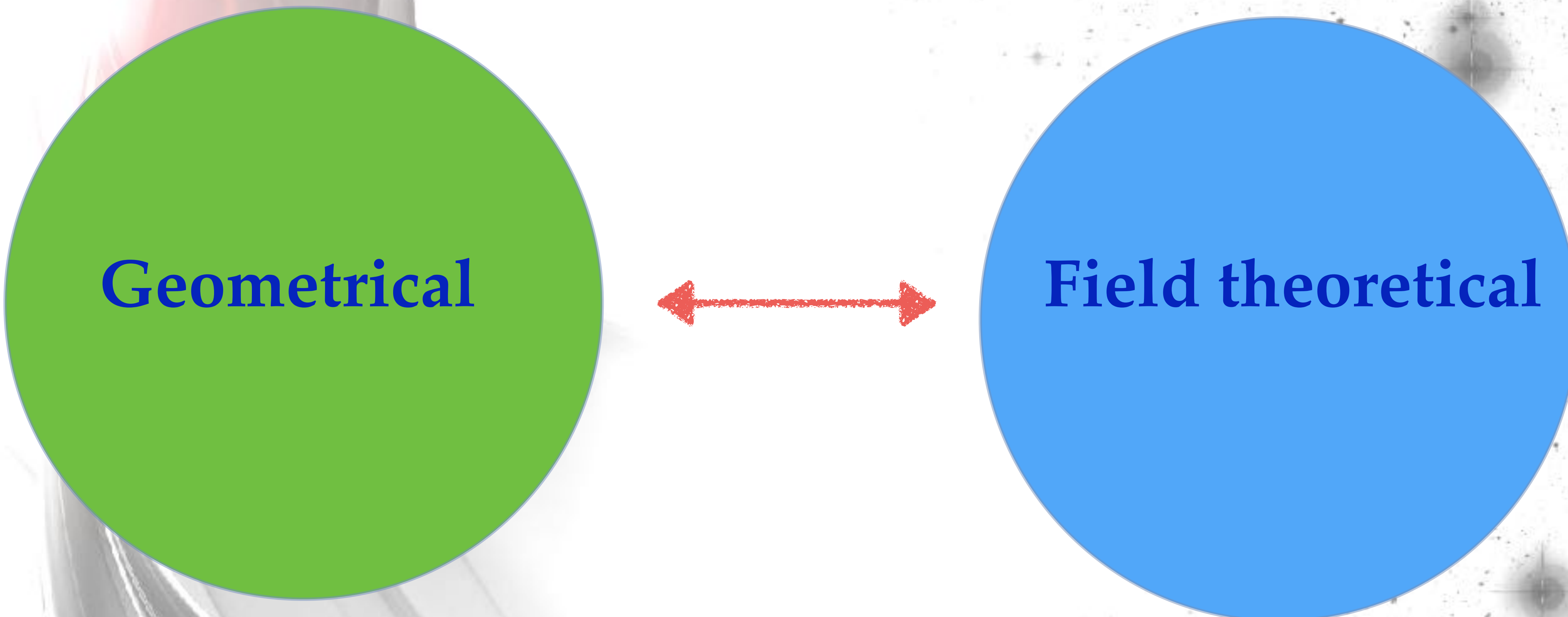
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

20th Oct 2019, 100th anniversary of I.M.Khalatnikov, Chernogolovka/Russia

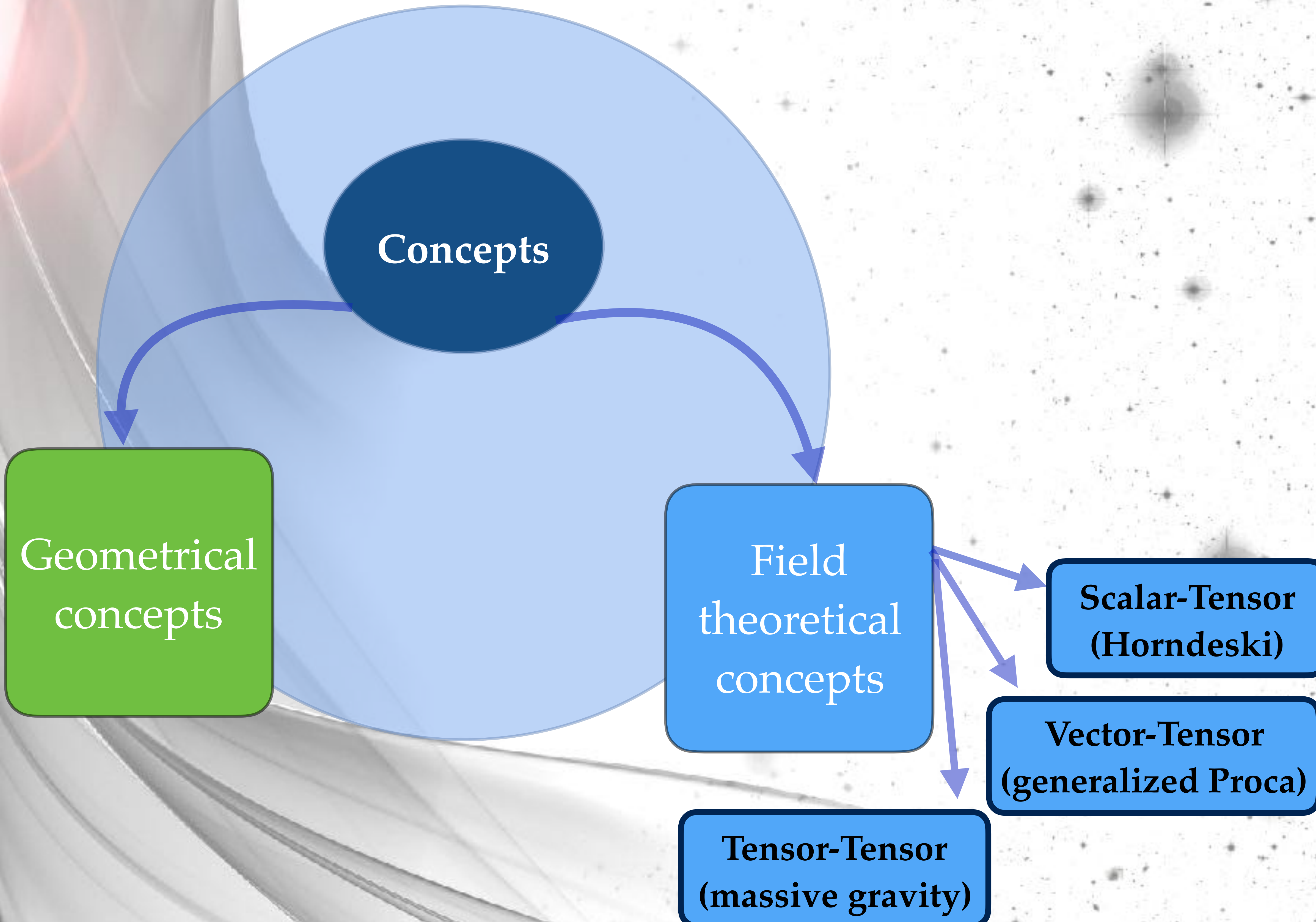
Q: Are you related to Werner Heisenberg?

LH: It is uncertain!

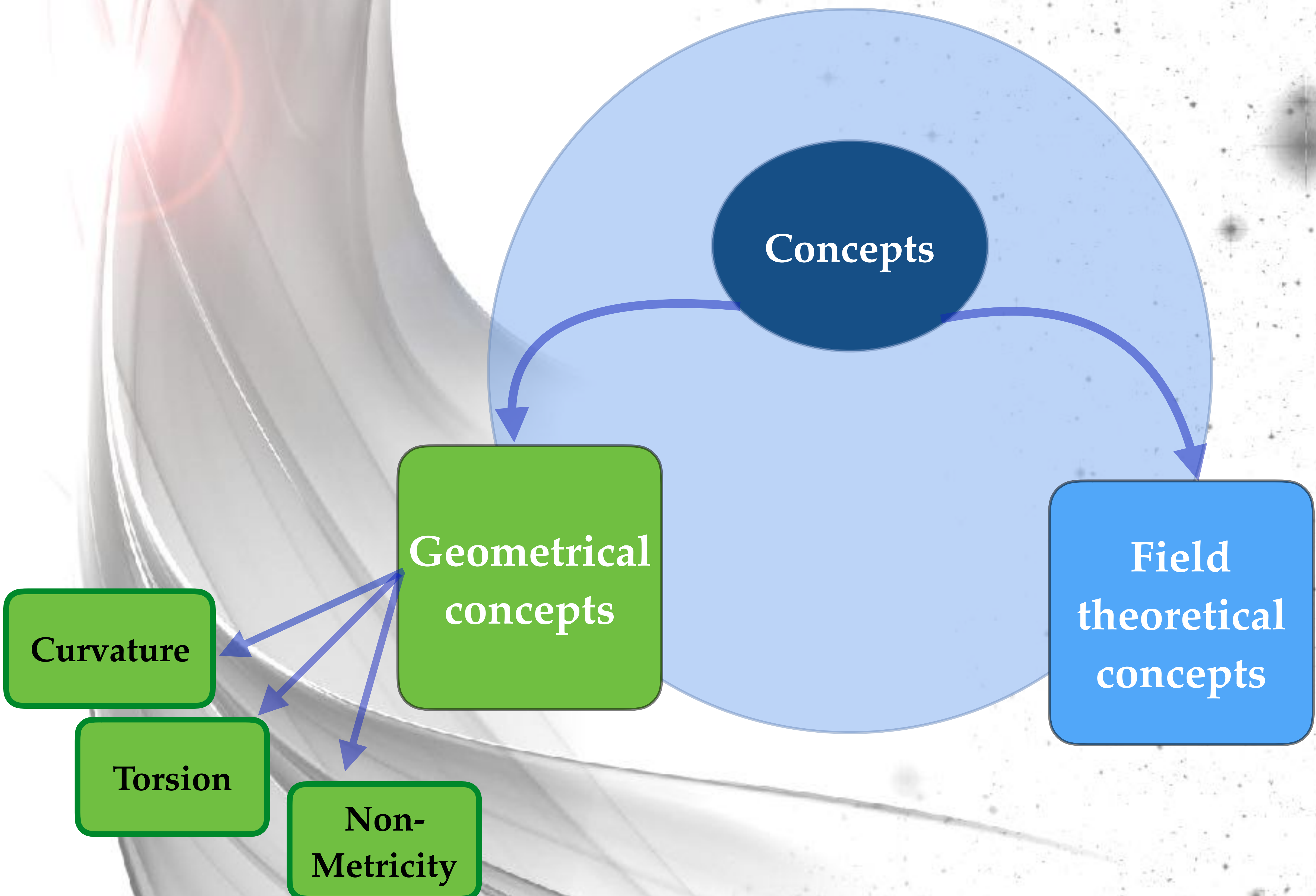
Different Interpretations of Gravity



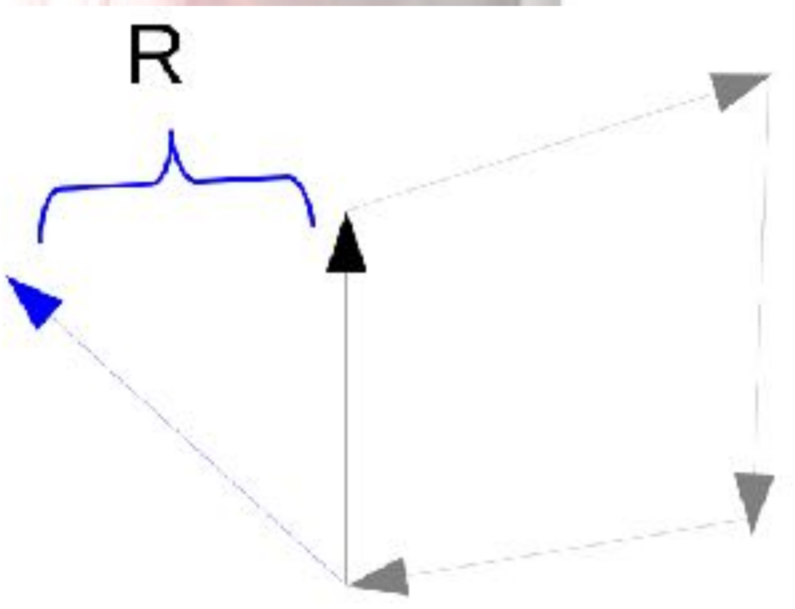
Gravity theories

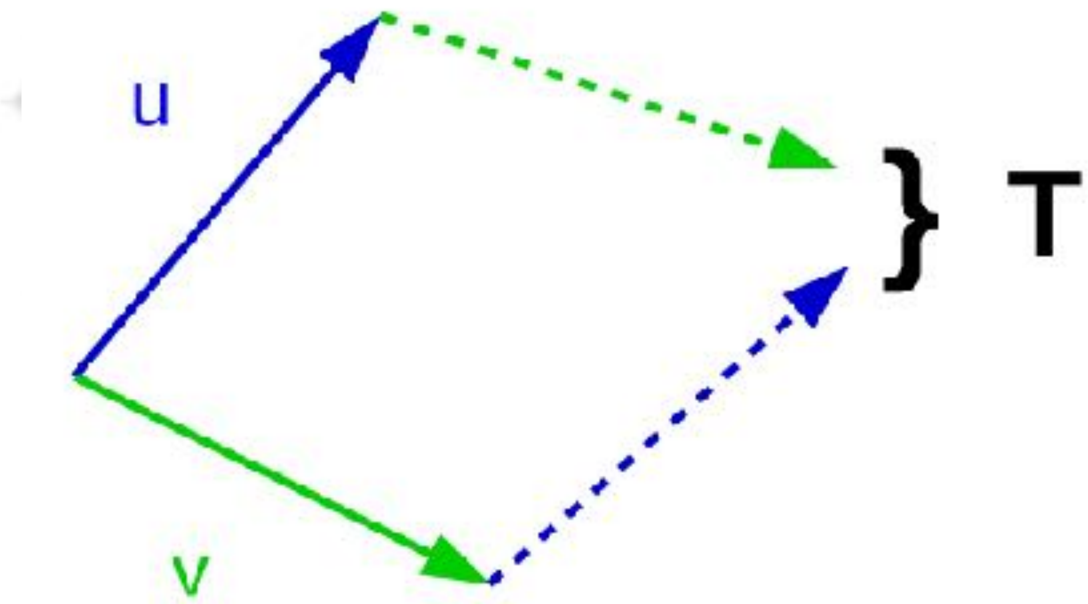


Gravity theories

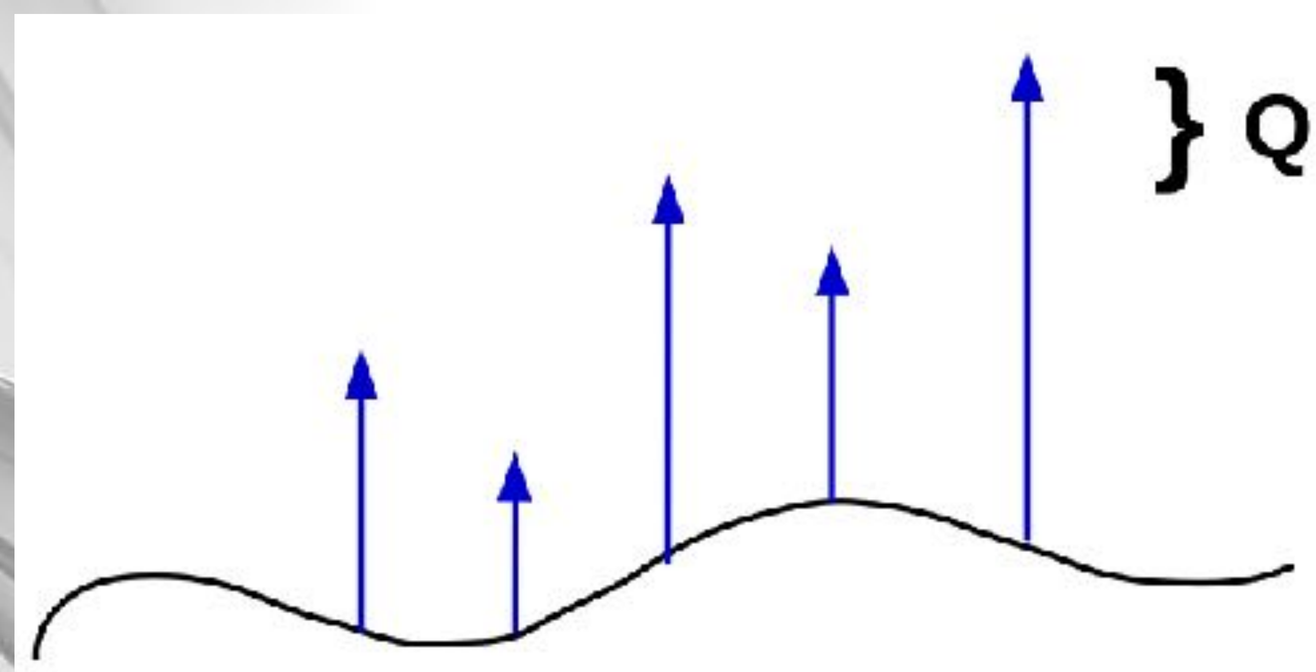


Geometrical objects





- **Riemann:** $R^{\alpha}_{\beta\mu\nu} \neq 0$
- **Torsion:** $T^{\alpha}_{\mu\nu} \neq 0$



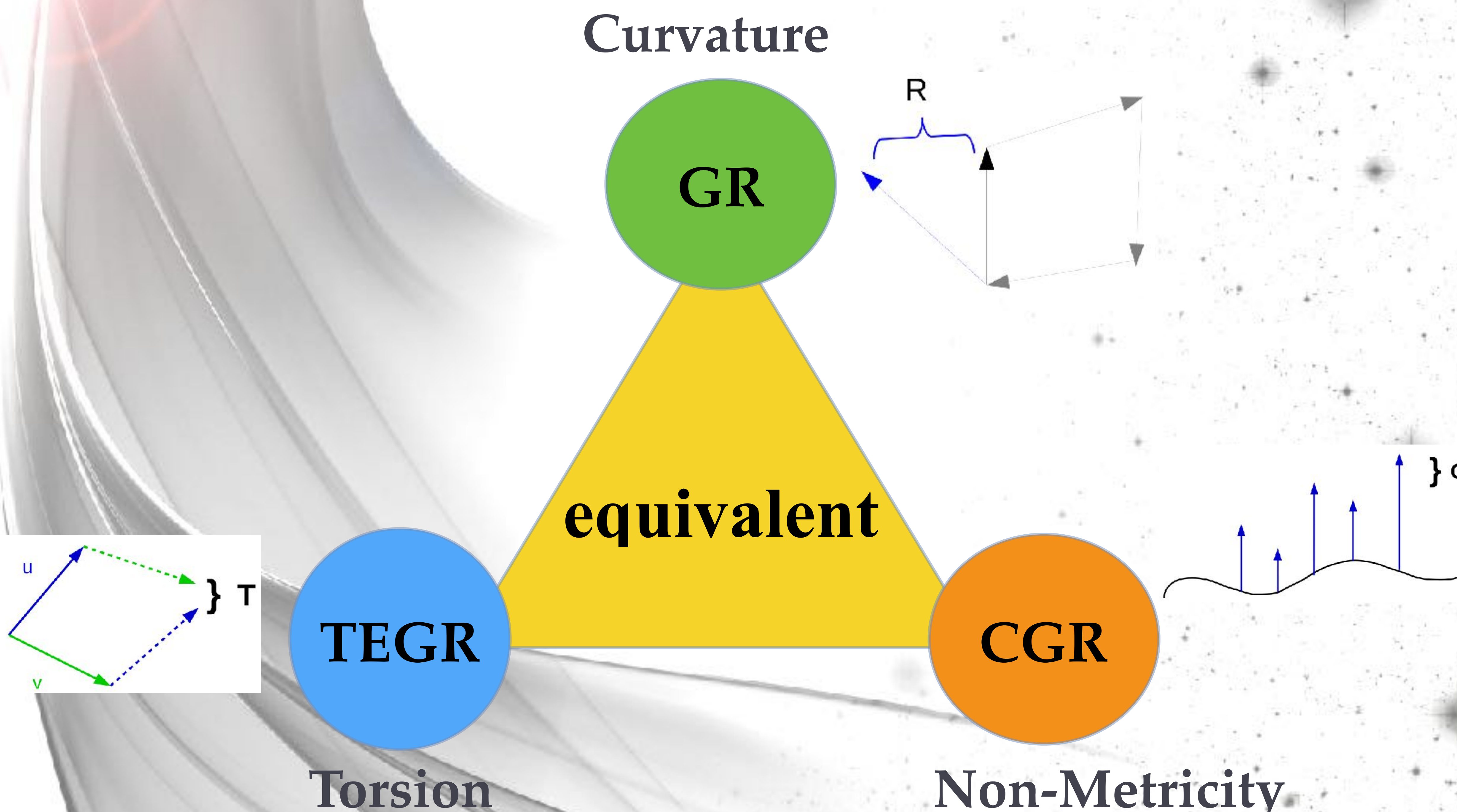
- **Non-metricity:** $Q^{\alpha}_{\mu\nu} \neq 0$

Geometrical Trinity of GR!

L.H & J.Beltran, T.Koivisto

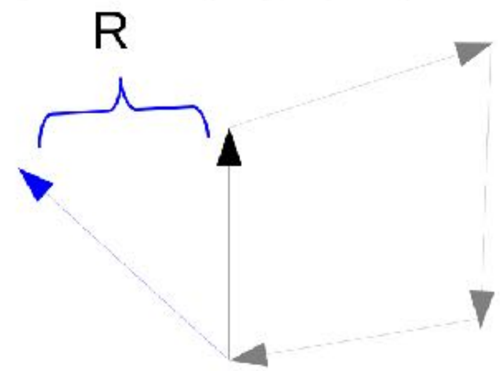
Universe 5 (2019) 7, 173,

arXiv:1903.06830



GR

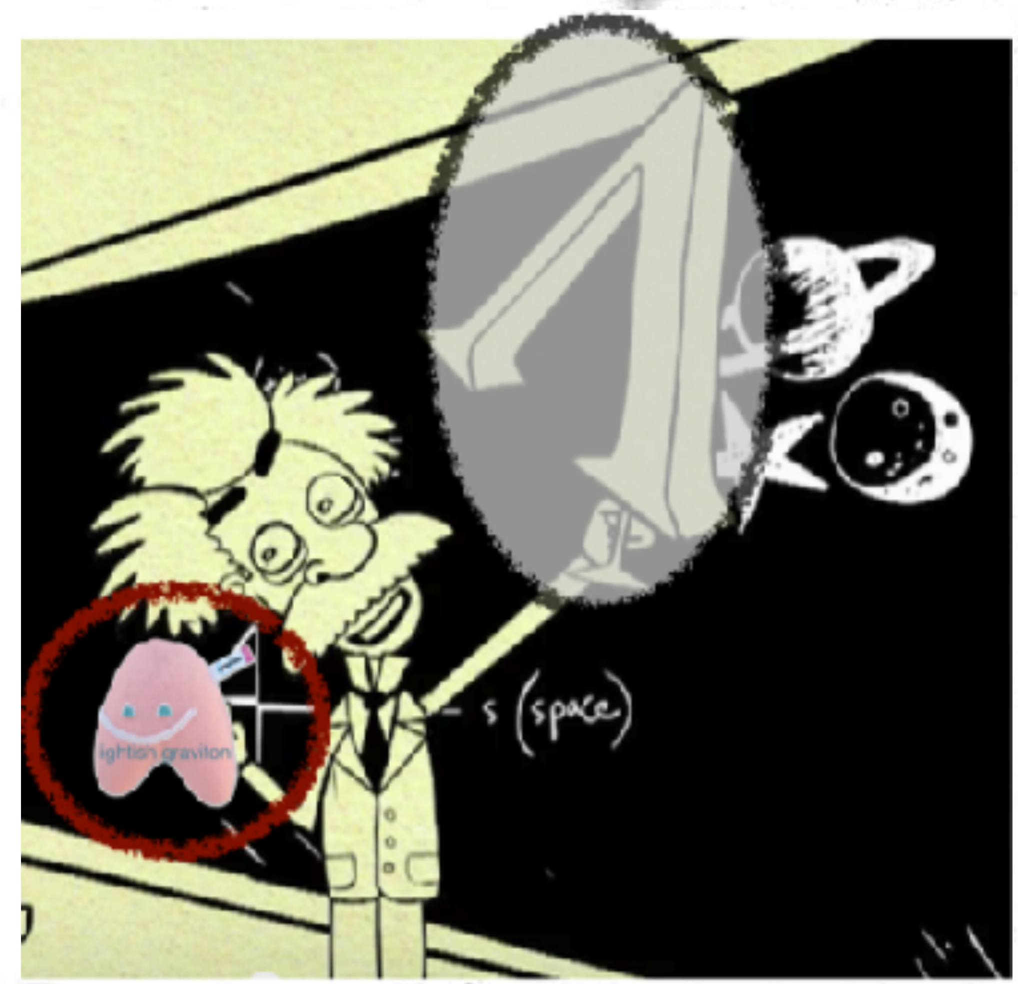
General Relativity (Curvature)



(pseudo-) Riemanian manifold

$g_{\mu\nu}$

$$\Gamma^{\alpha}_{\mu\nu} = \left\{ {\alpha \atop \mu\nu} \right\}$$



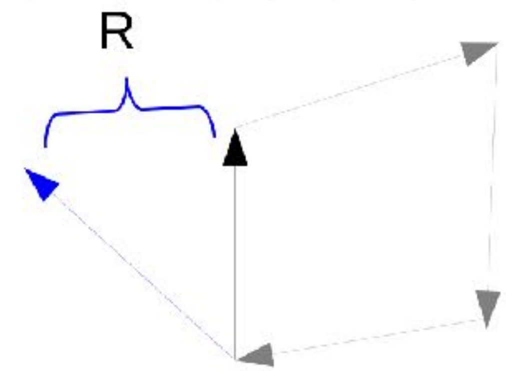
$$\left\{ {\alpha \atop \mu\nu} \right\} = \frac{1}{2} g^{\alpha\lambda} (\partial_\nu g_{\lambda\mu} + \partial_\mu g_{\lambda\nu} - \partial_\lambda g_{\mu\nu})$$

- **Riemann:** $R^{\alpha}_{\beta\mu\nu} \neq 0$
- **Torsion:** $T^{\alpha}_{\mu\nu} = 0$
- **Non-metricity:** $Q^{\alpha}_{\mu\nu} = 0$

$$Q^{\alpha}_{\mu\nu} = \nabla_\alpha g_{\mu\nu}$$

General Relativity (Curvature)

GR



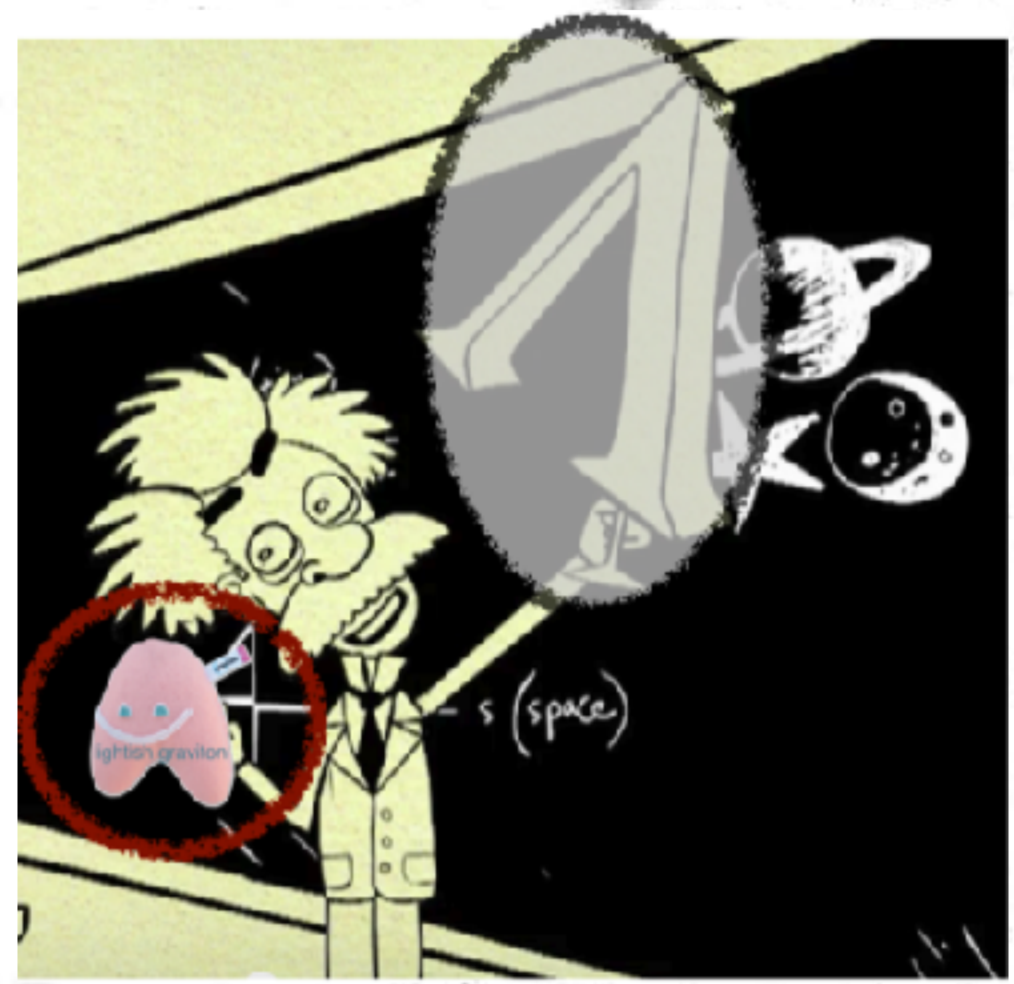
(pseudo-) Riemanian manifold

$g_{\mu\nu}$

$$\Gamma^{\alpha}_{\mu\nu} = \left\{ {\alpha \atop \mu\nu} \right\}$$

$$\left\{ {\alpha \atop \mu\nu} \right\} = \frac{1}{2} g^{\alpha\lambda} (\partial_\nu g_{\lambda\mu} + \partial_\mu g_{\lambda\nu} - \partial_\lambda g_{\mu\nu})$$

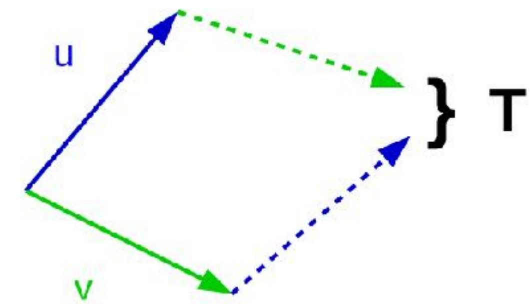
$$\mathcal{L}_g = \sqrt{-g} R + \sqrt{-g} \lambda$$



$$g_{\mu\nu} \ \ (10 \text{ components}) \ - 2 \times 4 \ \ (\text{gauge Diffs}) = 2 \ \ \text{dof}$$

TEGR (Torsion)

TEGR



a manifold based on torsion

$\Gamma^{a}_{\mu\nu}$

$$\Gamma^{a}_{\mu\nu} = \left\{ {}^{\alpha}_{\mu\nu} \right\} + K^{\alpha}_{\mu\nu}(T)$$

$$K^{\alpha}_{\mu\nu} = \frac{1}{2} T^{\alpha}_{\mu\nu} + T_{(\mu}{}^{\alpha}{}_{\nu)}$$ **contorsion tensor**

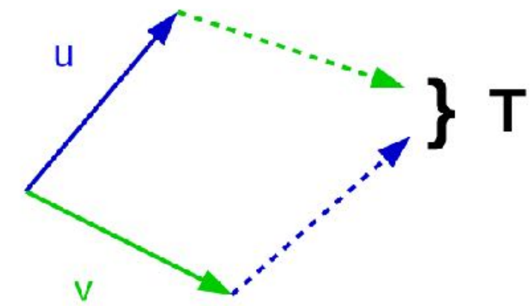
- **Riemann:** $R^{\alpha}_{\beta\mu\nu} = 0$
- **Torsion:** $T^{\alpha}_{\mu\nu} \neq 0$
- **Non-metricity:** $Q^{\alpha}_{\mu\nu} = 0$

$$T^{\lambda}_{\mu\nu} = (\Gamma^{\lambda}_{\mu\nu} - \Gamma^{\lambda}_{\nu\mu})$$

Torsion

TEGR (Torsion)

TEGR



a manifold based on torsion

$\Gamma^{a}_{\mu\nu}$

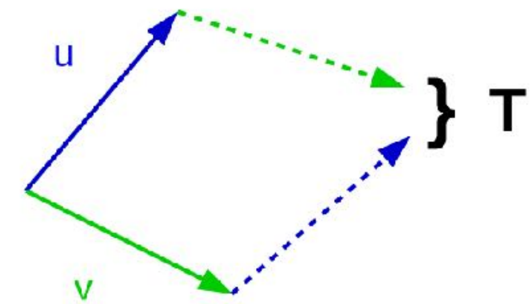
$$\Gamma^{a}_{\mu\nu} = \left\{ {\alpha \atop \mu\nu} \right\} + K^{\alpha}_{\mu\nu}(T)$$

- **Riemann:** $R^{\alpha}_{\beta\mu\nu} = 0$ $\Rightarrow$ $\Gamma^{\alpha}_{\mu\nu} = (\Lambda^{-1})^{\alpha}_{\lambda}\partial_{\mu}\Lambda^{\lambda}_{\nu}$
- **Non-metricity:** $Q^{\alpha}_{\mu\nu} = 0$ $\Rightarrow$ $g^{\lambda(\mu}\partial_{\alpha}\Lambda^{\nu)}_{\rho}(\Lambda^{-1})^{\rho}_{\lambda} = \frac{1}{2}\partial_{\alpha}g^{\mu\nu}$

$$T^{\alpha}_{\mu\nu} = 2(\Lambda^{-1})^{\alpha}_{\lambda}\partial_{[\mu}\Lambda^{\lambda}_{\nu]}$$

TEGR (Torsion)

TEGR



a manifold based on torsion

$\Gamma^{a}_{\mu\nu}$

$$\Gamma^{a}_{\mu\nu} = \left\{ {}^{\alpha}_{\mu\nu} \right\} + K^{\alpha}_{\mu\nu}(T)$$

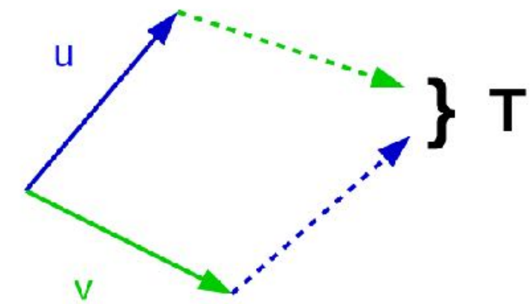
$$\mathcal{S} = \frac{1}{2}\sqrt{-g}\left(c_1 T_{\alpha}{}^{\mu\nu} T^{\alpha}{}_{\mu\nu} + c_2 T_{\alpha}{}^{\mu\nu} T_{\mu}{}^{\alpha}{}_{\nu} + c_3 T_{\mu} T^{\mu}\right) + \lambda_{\alpha}^{\beta\mu\nu} R^{\alpha}_{\beta\mu\nu} + \tilde{\lambda}_{\alpha}^{\mu\nu} Q_{\alpha}{}^{\mu\nu}$$

$$c_1 = \frac{1}{4}, \quad c_2 = \frac{1}{2}, \quad c_3 = -1 \qquad \rightarrow \text{equivalent to GR!}$$

Λ^{α}_{μ} (16 components) $- 2 \times 4$ (transl.gauge) $- 6$ (Lor. rot+boosts) $= 2$ dof

TEGR (Torsion)

TEGR



a manifold based on torsion

$\Gamma^{a}_{\mu\nu}$

$$\Gamma^{a}_{\mu\nu} = \left\{ {\alpha \atop \mu\nu} \right\} + K^{\alpha}_{\mu\nu}(T)$$

$$\mathcal{S} = \frac{1}{2}\sqrt{-g}\left(c_1 T_{\alpha}{}^{\mu\nu} T^{\alpha}{}_{\mu\nu} + c_2 T_{\alpha}{}^{\mu\nu} T_{\mu}{}^{\alpha}{}_{\nu} + c_3 T_{\mu} T^{\mu}\right) + \lambda_{\alpha}^{\beta\mu\nu} R^{\alpha}_{\beta\mu\nu} + \tilde{\lambda}_{\alpha}^{\mu\nu} Q_{\alpha}{}^{\mu\nu}$$

$$c_1 = \frac{1}{4}, \quad c_2 = \frac{1}{2}, \quad c_3 = -1 \qquad \rightarrow \text{equivalent to GR!}$$

$$R = \mathcal{R} + \mathring{\mathbb{T}} + 2\mathcal{D}_{\alpha} T^{\alpha}$$

$R = 0$

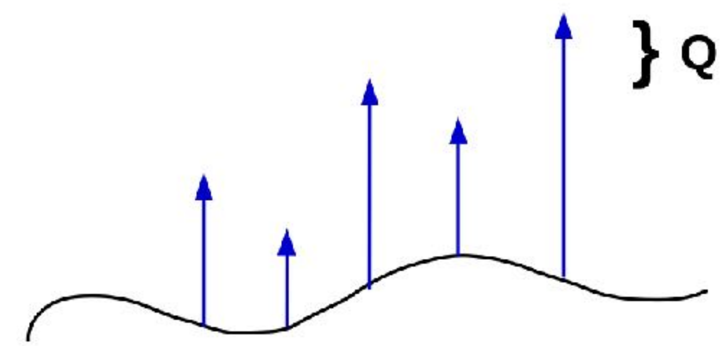
$$\Rightarrow \quad -\mathcal{R} = \mathring{\mathbb{T}} + 2\mathcal{D}_{\alpha} T^{\alpha}$$

CGR (Non-metricity)

CGR

L.H & J.Beltran, T.Koivisto
PRD98 (2018) 4, 044048,
arXiv:1803.10185

a manifold based on non-metricity



$g_{\mu\nu}, \Gamma^{a}_{\mu\nu}$

$$\Gamma^{a}_{\mu\nu} = \left\{ {\alpha \atop \mu\nu} \right\} + L^{\alpha}_{\mu\nu}(Q)$$

$$L^{\alpha}_{\mu\nu} = \frac{1}{2} Q^{\alpha}_{\mu\nu} - Q_{(\mu \ \ \nu)}{}^{\alpha}$$ **disformation tensor**

- **Riemann:** $R^{\alpha}_{\beta\mu\nu} = 0$
- **Torsion:** $T^{\alpha}_{\mu\nu} = 0$
- **Non-metricity:** $Q^{\alpha}_{\mu\nu} \neq 0$

$$\nabla_{\alpha} g_{\mu\nu} = Q_{\alpha\mu\nu}$$

Non-metricity

CGR (Non-metricity)

CGR

L.H & J.Beltran, T.Koivisto
PRD98 (2018) 4, 044048,
arXiv:1803.10185

a manifold based on non-metricity

$g_{\mu\nu}, \Gamma^{a}_{\mu\nu}$ $\quad \Gamma^{a}_{\mu\nu} = \left\{ {}^{\alpha}_{\mu\nu} \right\} + L^{\alpha}_{\mu\nu}(Q)$

- Riemann: $R^{\alpha}_{\beta\mu\nu} = 0 \rightarrow \Gamma^{\alpha}_{\mu\nu} = (\Lambda^{-1})^{\alpha}_{\lambda}\partial_{\mu}\Lambda^{\lambda}_{\nu}$
- Torsion: $T^{\alpha}_{\mu\nu} = 0 \rightarrow \Lambda^{\rho}_{\nu} = \partial_{\nu}\xi^{\rho}$

$$\Gamma^{\alpha}_{\mu\nu} = \left(\frac{\partial x^{\alpha}}{\partial \xi^{\rho}}\right)\partial_{\mu}\partial_{\nu}\xi^{\rho}$$ **Diffs**

the connection can be set to zero by means of a gauge choice

$$\Gamma^{\alpha}_{\mu\nu} = 0$$

CGR (Non-metricity)

CGR

L.H & J.Beltran, T.Koivisto
PRD98 (2018) 4, 044048,
arXiv:1803.10185

a manifold based on non-metricity

$g_{\mu\nu}, \Gamma^{a}_{\mu\nu}$

$$\Gamma^{a}_{\mu\nu} = \left\{ {\alpha \atop \mu\nu} \right\} + L^{\alpha}_{\mu\nu}(Q)$$

$$\Gamma^{\alpha}_{\mu\nu} = \left(\frac{\partial x^{\alpha}}{\partial \xi^{\rho}} \right) \partial_{\mu}\partial_{\nu}\xi^{\rho}$$

Diffs

the connection can be set to zero by means of a gauge choice

$$\Gamma^{\alpha}_{\mu\nu} = 0$$

- All points are equivalent!
- No inertial effects at all!
- From the full GL(4,R) we have reduced it to Diffs.

CGR (Non-metricity)

CGR

L.H & J.Beltran, T.Koivisto

PRD98 (2018) 4, 044048,
arXiv:1803.10185

a manifold based on non-metricity

$g_{\mu\nu}, \Gamma^a_{\mu\nu}$

$$\Gamma^a_{\mu\nu} = \left\{ {\alpha \atop \mu\nu} \right\} + L^\alpha_{\mu\nu}(Q)$$

$$\mathcal{S} = \int d^4x \sqrt{-g} \sum_{i=1}^{5} c_i Q_i^2 + \lambda_\alpha{}^{\beta\mu\nu} R^\alpha{}_{\beta\mu\nu} + \tilde{\lambda}_\alpha{}^{\mu\nu} T^\alpha{}_{\mu\nu}$$

$$c_1 = -c_3 = -\frac{1}{2} c_2 = -\frac{1}{2} c_5 = -\frac{1}{4} \text{ and } c_4 = 0 \quad \rightarrow \text{equivalent to GR!}$$

$$\sum_{i=1}^{5} c_i Q_i^2 = \frac{1}{4} Q_{\alpha\beta\mu} Q^{\alpha\beta\mu} - \frac{1}{2} Q_{\alpha\beta\mu} Q_{\beta\mu\alpha} - \frac{1}{4} Q_\alpha Q^\alpha + \frac{1}{2} Q_\alpha \tilde{Q}^\alpha$$

$$Q_\mu = Q_\mu{}^\alpha{}_\alpha \text{ and } \tilde{Q}^\mu = Q_\alpha{}^{\mu\alpha}$$

CGR (Non-metricity)

CGR

L.H & J.Beltran, T.Koivisto
PRD98 (2018) 4, 044048,
arXiv:1803.10185

a manifold based on non-metricity

$g_{\mu\nu}, \Gamma^a_{\mu\nu}$

$$\Gamma^a_{\mu\nu} = \left\{ {\alpha \atop \mu\nu} \right\} + L^\alpha_{\mu\nu}(Q)$$

$$\mathcal{S} = \int d^4x \sqrt{-g} \sum_{i=1}^{5} c_i Q_i^2 + \lambda_\alpha{}^{\beta\mu\nu} R^\alpha{}_{\beta\mu\nu} + \tilde{\lambda}_\alpha{}^{\mu\nu} T^\alpha{}_{\mu\nu}$$

$$\sum_{i=1}^{5} c_i Q_i^2 = \frac{1}{4} Q_{\alpha\beta\mu} Q^{\alpha\beta\mu} - \frac{1}{2} Q_{\alpha\beta\mu} Q_{\beta\mu\alpha} - \frac{1}{4} Q_\alpha Q^\alpha + \frac{1}{2} Q_\alpha \tilde{Q}^\alpha$$

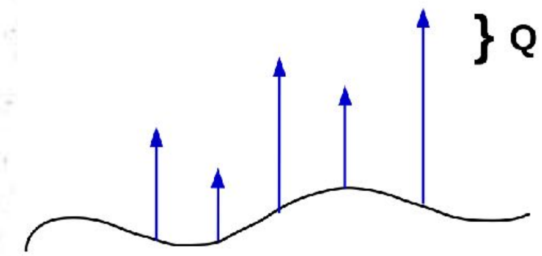
$$R = \mathcal{R} + \mathcal{Q} + \mathcal{D}_\alpha (Q^\alpha - \tilde{Q}^\alpha)$$

$\uparrow$ $= 0$

$$\rightarrow \quad -\mathcal{R} = \mathcal{Q} + \mathcal{D}_\alpha (Q^\alpha - \tilde{Q}^\alpha)$$

CGR

CGR (Non-metricity)



a manifold based on non-metricity

$g_{\mu\nu}, \Gamma^{a}_{\mu\nu}$

$$\Gamma^{a}_{\mu\nu} = \left\{ {\alpha \atop \mu\nu} \right\} + L^{\alpha}_{\mu\nu}(Q)$$

$$\sum_{i=1}^{5} c_i Q_i^2 = \frac{1}{4} Q_{\alpha\beta\mu} Q^{\alpha\beta\mu} - \frac{1}{2} Q_{\alpha\beta\mu} Q_{\beta\mu\alpha} - \frac{1}{4} Q_\alpha Q^\alpha + \frac{1}{2} Q_\alpha \tilde{Q}^\alpha$$

in the coincident gauge

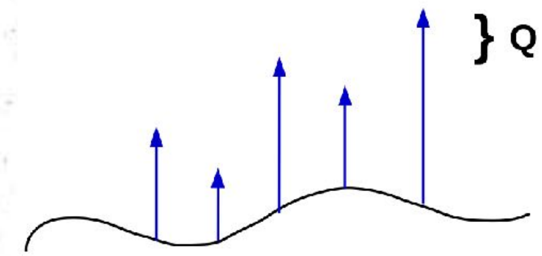
$$\Gamma^{\alpha}_{\mu\nu} = \left\{ {\alpha \atop \mu\nu} \right\} + L^{\alpha}_{\mu\nu}(Q) = 0 \;\; \rightarrow \;\; -\left\{ {\alpha \atop \mu\nu} \right\} = L^{\alpha}_{\mu\nu}$$

$$\sum_{i=1}^{5} c_i Q_i^2 = g^{\mu\nu} \left(\left\{ {\alpha \atop \beta\mu} \right\} \left\{ {\beta \atop \nu\alpha} \right\} - \left\{ {\alpha \atop \beta\alpha} \right\} \left\{ {\beta \atop \mu\nu} \right\} \right)$$

the action is the {}{} part of R({})

CGR (Non-metricity)

CGR



a manifold based on non-metricity

$g_{\mu\nu}, \Gamma^a_{\mu\nu}$

$$\Gamma^a_{\mu\nu} = \left\{ {\alpha \atop \mu\nu} \right\} + L^\alpha_{\mu\nu}(Q) \quad = 0$$

$$\sum_{i=1}^{5} c_i Q_i^2 = g^{\mu\nu}\left(\left\{ {\alpha \atop \beta\mu} \right\} \left\{ {\beta \atop \nu\alpha} \right\} - \left\{ {\alpha \atop \beta\alpha} \right\} \left\{ {\beta \atop \mu\nu} \right\} \right)$$

- No need for GHY boundary term for a well-defined variational principle
- More direct contact with (the most fundamental) field theory description (Deser's resummation approach)
- Improved and unambigous entropy of BHs
- New tool to canonically quantize GR?

Modified gravity (geometrical perspective)

- Promote the scalars to general functions

$$\int \sqrt{-q}\mathcal{R} \quad \rightarrow \quad \int \sqrt{-q} f(\mathcal{R})$$

$$\int \sqrt{-q}\mathring{\mathbb{T}} \quad \rightarrow \quad \int \sqrt{-q} f(\mathring{\mathbb{T}})$$

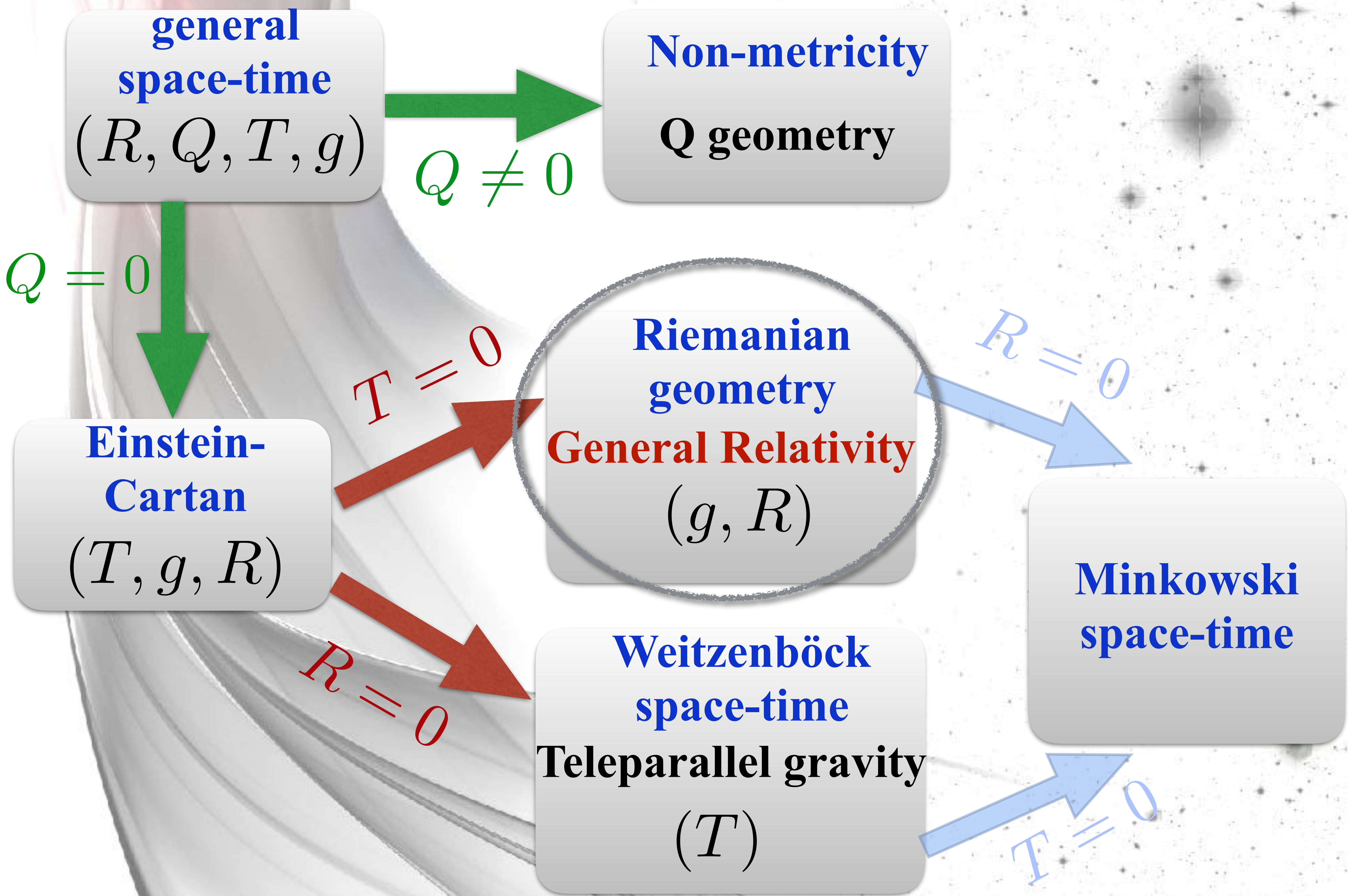
$$\int \sqrt{-q}\mathring{\mathcal{Q}} \quad \rightarrow \quad \int \sqrt{-q} f(\mathring{\mathcal{Q}})$$

- Other consistent quadratic actions? c_i

$$\mathcal{S} = \int d^4x \sqrt{-g} \sum_{i=1}^{5} c_i Q_i^2 + \lambda_\alpha{}^{\beta\mu\nu} R^\alpha{}_{\beta\mu\nu} + \tilde{\lambda}_\alpha{}^{\mu\nu} T^\alpha{}_{\mu\nu}$$

$$\mathcal{S} = \frac{1}{2}\sqrt{-g}\left(c_1 T_\alpha{}^{\mu\nu} T^\alpha{}_{\mu\nu} + c_2 T_\alpha{}^{\mu\nu} T_\mu{}^\alpha{}_\nu + c_3 T_\mu T^\mu\right) + \lambda_\alpha^{\beta\mu\nu} R^\alpha_{\beta\mu\nu} + \tilde{\lambda}_\alpha^{\mu\nu} Q_\alpha{}^{\mu\nu}$$

Geometrical setup

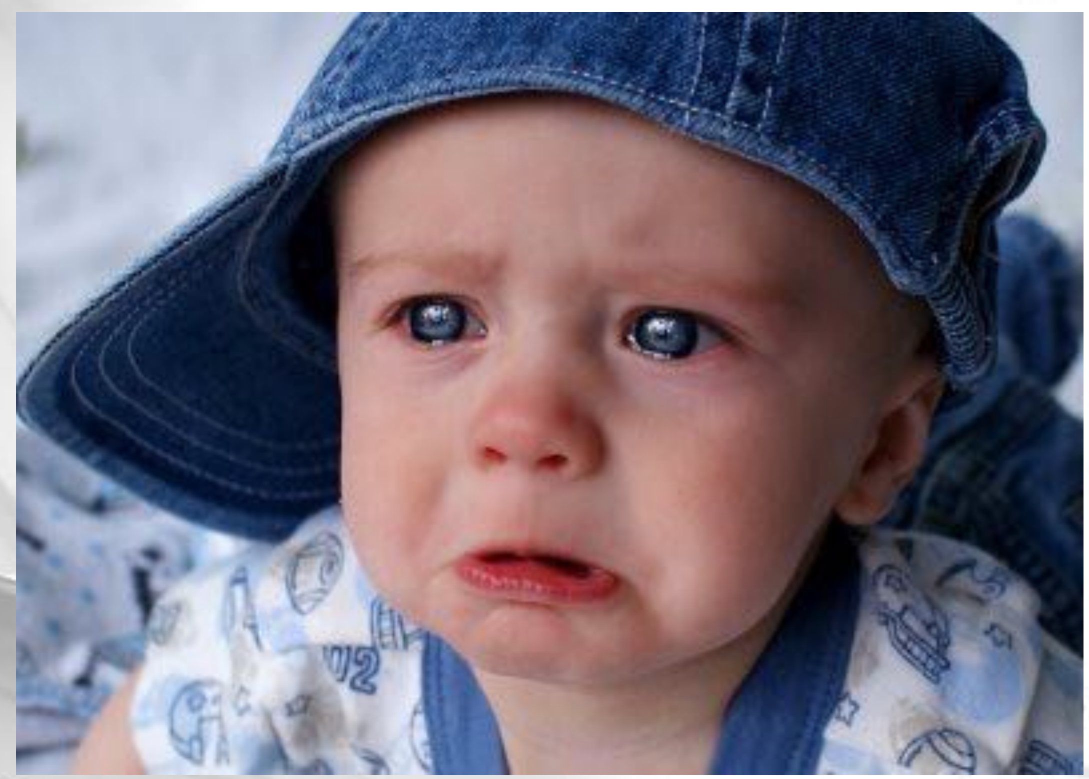


General Relativity (field theory perspective)

Imagine a universe in which Einstein did not exist!

General Relativity (field theory perspective)

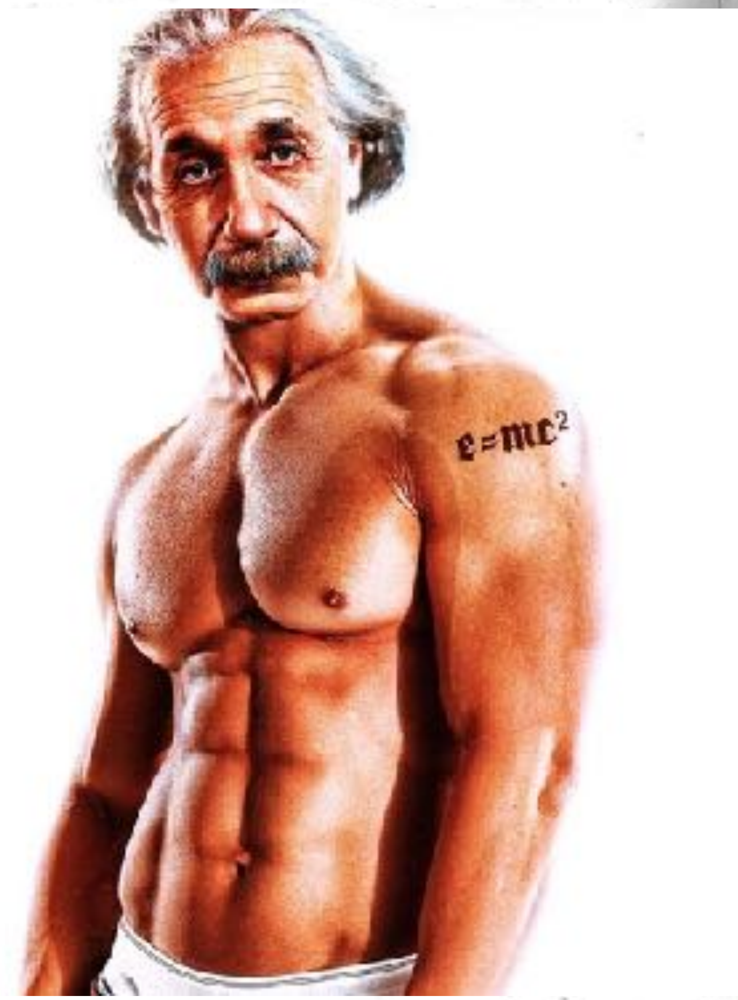
Imagine a universe in which Einstein did not exist!



General Relativity (field theory perspective)

Imagine a modern version of Einstein

Who would have learned
all the standard techniques
of field theory description



We would have eventually constructed GR from field theory perspective, just years later

General Relativity (field theory perspective)

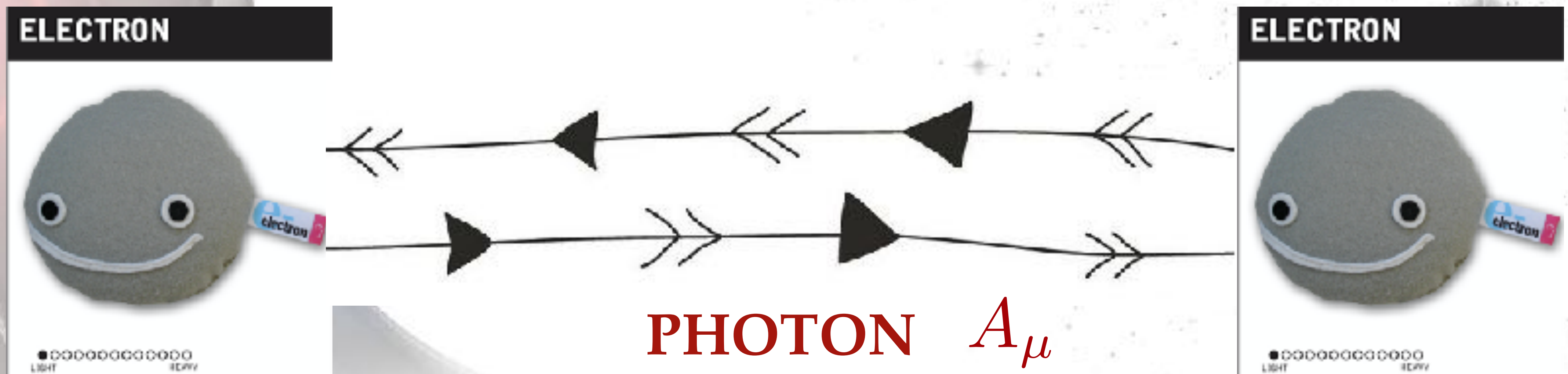
Imagine a modern version of Einstein

Who would have learned
all the standard techniques
of field theory description

We would have eventually constructed GR from field theory perspective, just years later

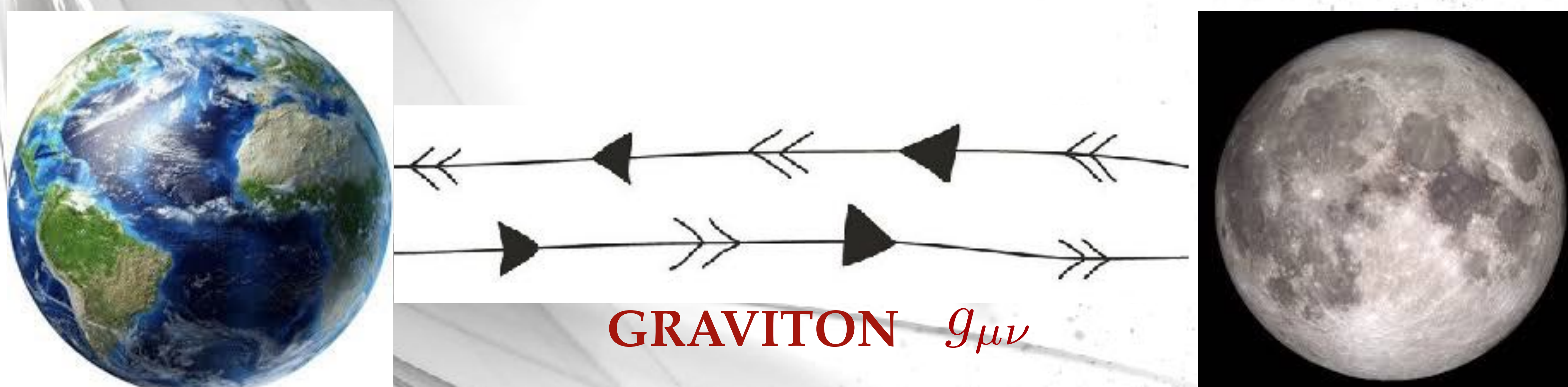
Field Theory Perspective

- Electromagnetic interactions



PHOTON A_μ

- Gravitational interactions



GRAVITON $g_{\mu\nu}$

GR corresponds to interactions based on a massless spin 2 field

massless Spin 2 Field

In 4 dimensions the only Lovelock invariants are

- Cosmological constant $\sqrt{-g}$
- Ricci scalar R
- Gauss-Bonnet term $\mathcal{L}_{GB}$

  (topological)

$$\mathcal{L}_{GB} = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$

$$\mathcal{L}_g = \sqrt{-g}M_{\rm Pl}^2 R + \sqrt{-g}\Lambda$$

GR

massless Spin 2 Field

Related to the Lovelock invariants one can construct **divergenceless** tensors

equations of motions

- **Cosmological constant** $\sqrt{-g}$ $\longrightarrow$ $g_{\mu\nu}$
- **Ricci scalar** R $\longrightarrow$ $G_{\mu\nu}$
- **Gauss-Bonnet term** (topological) $\mathcal{L}_{GB}$ $\not\longrightarrow$ None

massless Spin 2 Field

How can the matter fields couple to gravity?

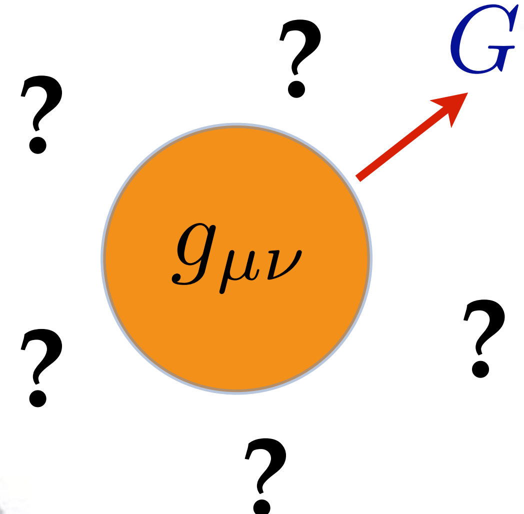
	Lovelock invariants			Divergenceless tensors	
They can couple to:	$\sqrt{-g}$	(minimal)	and	$g_{\mu\nu}$	(minimal)
	R	(non-minimal)	and	$G_{\mu\nu}$	(non-minimal)

Actually one can construct yet another divergenceless tensor in 4dim

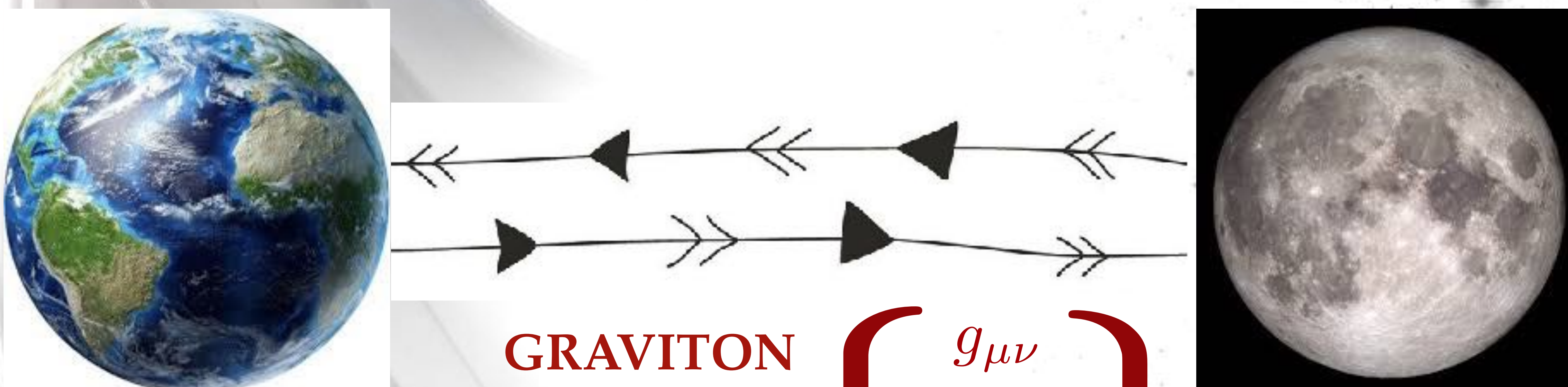
$$\mathcal{L}^{\mu\nu\alpha\beta} = -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\delta\gamma}R_{\rho\sigma\delta\gamma}$$ (non-minimal)

↑ double dual Riemann tensor

Modified Gravity

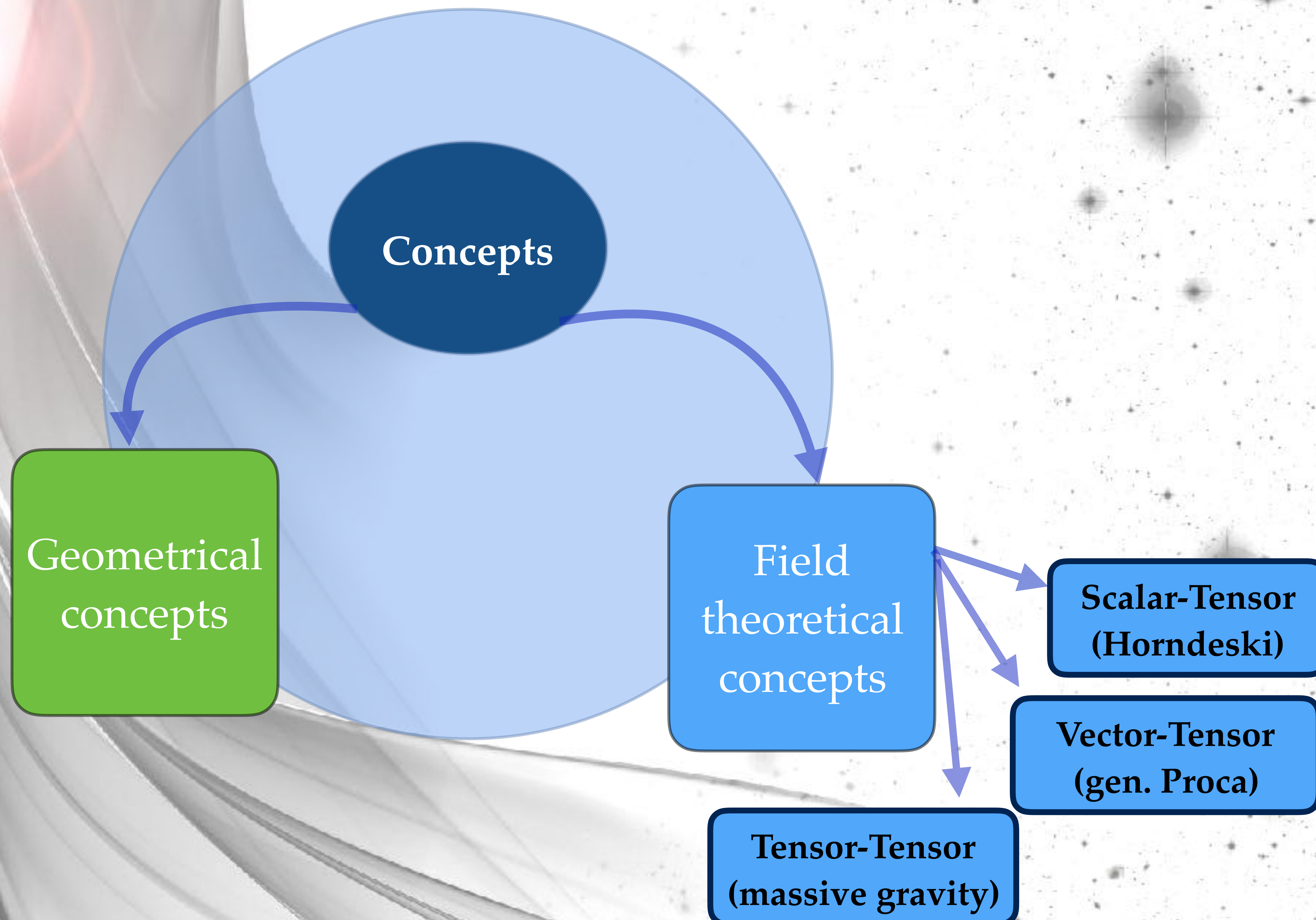


$$G_{\mu\nu} = \frac{T_{\mu\nu}}{M_{\rm Pl}^2}$$

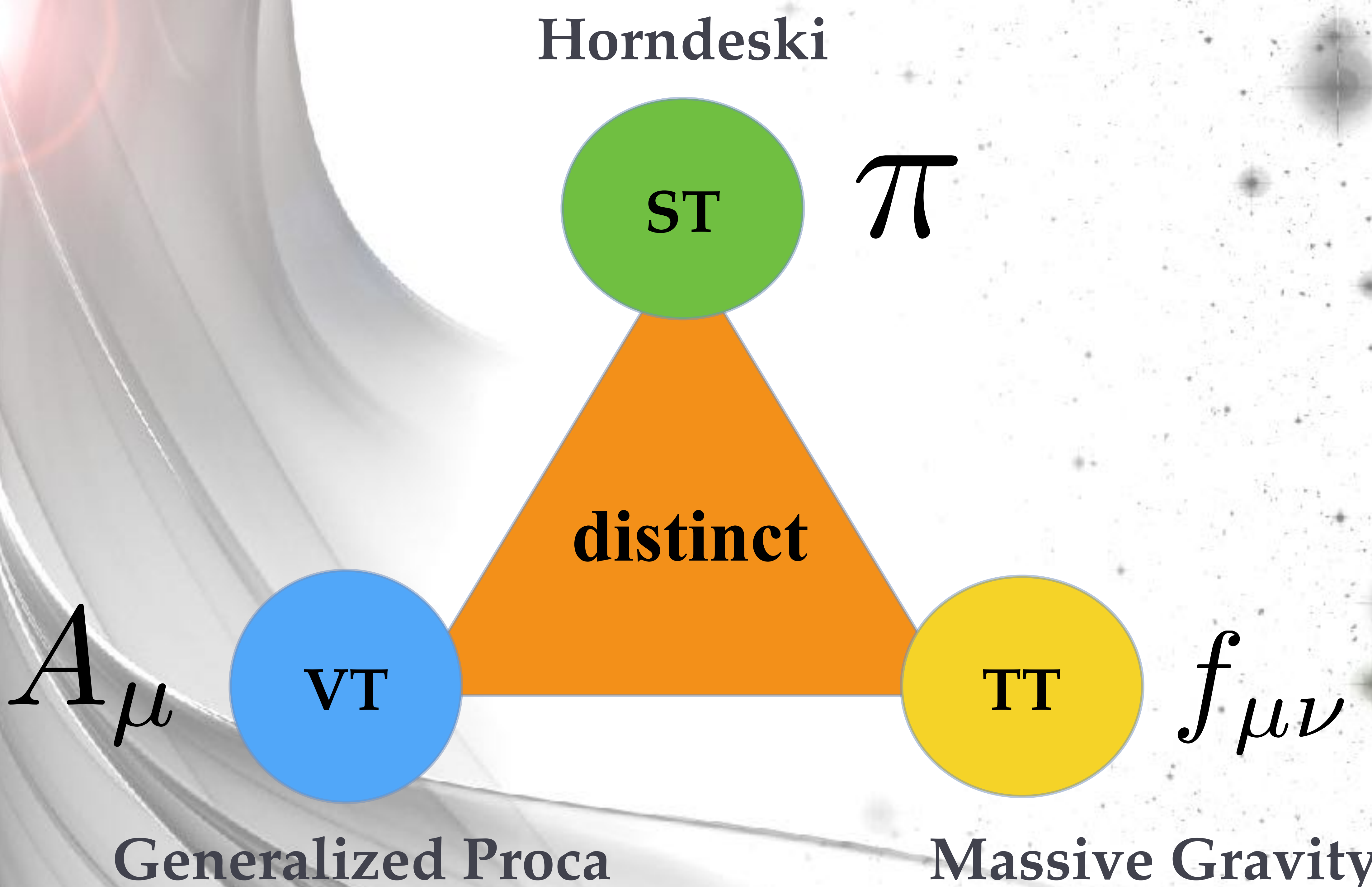


GRAVITON $\left\{ \begin{array}{c} g_{\mu\nu} \\ f_{\mu\nu} \\ A_{\mu} \\ \phi \end{array} \right\}$

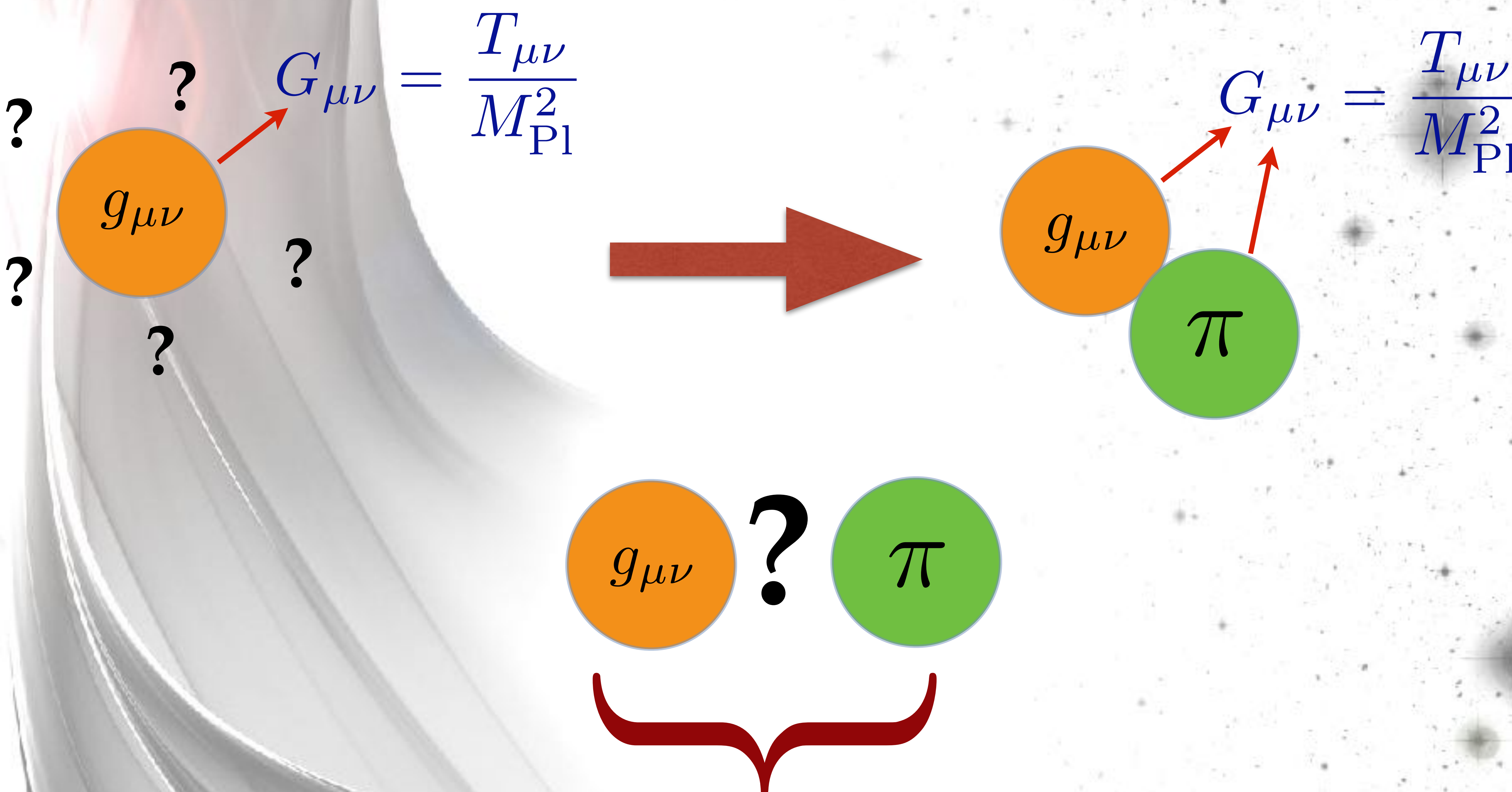
Gravity theories



Field Theoretical Trinity of Gravity



Horndeski theory (scalar-tensor theory)



how do they couple???

Scalar-Tensor-interactions

The generalisation to curved space-time yield the rediscovery of Horndeski interactions

G.W.Horndeski Int. J. Theo.Phys. 10, 363-384 (1974)
C.Deffayet, Esposito-Farese,Vikman
Phys. Rev.D (79), 084003 (2009)
T. Kobayashi, M. Yamaguchi, J. Yokoyama,
Prog. Theor. Phys. 126 (2011),511-529

$$\Pi_{\mu\nu} = \nabla_\mu \partial_\nu \pi$$

$$X = -\frac{1}{2}(\partial\pi)^2$$

$$\mathcal{L}_2 = K(\pi, X)$$

$$\mathcal{L}_3 = G_3(\pi, X)[\Pi]$$

$$\mathcal{L}_4 = G_4(\pi, X)R + G_{4,X}([\Pi]^2 - [\Pi^2])$$

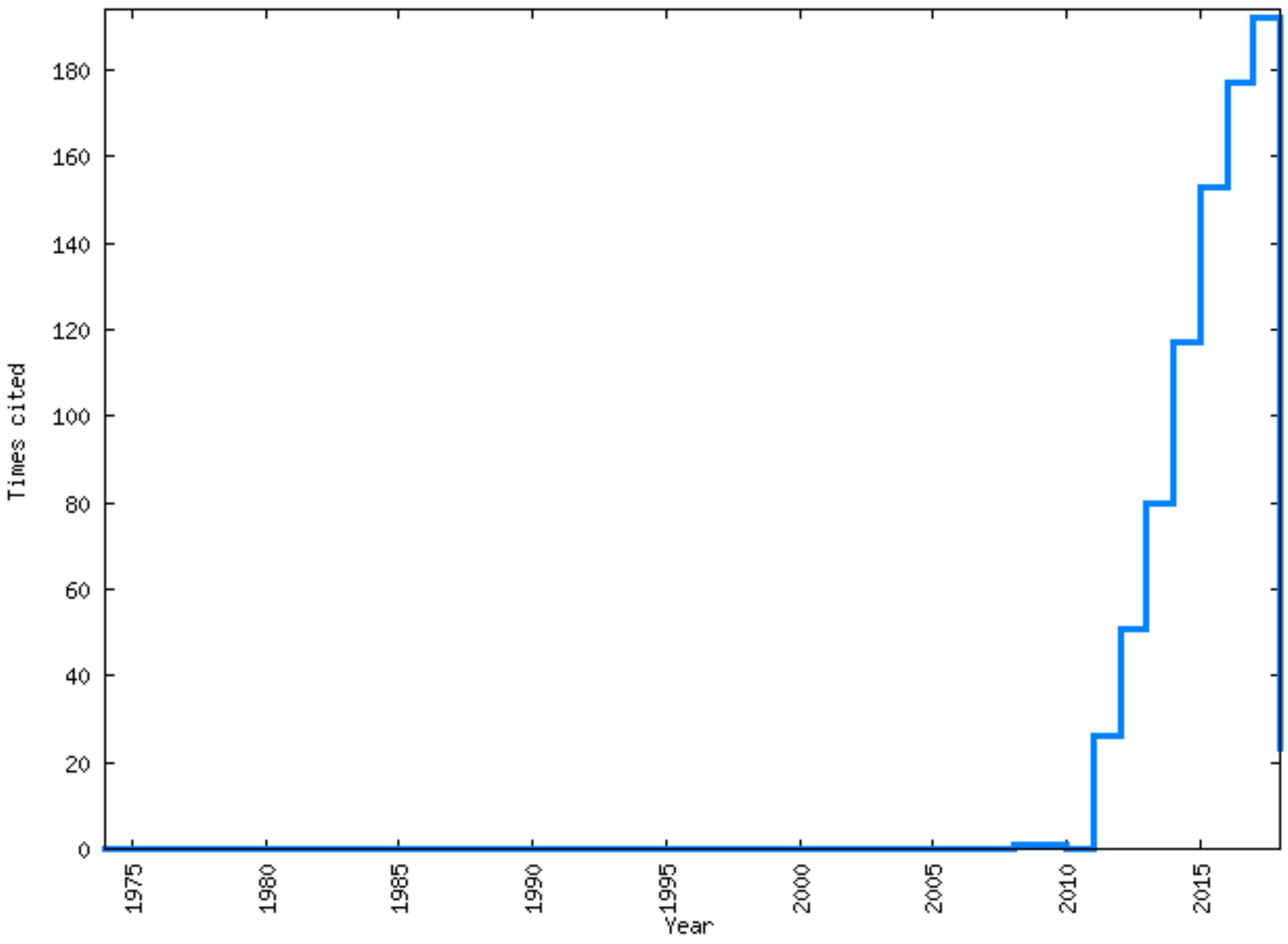
$$\mathcal{L}_5 = G_5(\pi, X)G_{\mu\nu}\Pi^{\mu\nu} - \frac{G_{5,X}}{6}([\Pi]^3 - 3[\Pi][\Pi^2] + 2[\Pi^2])$$

Non-minimal couplings to gravity have to be added to maintain second order equations of motions

Scalar-Tensor-interactions

The generalisation to curved space-time yield the rediscovery of Horndeski interactions

G.W.Horndeski Int. J. Theo.Phys. 10, 363-384 (1974)
C.Deffayet, Esposito-Farese,Vikman Phys. Rev.D (79), 084003 (2009)



$$\Pi_{\mu\nu} = \nabla_\mu \partial_\nu \pi$$

$$X = -\frac{1}{2}(\partial\pi)^2$$

$$- 3[\Pi][\Pi^2] + 2[\Pi^2])$$

e added to maintain

Scalar-Tensor-interactions

Allowing higher order derivatives in the equations of motion

→ **Beyond Horndeski**

J.Gleyzes, D.Langlois, F.Piazza, F.Vernizzi Phys. Rev.Lett 114(2015), 21,211101

HEP Encontrados **11** registros

1. **Lagrange Multipliers and Third Order Scalar-Tensor Field Theories**
   Gregory W. Horndeski. Aug 9, 2016. 43 pp.
   e-Print: arXiv:1608.03212 [gr-qc] | PDF
   References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
   ADS Abstract Service
   Registro completo

2. **The Relationship Between Inertial and Gravitationa**
   Gregory W. Horndeski. Feb 28, 2016. 7 pp.
   e-Print: arXiv:1602.08516 [physics.gen-ph] | PDF
   References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac |
   ADS Abstract Service
   Registro completo

3. **A Simple Theory of Quantum Gravity**
   Gregory W. Horndeski. Aug 22, 2015. 54 pp.
   e-Print: arXiv:1508.06180 [physics.gen-ph] | PDF
   References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac |
   ADS Abstract Service
   Registro completo - Citado por 1 registro

4. **Effective Determinism In A Classical Field Theory With Space - Like Characteristics**
   J. Isenberg (Oregon U.), G. Horndeski (Waterloo U.). 1986. 7 pp.
   Published in **J.Math.Phys. 27 (1986) 739-745**
   DOI: 10.1063/1.527176
   References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
   ADS Abstract Service
   Registro completo - Citado por 3 registros

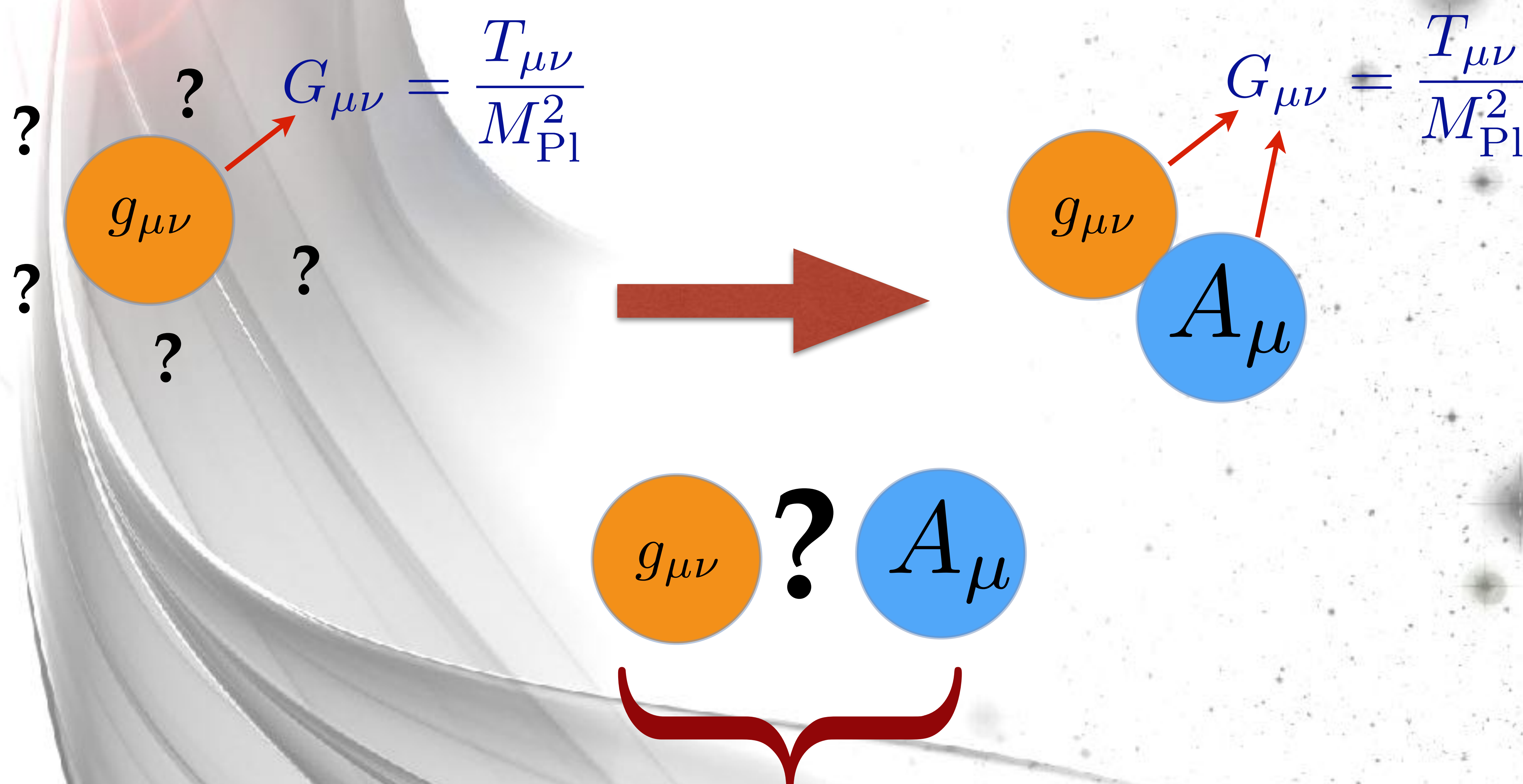
ACKNOWLEDGEMENTS

I wish to thank Dr. A.Guarnizo Trillera for presenting me with a copy of his Ph.D. thesis [27]. This thesis provided me with an introduction to the efforts of those people who were trying to find third-order scalar-tensor field theories, the so-called "Beyond Horndeski Theories." Well, so long as I am alive, no one goes beyond Horndeski without me! And that explains the inception of this paper.

Scalar-Tensor-interactions

- Horndeski
- Beyond Horndeski
- Beyond Beyond Horndeski
- DHOST

Vector-Tensor Theories



how do they couple???

Generalized Proca action

Interactions on curved space-time requires the presence of non-minimal couplings to gravity

L.H & J.Beltran, Phys.Lett.B757 (2016) 405-411, arXiv:1602.03410

L. H., JCAP 1405, 015 (2014), arXiv:1402.7026

G.Tasinato JHEP 1404 (2014)067 arXiv:1402.6450

Allys, Peter, Rodriguez, JCAP 1602 (2016) 02, 004

$$Y = -\frac{1}{2}A_\mu A^\mu$$

$$\mathcal{L}_2 = G_2(A_\mu, F_{\mu\nu}, \tilde{F}_{\mu\nu})$$

$$\mathcal{L}_3 = G_3(Y)\nabla_\mu A^\mu$$

$$\mathcal{L}_4 = G_4(Y)R + G_{4,Y}\left[(\nabla_\mu A^\mu)^2 - \nabla_\rho A_\sigma \nabla^\sigma A^\rho\right]$$

$$\mathcal{L}_5 = G_5(Y)G_{\mu\nu}\nabla^\mu A^\nu - \frac{1}{6}G_{5,Y}\Big[(\nabla\cdot A)^3 + 2\nabla_\rho A_\sigma \nabla^\gamma A^\rho \nabla^\sigma A_\gamma - 3(\nabla\cdot A)\nabla_\rho A_\sigma \nabla^\sigma A^\rho\Big] - \tilde{G}_5(Y)\tilde{F}^{\alpha\mu}\tilde{F}^\beta{}_\mu \nabla_\alpha A_\beta$$

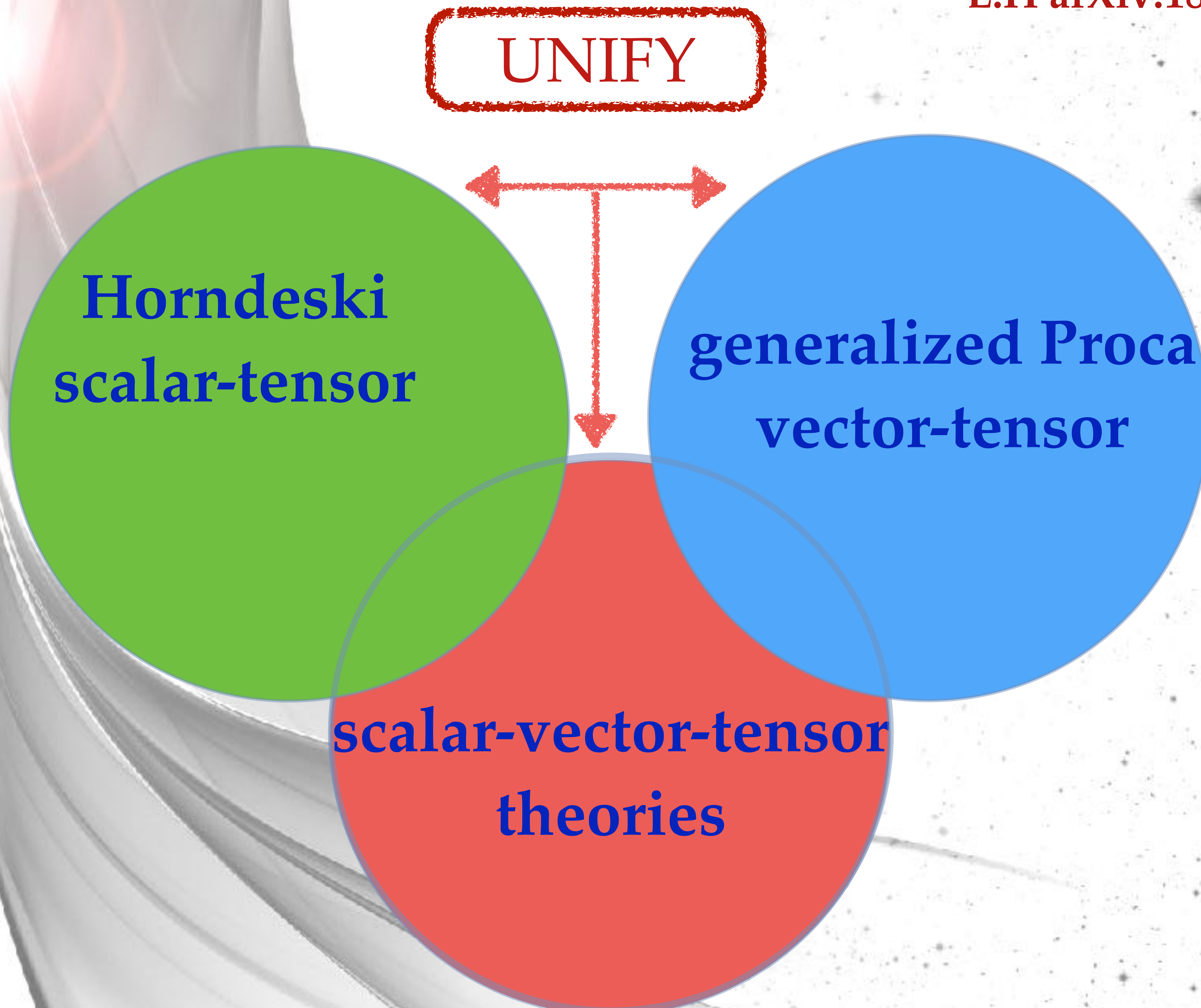
$$\mathcal{L}_6 = G_6(Y)L^{\mu\nu\alpha\beta}\nabla_\mu A_\nu \nabla_\alpha A_\beta + \frac{G_{6,Y}}{2}\tilde{F}^{\alpha\beta}\tilde{F}^{\mu\nu}\nabla_\alpha A_\mu \nabla_\beta A_\nu$$

Vector-Tensor Theories

- Generalized Proca
- Extended Vector Theories
- Beyond Generalized Proca

Scalar-Vector-Tensor Theories

L.H arXiv:1801.01523

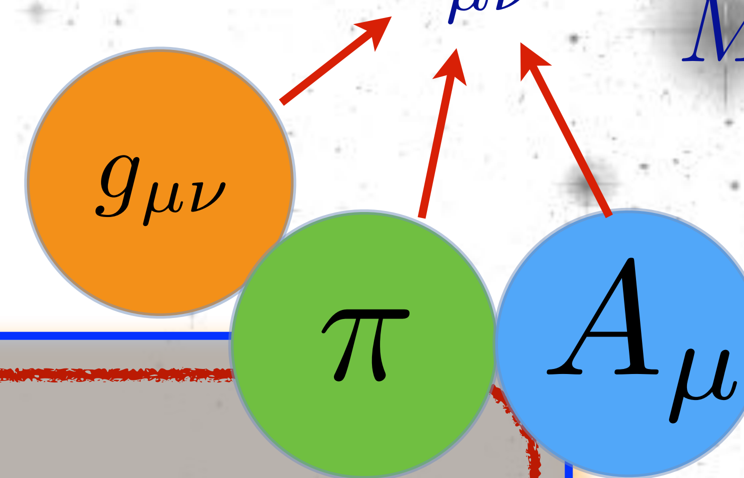


Scalar-Vector-Tensor Theories

L.H arXiv:1801.01523

Gauge-invariant

$$G_{\mu\nu} = \frac{T_{\mu\nu}}{M_{\rm Pl}^2}$$



$$\mathcal{L}^2_{\rm SVT} = f_2(\pi, X, F, \tilde{F}, Y)$$

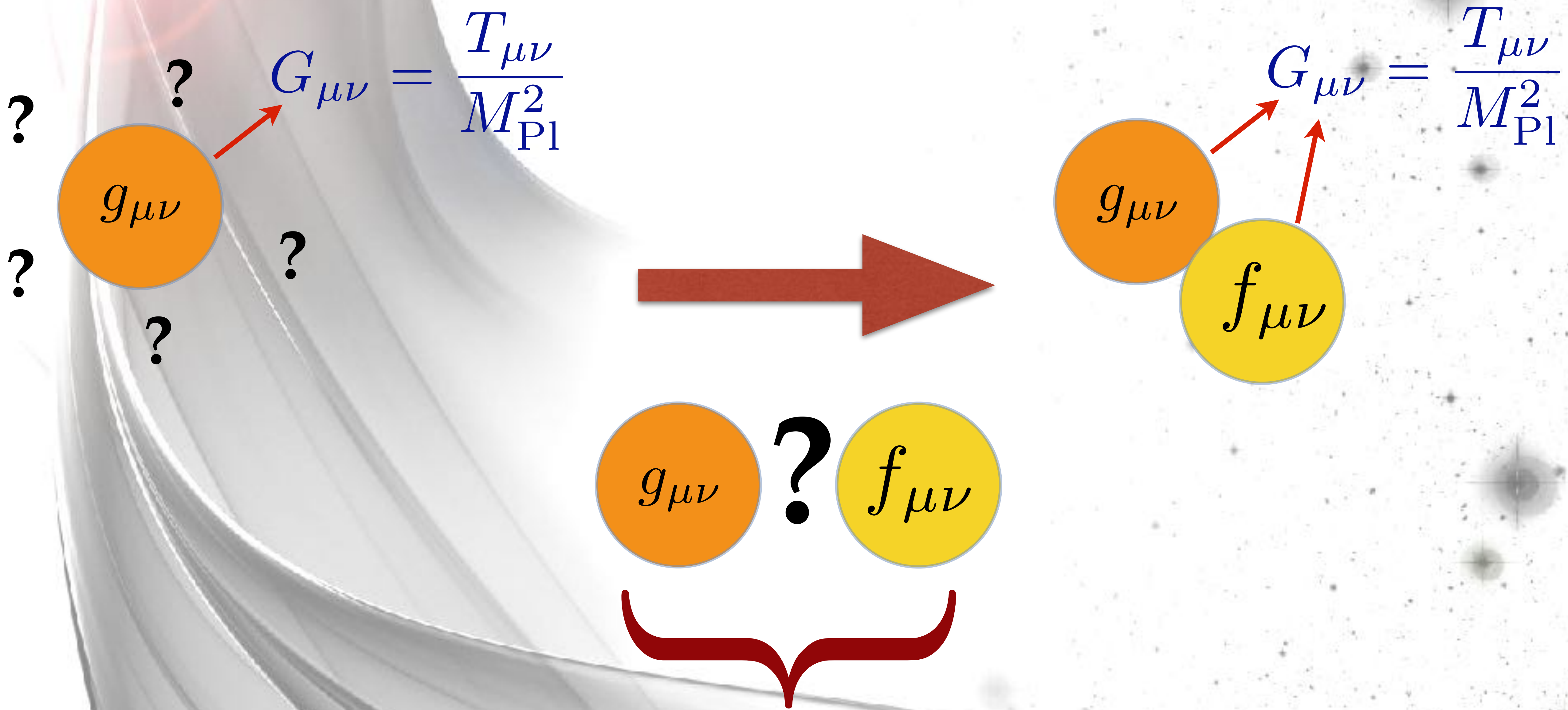
$$\mathcal{L}^3_{\rm SVT} = \mathcal{M}_3^{\mu\nu}\nabla_\mu\partial_\nu\pi$$

$$\mathcal{L}^4_{\rm SVT} = \mathcal{M}_4^{\mu\nu\alpha\beta}\nabla_\mu\partial_\alpha\pi\nabla_\nu\partial_\beta\pi + f_4(\pi, X)L^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}$$

$$\mathcal{M}_3^{\mu\nu} = \left(f_3(\pi, X)g_{\rho\sigma} + \tilde{f}_3(\pi, X)\partial_\rho\pi\partial_\sigma\pi\right)\tilde{F}^{\mu\rho}\tilde{F}^{\nu\sigma}$$

$$\mathcal{M}_4^{\mu\nu\alpha\beta} = \left(\frac{1}{2}f_{4,X} + \tilde{f}_4(\pi)\right)\tilde{F}^{\mu\nu}\tilde{F}^{\alpha\beta}$$

Massive Gravity (tensor-tensor theory)



how do they couple???

Massive Gravity (tensor-tensor theory)

$\rightarrow$ $m^2 g^{\mu\nu} f_{\mu\nu}?$

The building block for a consistent theory of massive gravity

$$m^2\sqrt{g^{-1}f}$$

$$S = \int \frac{M_{\rm Pl}^2}{2}\sqrt{-g}R + \mathcal{L}_{\rm matter} \; + \; m^2\mathcal{U}\left(\sqrt{g^{-1}f}\right)$$

$\uparrow$ $\partial^2 g$

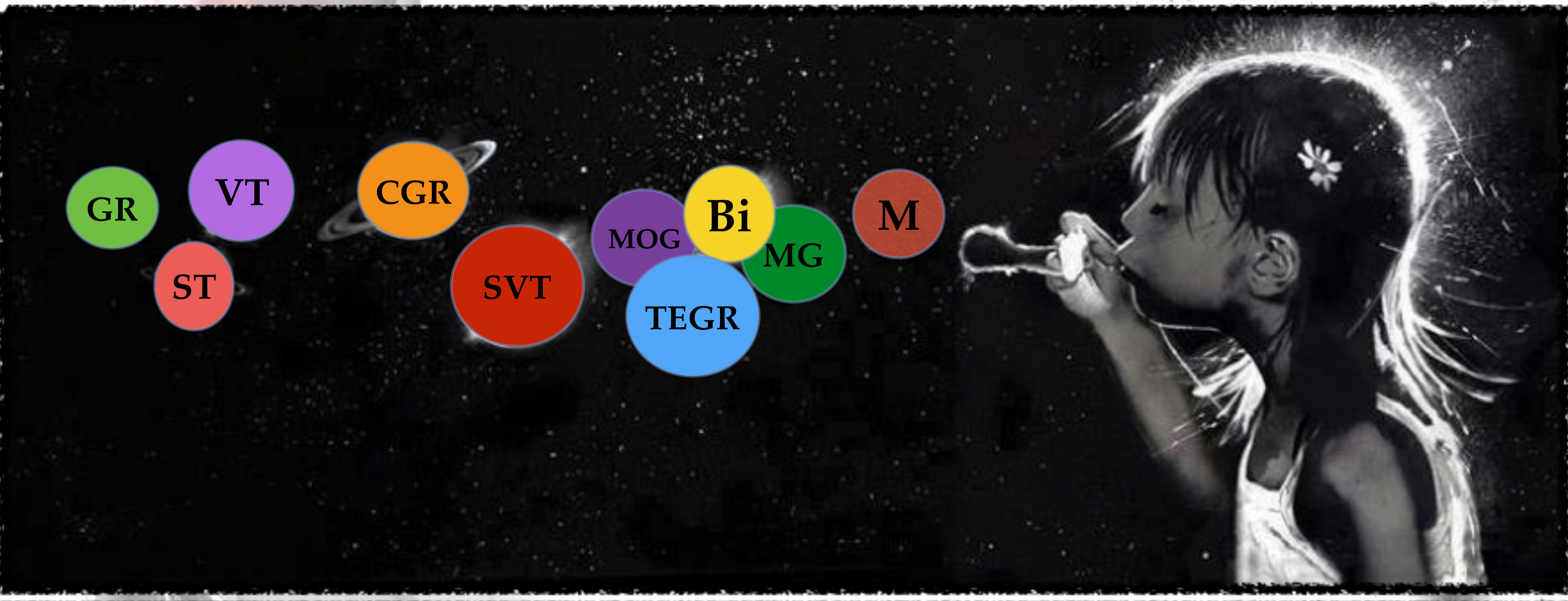
$$S_{MG} = \int d^4x\sqrt{-g}\sum_{n=0}^{4}\frac{\beta_n}{n!(4-n)!}e_n(\sqrt{g^{-1}f})$$

C. de Rham, G. Gabadaze, A.J.Tolley, PRL106 (2011)

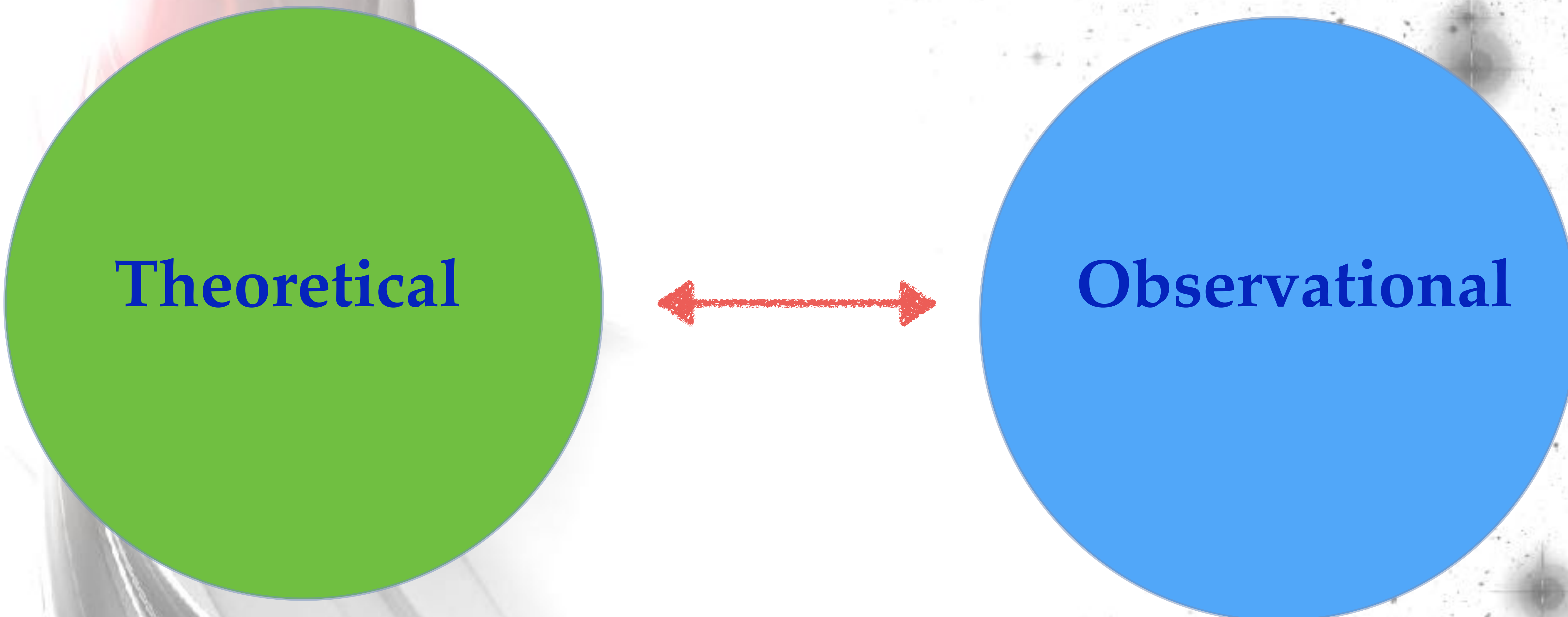
S.F. Hassan, R.A. Rosen JHEP1107 (2011)

$\downarrow$ elementary symmetric polynomials

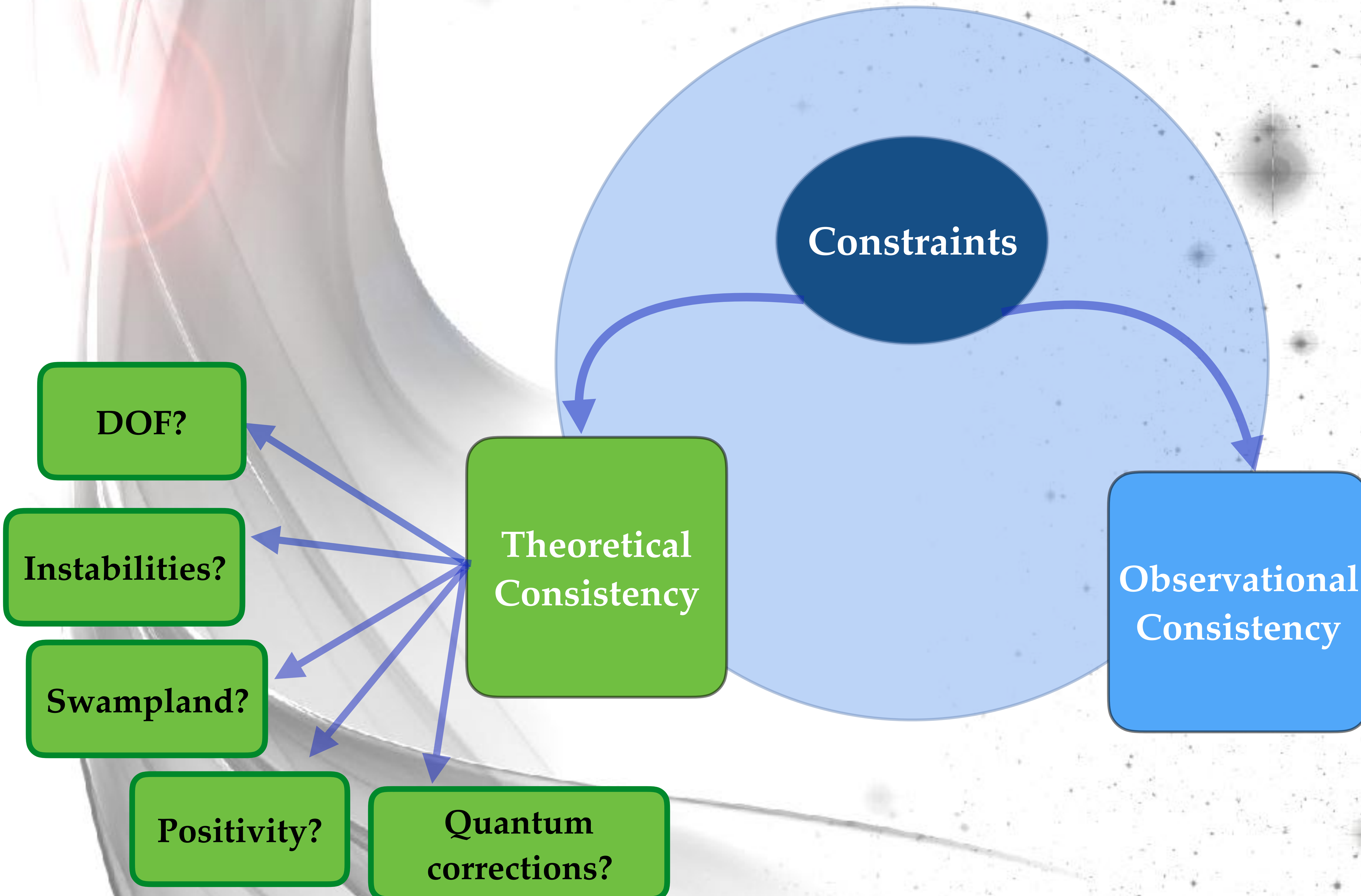
Huge Landscape of Theories



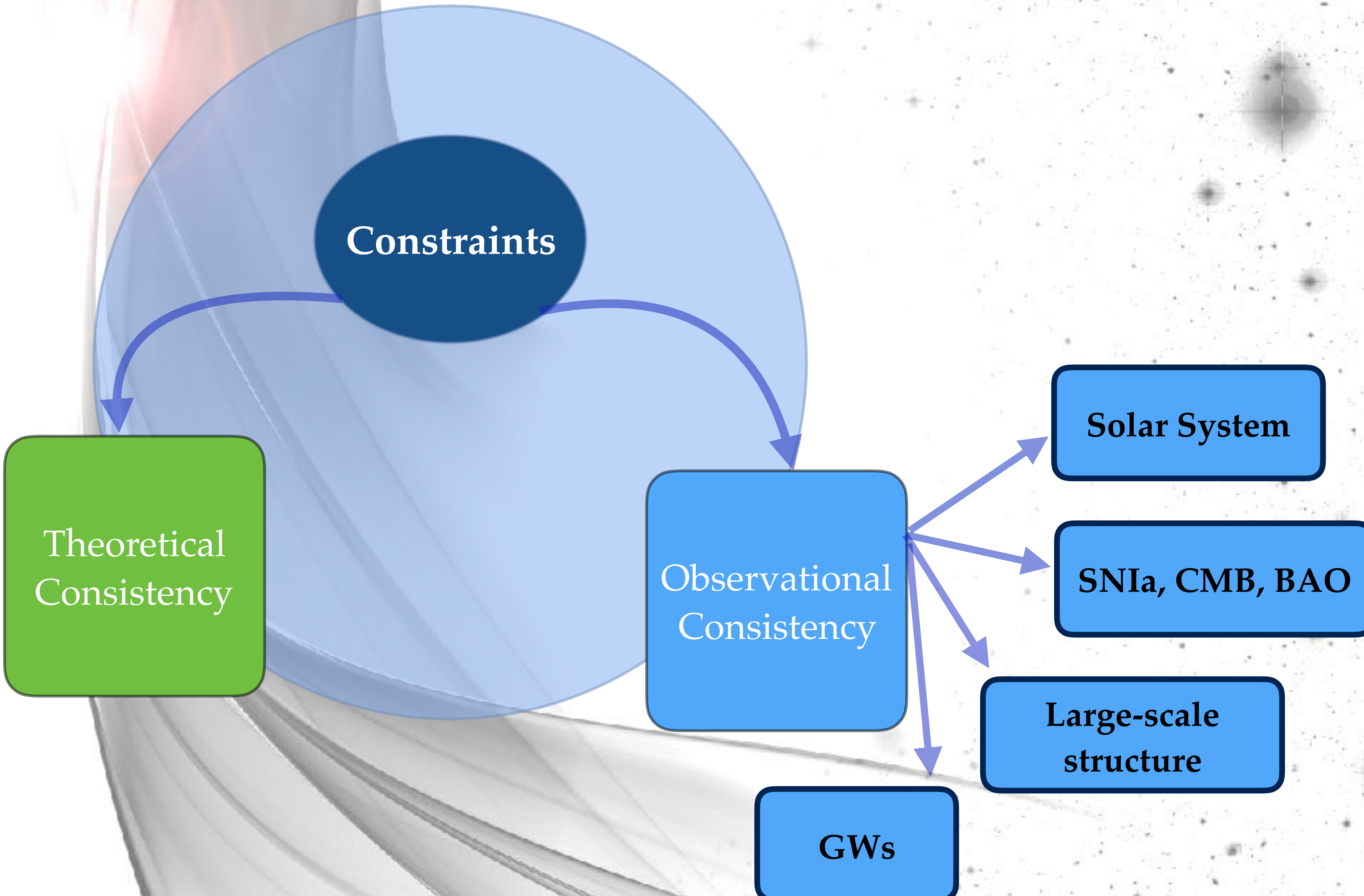
Constraints???



Constraints!!!



Constraints!!!



All you ever wanted to know about generalizations of gravity, but were afraid to ask.

arXiv:1807.01725

A systematic approach to generalisations of General Relativity and their cosmological implications

Lavinia Heisenberg

Institute for Theoretical Studies, ETH Zurich, Clausiusstrasse 47, 8092 Zurich, Switzerland.

Abstract

A century ago, Einstein formulated his elegant and elaborate theory of General Relativity, which has so far withstood a multitude of empirical tests with remarkable success. Notwithstanding the triumphs of Einstein's theory, the tenacious challenges of modern cosmology and of particle physics have motivated the exploration of further generalised theories of spacetime. Even though Einstein's interpretation of gravity in terms of the curvature of spacetime is commonly adopted, the assignment of geometrical concepts to gravity is ambiguous because General Relativity allows three entirely different, but equivalent approaches of which Einstein's interpretation is only one. From a field-theoretical perspective, however, the construction of a consistent theory for a Lorentz-invariant massless spin-2 particle uniquely leads to General Relativity. Keeping Lorentz invariance then implies that any modification of General Relativity will inevitably introduce additional propagating degrees of freedom into the gravity sector. Adopting this perspective, we will review the recent progress in constructing consistent field theories of gravity based on additional scalar, vector and tensor fields. Within this conceptual framework, we will discuss theories with Galileons, with Lagrange densities as constructed by Horndeski and beyond, extended to DHOST interactions, or containing generalized Proca fields and extensions thereof, or several Proca fields, as well as bigravity theories and scalar-vector-tensor theories. We will review the motivation of their inception, different formulations, and essential results obtained within these classes of theories together with their empirical viability.

Keywords: Modified Gravity, Massive Gravity, Scalar-Tensor theories, Generalized Proca, Multi-Proca, Scalar-Vector-Tensor theories, Cosmology