Debye mechanism of giant microwave absorption in superconductors

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conclusion

The Debye contribution to the conductivity of superconductors is proportional to the inelastic relaxation time rather than to the elastic one.
It may be many orders of magnitude larger than the conventional contribution.

Such a contribution to the linear conductivity exists only in the presence of supercurrent.

The Debye contribution to the non-linear conductivity exists even in the absence of the supercurrent.

Measurements of this effect may provide information about the inelastic relaxation time in superconductors

Characteristic relaxation times in metals

Momentum relaxation time τ_{el} is controlled by elastic electron-impurity scattering.

Inelastic relaxation time τ_{in} is due to electron-electron and electron-phonon scattering.

At low temperatures the inelastic relaxation time in metals is much longer than the elastic one

$$\tau_{in} \gg \tau_{el}$$

Microwave absorption in normal metals

$$\mathbf{E} = \mathbf{E}_\omega e^{i\omega t}$$

$$\omega \ll \frac{1}{\tau_{el}} \qquad\qquad \tau_{in} \gg \tau_{el}$$

At small frequencies the microwave absorption coefficient is controlled by the *dc* Drude conductivity

$$\sigma_D = \frac{e^2}{3}\nu_N v_F^2 \tau_{el}$$

Microwave field penetrates into metallic samples to skin depth.

The long inelastic relaxation time plays no role.

Microwave absorption in superconductors is controlled by the conductivity σ

$$\mathbf{j} = \frac{e}{m} N_s \mathbf{p}_s + \sigma \mathbf{E}. \qquad \omega \ll \Delta \ll T_c$$

Near T_c relaxation times of quasiparticles in superconductors are of the same order as in the normal state. As a result the real part of the linear conductivity is close to the Drude value.

$$\sigma = \sigma_D \left(1 + \frac{\Delta}{T} \ln \frac{T}{\omega}\right)$$

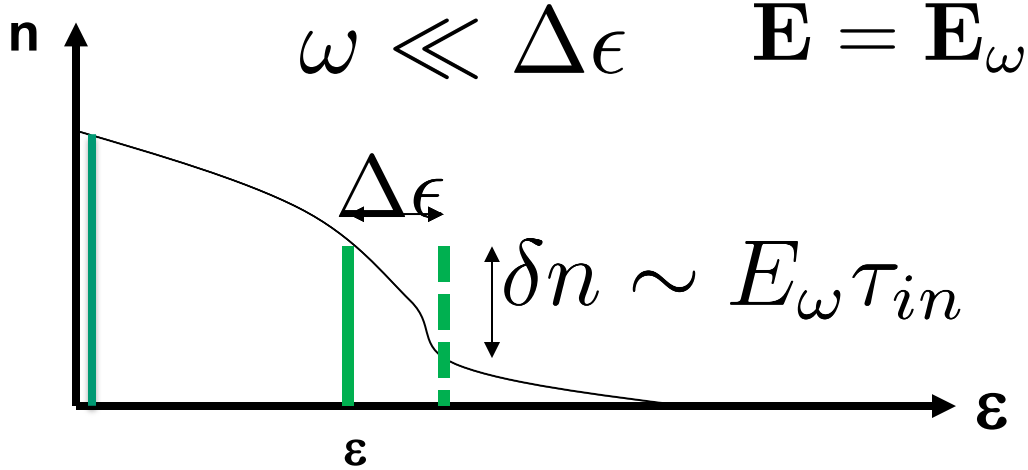
Mattis & Bardeen 1958, Schrieffer 1964; Larkin & Ovchinnikov 1977; Aronov & Gurevich 1981

The log-divergence is related to the singularity of the density of states at $\varepsilon=\Delta$

The long inelastic relaxation time plays no role

Debye mechanism of microwave absorption in molecular gases

$$\omega \ll \Delta\epsilon \qquad \mathbf{E} = \mathbf{E}_\omega e^{i\omega t}$$



Conductivity from the entropy production:

$$T\frac{\partial S}{\partial t} \sim T\frac{(\delta n)^2}{\tau_{in}} = \sigma_{DB}E^2 \Rightarrow \sigma_{DB} \sim \tau_{in}$$

The microwave absorption coefficient is proportional to the inelastic relaxation time.

Similar mechanism is known under the name of Pollak-Gablle mechanism of the microwave hopping conductivity regime and in the Mandelstam-Leontovich mechanism of large second viscosity in gases and liquids.

Debye mechanism of the microwave absorption in superconductors.

$$p_s = \frac{\hbar}{2}\nabla\chi - \frac{e}{c}A \qquad p_s(t) = \bar{p}_s + \delta p_s(t)$$

$$\delta\dot{p}_s(t) = e\mathrm{E}(t).$$

In the presence of microwave the quasiparticle density of states $\nu(\epsilon, p_s(t))$ depends on time.

The motion of quasiparticle levels creates non-equilibrium quasiparticle distribution. Its relaxation increases entropy and contributes to the microwave absorption.

Since the density of states is even in p_S, this contribution exists only in the presence of DC supercurrent or in the non-linear regime.

Quasiparticle dynamics in the presence of uniform microwave field E(t)

Two continuity equations:

Number of quasiparticle states in a sample is conserved.

$$\frac{\partial \nu(\epsilon, \mathbf{p}_s(t))}{\partial t} + \frac{\partial (v_\nu \nu)}{\partial \epsilon} = 0$$

$v_\nu(\epsilon, \boldsymbol{p}_s) = e\boldsymbol{E} \cdot \boldsymbol{V}(\epsilon, \boldsymbol{p}_s)$ is the level's velocity In the energy space

$$\boldsymbol{V}(\epsilon, \boldsymbol{p}_s) = -\frac{1}{\nu(\epsilon, p_s)} \int_0^\epsilon d\tilde{\epsilon} \frac{\partial \nu(\tilde{\epsilon}, p_s)}{\partial \boldsymbol{p}_s}$$

In the absence of relaxation processes the quasiparticle distribution follows the energy levels and number of quasiparticles is conserved

$$\frac{\partial \nu n}{\partial t} + \frac{\partial (v_\nu \nu n)}{\partial \epsilon} = 0$$

kinetic equation in the presence of level motion

$$\partial_t n(\epsilon, t) + e\boldsymbol{E}(t) \cdot \boldsymbol{V}(\epsilon, \boldsymbol{p}_s)\, \partial_\epsilon n(\epsilon, t) = I\{n\},$$

$$\boldsymbol{V}(\epsilon, \boldsymbol{p}_s) = -\frac{1}{\nu(\epsilon, p_s)} \int_0^\epsilon d\tilde{\epsilon} \frac{\partial \nu(\tilde{\epsilon}, p_s)}{\partial \boldsymbol{p}_s}$$

$$I\{n\} = -\frac{\delta n}{\tau_{in}} = -\frac{n - n_F}{\tau_{in}}$$

Near T_c the relaxation time approximation for the scattering integral is “exact”: The reason is that relevant to the Debye mechanism energies are much smaller than temperature while the energy transfer in the relaxation processes is of order T.

absorption power of microwave

$$\delta n_\omega(\epsilon) = \frac{1}{i\omega + 1/\tau_{in}} \frac{dn_F}{d\epsilon} e\mathbf{E}\mathbf{V} \qquad \boldsymbol{V}(\epsilon, \boldsymbol{p}_s) = -\frac{1}{\nu(\epsilon, \boldsymbol{p}_s)} \int_0^\epsilon d\tilde{\epsilon} \frac{\partial \nu(\tilde{\epsilon}, \boldsymbol{p}_s)}{\partial \boldsymbol{p}_s}$$

$$W = \int_0^\infty d\epsilon \langle \nu(\epsilon, p_s(t)) n(\epsilon, t) e\boldsymbol{E}(t) \cdot \boldsymbol{V}(\epsilon, \boldsymbol{p}_s(t)) \rangle =$$

$$T\frac{\partial S}{\partial t} = T\int \frac{(\delta n)^2}{n_F(1-n_F)\tau_{in}} d\epsilon = \sigma_{DB}\mathbf{E}_\omega^2$$

Debye contribution to conductivity

Brackets <..> stand for the averaging over the period of oscillations

The Debye contribution to the conductivity can be expressed in terms of the p_s dependence of the density of states.

$$T_c - T \ll T_c$$

$$\frac{\sigma_{\mathrm{DB}}}{\sigma_{\mathrm{D}}} = \frac{3\tau_{\mathrm{in}}}{4\tau_{\mathrm{el}}} \frac{1}{\left[1+(\omega\tau_{\mathrm{in}})^2\right]} \int_0^\infty \frac{d\epsilon}{T} \frac{\nu(\epsilon,\bar{p}_{\mathrm{s}})V^2(\epsilon,\bar{p}_{\mathrm{s}})}{\nu_n v_{\mathrm{F}}^2}$$

$$V(\epsilon,p_{\mathrm{s}}) = -\frac{1}{\nu(\epsilon,p_{\mathrm{s}})} \int_0^\epsilon d\tilde{\epsilon} \frac{\partial \nu(\tilde{\epsilon},p_{\mathrm{s}})}{\partial p_{\mathrm{s}}}$$

The density of states in non-magnetic conventional superconductors is quadratic in the superfluid momentum.

Therefore Debye contribution exists either in the presence of supercurrent or in the non-linear regime

In the presence of superfluid current, or superfluid momentum, the singularity of the quasiparticle density of states in s-wave superconductors is broadened

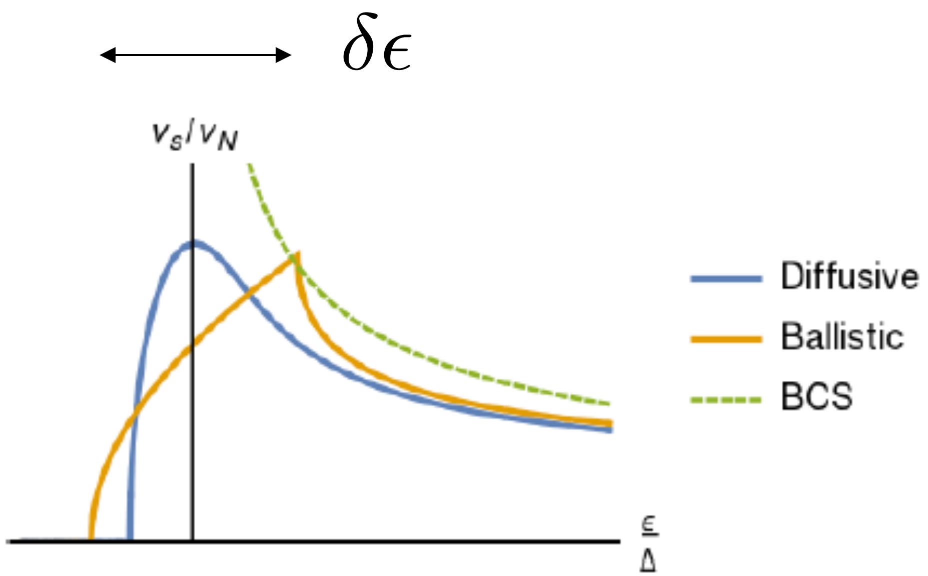


Figure 1: Schematic plots of $\nu(\epsilon, p_s)$ at: $p_s = 0$ - dashed green line, in the diffusive regime $\Delta v_F p_s \tau_{el}^2 \ll 1$ - blue line, and in the ballistic regime $\Delta v_F p_s \tau_{el}^2 \gg 1$ – orange line.

The value of the broadening width $\delta\varepsilon$ depends on the degree of disorder.

Ballistic limit, pure s-wave superconductors

spectrum:

$$\epsilon(\boldsymbol{k}) = \sqrt{\xi_{\boldsymbol{k}}^2 + |\Delta|^2} + v_{\boldsymbol{k}} \cdot \bar{\boldsymbol{p}}_{\mathrm{s}}$$

broadening width:

$$\delta\epsilon \sim v_F p_{\mathrm{s}}$$

density of stats

$$\frac{\nu(\epsilon, p_s)}{\nu_n} = \sqrt{\frac{\Delta}{2 v_{\mathrm{F}} p_{\mathrm{s}}}} \left[\theta(z+1)\sqrt{z+1} - \theta(z-1)\sqrt{z-1}\right]$$

$$z = (\epsilon - \Delta)/v_F p_{\mathrm{s}}$$

Debye contribution to the linear conductivity of s-wave superconductors in the ballistic regime is proportional to the inelastic relaxation time .

Its dependence of the supercurrent density is a non-analytic

$$T_c - T \ll T_c$$

$$\frac{\sigma_{\mathrm{DB}}}{\sigma_{\mathrm{D}}} = \frac{8}{45} \frac{\tau_{\mathrm{in}}}{\tau_{\mathrm{el}}\left[1 + (\omega\tau_{\mathrm{in}})^2\right]} \frac{\Delta}{T} \sqrt{\frac{v_{\mathrm{F}}\bar{p}_{\mathrm{s}}}{\Delta}}.$$

Ovchinnikov, Isaakyan 1978

The range of applicability of the ballistic regime:

The level's broadening due to elastic scatting should be smaller than $\delta\varepsilon$

$$\frac{1}{\tau_{el}(\epsilon)} = \sqrt{\frac{\epsilon - \Delta}{\Delta}}\frac{1}{\tau_{el}}$$

$$v_F \bar{p}_s \tau_{el}(\epsilon) \gg 1$$

⇓

$$v_F \bar{p}_s \tau_{el}^2 \Delta \gg 1$$

Quasiparticle density of states in the diffusive regime

$$v_F \bar{p}_s \tau_{el}^2 \Delta \ll 1$$

$$\delta\epsilon \sim (\Delta D^2 \bar{p}_s^4)^{1/3}$$

$$\mathbf{V} \sim (\Delta D^2 \bar{p}_s)^{1/3}$$

$D = v_F^2 \tau_{el}/3$ is a diffusion coefficient

Debye contribution to the conductivity in the s-wave superconductors in the diffusive regime is proportional to the inelastic relaxation time.

Its dependence of supercurrent is a non-analytic.

$$T_c - T \ll T_c$$

$$\frac{\sigma_{\rm DB}}{\sigma_{\rm D}} = I_{\rm d}\frac{\tau_{\rm in}}{\tau_{\rm el}}\frac{\Delta}{T}\frac{\tau_{\rm el}\left(\Delta D^2 \bar{p}_{\rm s}^4\right)^{1/3}}{1+\left(\omega\tau_{\rm in}\right)^2},$$

Ovchinnikov, Isaakyan 1978

The origin of the non-analytic p_s –dependence of the conductivity is the singular energy dependence of the density of states

$$\nu(\epsilon, 0) \rightarrow \nu_n \sqrt{\frac{\Delta}{2(\epsilon - \Delta)}} \text{ at } \epsilon \rightarrow \Delta$$

In real situations the singularity is broadened by inelastic scattering and by the gap anisotropy. Thus at very small values of p_s this dependence is analytic.

Microwave dynamics of high aspect ratio superconducting nanowires studied using self-resonance

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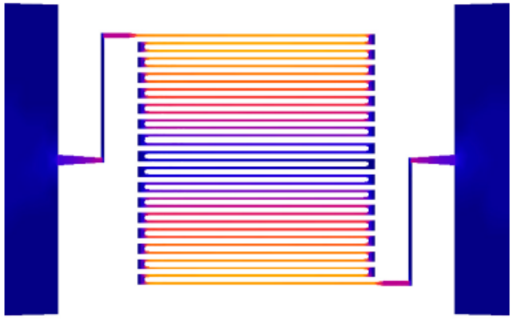
NbN, T/Tc =0.2

"At finite current, the measured sheet resistance is several orders of magnitude larger than the theory would suggest".



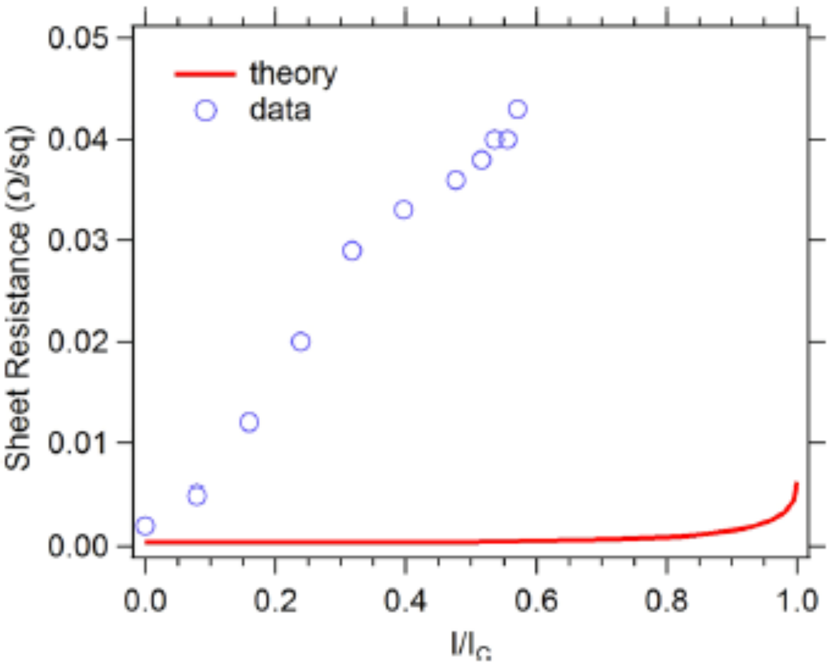
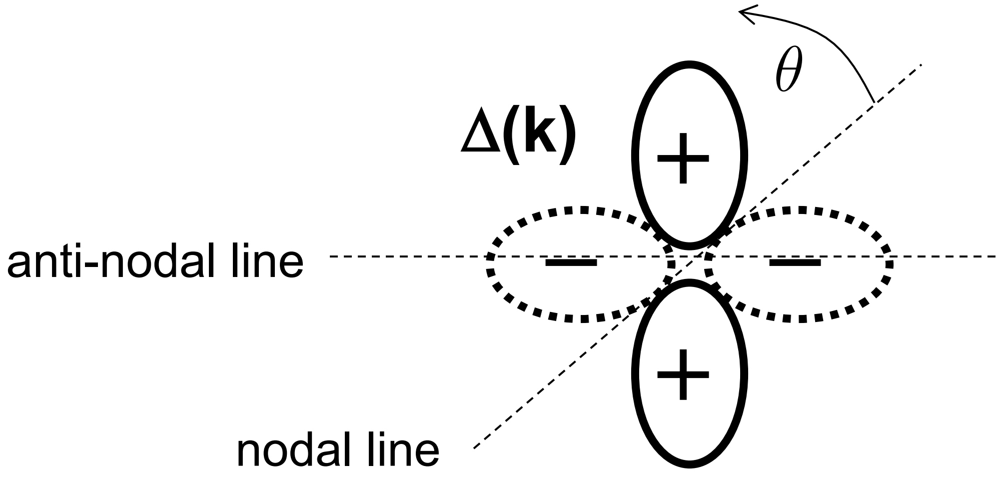
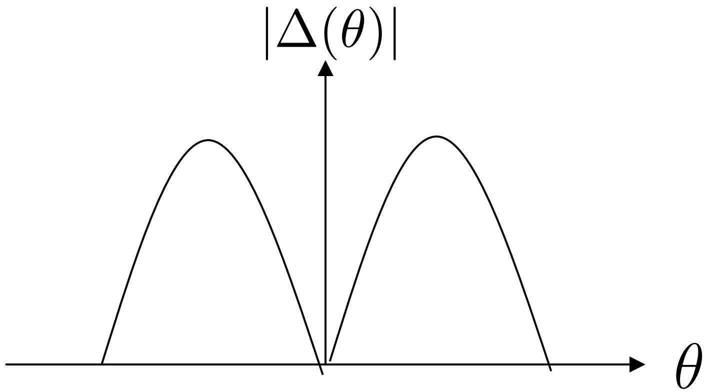


FIG. 11. Experimental and calculated values for the sheet resistance as a function of the normalized current. The data were taken at $T = 1.5$ K and the calculation assumes $t = 0.2$. We assume a critical current of 25 μA.

d-wave order parameter

$$\Delta(\mathbf{k}) = \Delta_0[\sin^2(k_x a) - \cos^2(k_y a)]$$





Density of states in D-wave superconductors

near an antinodal line $v_F p_s > 1/\tau_{el}$

$$\nu(\epsilon, p_s) = \frac{\nu_n}{\pi} \ln \frac{\Delta_0^2}{|(\epsilon - \Delta_0)^2 - (v_F p_s)^2|}$$

near a nodal line $v_F \bar{p}_s \gg \Gamma_{el}$

$$\nu(\epsilon) = \frac{\nu_n}{\Delta_0} \begin{cases} v_F p_s, & \text{for } \epsilon \le v_F p_s \\ \epsilon, & \text{for } \epsilon > v_F p_s \end{cases}$$

Debye contribution to the microwave absorption in D-wave superconductors at |T-T_c|<<T is controlled by quasiparticles with energies close to the gap

near the anti-node line.

It is proportional to the inelastic relaxation time , and its dependence of the supercurrent density is a non-analytic

$$T_c - T \ll T_c \qquad v_F p_s > 1/\tau_{el}$$

$$\frac{\sigma_{DB}}{\sigma_D} \sim \frac{\tau_{in}}{\tau_{el}} \frac{\Delta_0}{T} \left(\frac{v_F \bar{p}_s}{\Delta_0} \right) \ln \left(\frac{\Delta}{v_F \bar{p}_s} \right)$$

Near the antinode line the non-analytical energy dependence of the density of states is smeared by elastic scattering, and conductivity becomes an analytical fuction of the superfluid momentum

$$v_F p_s \ll 1/\tau_{el}$$

$$\frac{\sigma_{DB}}{\sigma_D} \sim \frac{\tau_{in}}{T_c}(v_F \bar{p}_s)^2 \ln(\Delta_0 \tau_{el})$$

Debye microwave absorption in s- and d-wave superconductors at low temperatures ($T \ll \Delta_0$)

At low temperatures quasi-particle concentration

$$x = \frac{1}{\Delta_0 \nu_n} \int \nu(\epsilon) n_F(\epsilon) d\epsilon$$

Is low. Therefore there are two relaxation times:

1) Scattering processes conserving the number of quasiparticles are characterized by the relaxation time τ_{st}

2) Processes which change the number of quasiparticles are characterized by the recombination time

$$\tau_r \sim \frac{\tau_{st}}{x} \gg \tau_{st}$$

Debye conductivity is controlled by the longest relaxation time.

$$\tau_r \gg \tau_{st}$$

$$n(\epsilon) = \frac{1}{1 + \exp(\frac{\mu-\epsilon}{T})}$$

quasiparticle conservation low

$$\mu \sim \frac{\tau_r e \boldsymbol{E}(t)}{n(\epsilon^*)} \int \cdot \boldsymbol{V}(\epsilon, \boldsymbol{p}_\mathrm{s})\, \partial_\epsilon n_F(\epsilon, t) d\epsilon$$

$$T\frac{\partial S}{\partial t} = T \int \frac{(\delta n)^2}{n_F(1-n_F)\tau_{in}} d\epsilon = \sigma_{DB} \mathbf{E}_\omega^2$$

Low temperature Debye conductivity of s-wave superconductors is of the same order at that near Tc in spite of the fact that the quasiparticle concentration is exponentially small

$$x = \frac{1}{\Delta_0 \nu_n} \int \nu(\epsilon) n_F(\epsilon) d\epsilon \sim e^{-\frac{\Delta(0)}{T}}$$

$$\frac{\sigma_{\rm DB}}{\sigma_{\rm D}} \sim \frac{\tau_{st}}{\tau_{\rm el}} \left[(\Delta \tau_{el}) \ (v_F \bar{p}_s \tau_{\rm el}) \right]^{2/3} \frac{1}{1 + (\omega \tau_r)^2}$$

In the absence of supercurrent the microwave conductivity is exponentially small !

Low temperature Debye conductivity of d-wave superconductors is controlled by quasi-particles **near the nodal lines**

$$\nu(\epsilon) = \frac{\nu_n}{\Delta_0} \begin{cases} v_F p_s, & \text{for } \epsilon \le v_F p_s \\ \epsilon, & \text{for } \epsilon > v_F p_s \end{cases}$$

Debye contribution to the microwave absorption coefficient is a non-monotonic function of the superfluid momentum.

$$\frac{\sigma_{DB}}{\sigma_D} \sim \frac{\tau_r}{\tau_{el}} \frac{1}{1+(\omega\tau_r)^2} \begin{cases} \left(\frac{v_F \bar{p}_s}{\Delta_0}\right)^2 \frac{\Delta_0}{T} \ln\left(\frac{T}{v_F \bar{p}_s}\right), & \text{for } T \gg v_F \bar{p}_s \\ \frac{T^2}{v_F \bar{p}_s \Delta_0}, & \text{for } v_F \bar{p}_s \gg T \end{cases}$$

Near the nodeal line the non-analytical energy dependence of the density of states is smeared by elastic scattering, and conductivity becomes an analytical fuction of the superfluid momentum

$$\frac{\sigma_{DB}}{\sigma_D} \sim \frac{\tau_r}{\tau_{el}} \frac{1}{1+(\omega\tau_r)^2} \frac{\Gamma_{el}}{\Delta_0} (\frac{T}{\Gamma_{el}})^2 \left(\frac{v_F \bar{p}_s}{\Gamma_{el}}\right)^2$$

$$\sigma(p_s = 0) \sim \sigma_D / \Delta_0 \tau \ll \sigma_D$$

P.A. Lee

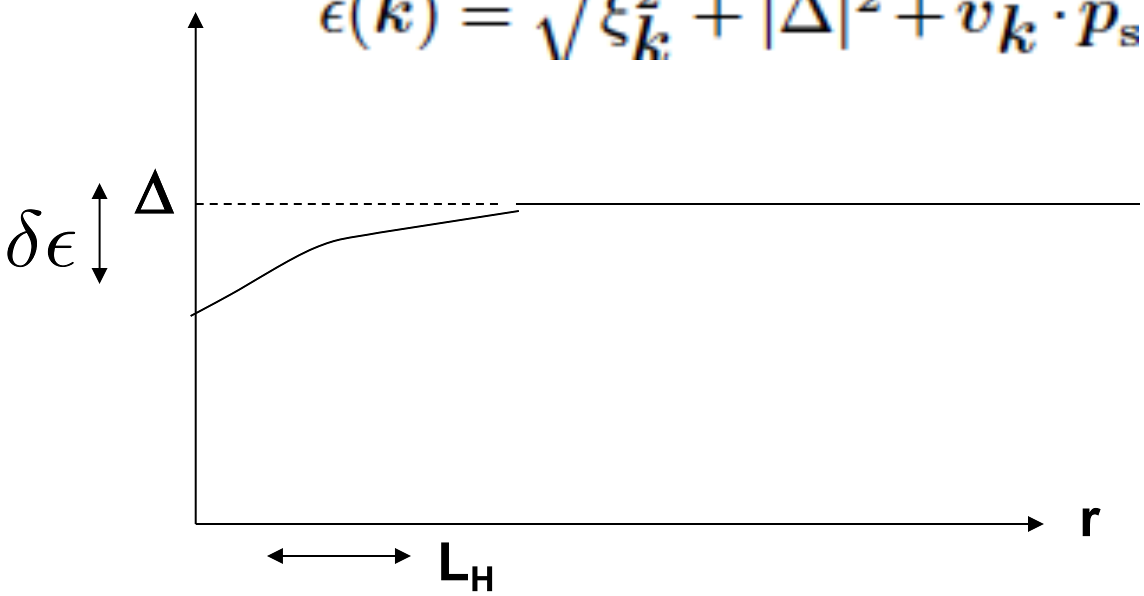
The Debye contribution to the non-linear conductivity, proportional to τ_{in} exists even in the absence of supercurrent

$$\bar{p}_s \to E_\omega / \omega$$

The threshold for the nonlinear conductivity is anomalously low. It is non-analytic in E_ω

Microwave field penetrates into bulk superconductors to distances of the order of the penetration length of the magnetic field L_H. Roughly, half of the quasiparticles relevant for the Debye mechanism have energies less than Δ.
So they are trapped near the surface.

$$\epsilon(\boldsymbol{k}) = \sqrt{\xi_{\boldsymbol{k}}^2 + |\Delta|^2} + v_{\boldsymbol{k}} \cdot \bar{p}_s$$

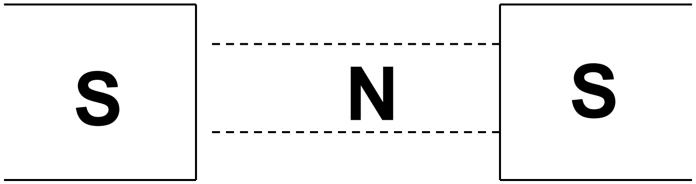


The Debye contribution to the conductivity of bulk samples is of the same order as in the case of films with thickness less than L_H

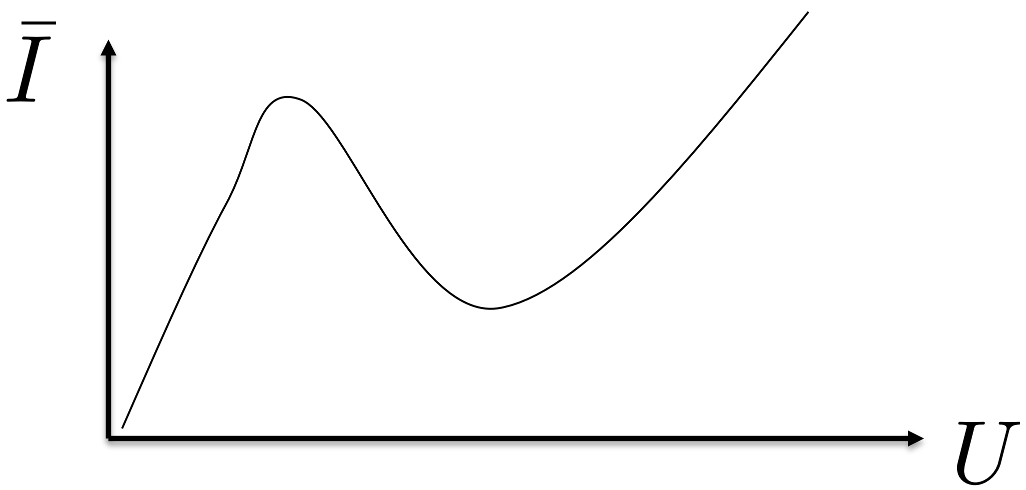
A closely related problem of conductance of superconductor-normal metal-superconductor junctions

S N Artemenko, A F Volkov, and A V Zaitsev, F. Zhou, B.Spivak

$$\bar{I} = G_{SNS} U$$



$$G_{SNS} \sim G_N \frac{E_T \tau_{in}}{1 + (eU\tau_{in})^2}, \qquad E_T = \sqrt{\frac{D}{L^2}}$$



Conclusion

The Debye contribution to the conductivity in superconductors is proportional to the inelastic relaxation time rather than to the elastic one. It controls the superfluid momentum dependence of the conductivity, and it may be much larger than the conventional contribution.

Such contribution to the linear conductivity exists only in the presence of supercurrent and it is strongly anisotropic.

The Debye contribution to the non-linear conductivity exists even in the absence of the supercurrent.

Energy relaxation rate may be determined from microwave absorption measurements.