The Floquet spectrum of superconducting multiterminal quantum dots

Benoît Douçot LPTHE Paris

In collaboration with: **Régis Mélin** Institut Néel Grenoble, Kang Yang (LPTHE), Denis Feinberg (Institut Néel).

Quasiparticles: where (almost) everything started!

THE THEORY OF A FERMI LIQUID

(The Properties of Liquid ^{3}He at Low Temperatures)

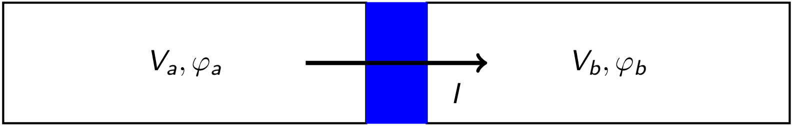
By A. A. ABRIKOSOV and I. M. KHALATNIKOV
Institute for Physical Problems, Moscow

Translated by M. G. Priestley

CONTENTS

Rep. Prog. Phys. **22**, 329 (1959)

The Josephson effect



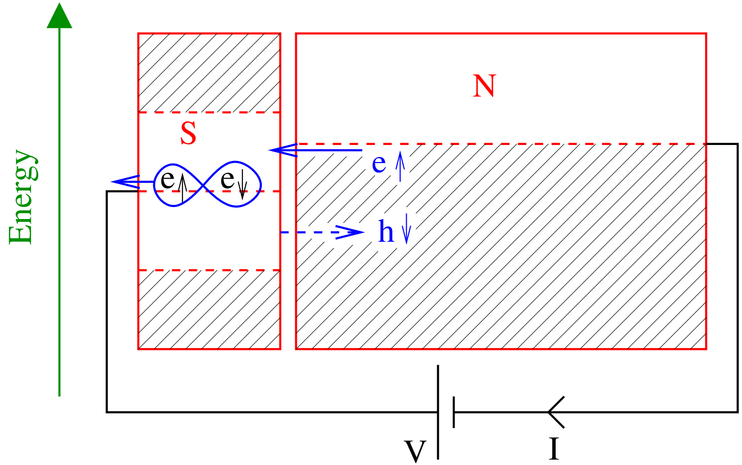
$$I = I_c \sin(\varphi_a - \varphi_b)$$

$$\frac{d}{dt}(\varphi_a - \varphi_b) = \frac{2e(V_a - V_b)}{\hbar}$$

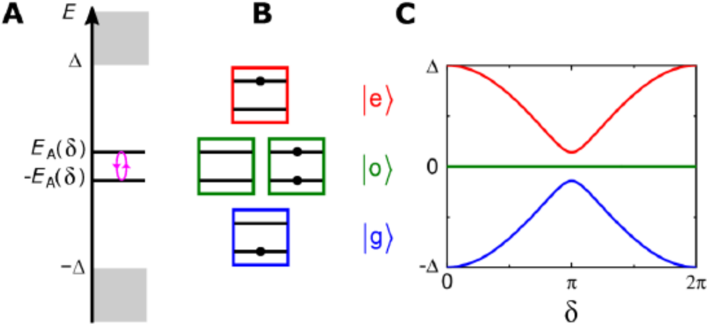
A dc voltage bias $V = V_a - V_b$ generates an ac current at the Josephson frequency ω_J:

$$\omega_J = \frac{2eV}{\hbar}$$

Andreev reflection

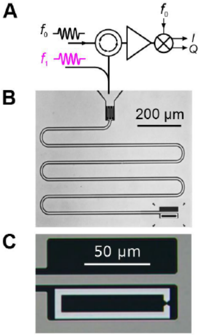


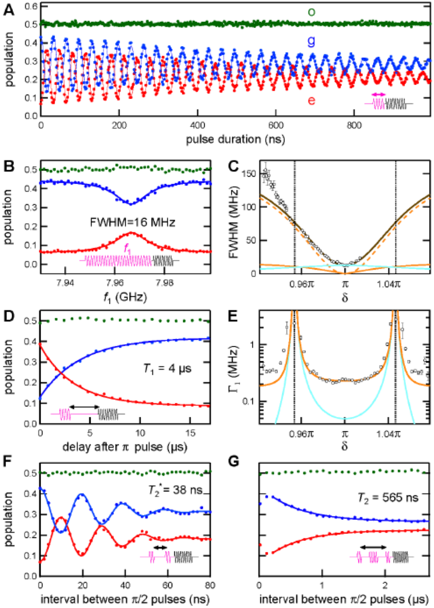
Andreev qubits in superconducting quantum point contacts



from C. Janvier et al. Science **349**, 1199, (2015)

Andreev bound-states in superconducting weak links

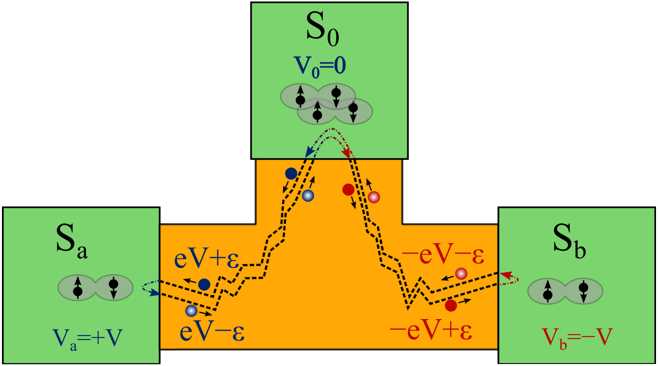




C. Janvier et al. Science **349**, 1199, (2015)

Quartets in Metallic Structures

A.H. Pfeffer, J.E. Duvauchelle, H. Courtois, R. Mélin, D. Feinberg and F. Lefloch, PRB ’14

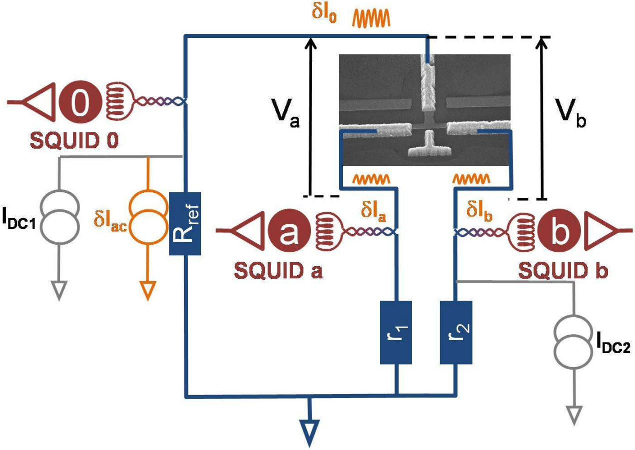


Theoretical calculation

- Perturbative expansion in the tunnel amplitudes

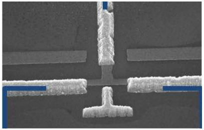
$\Rightarrow$ Diffusion modes, evaluated in the ladder approximation

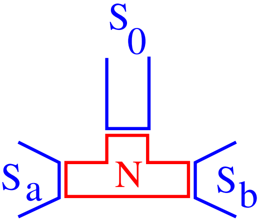
Experimental Set-up
A.H. Pfeffer, J.E. Duvauchelle, H. Courtois, and F. Lefloch

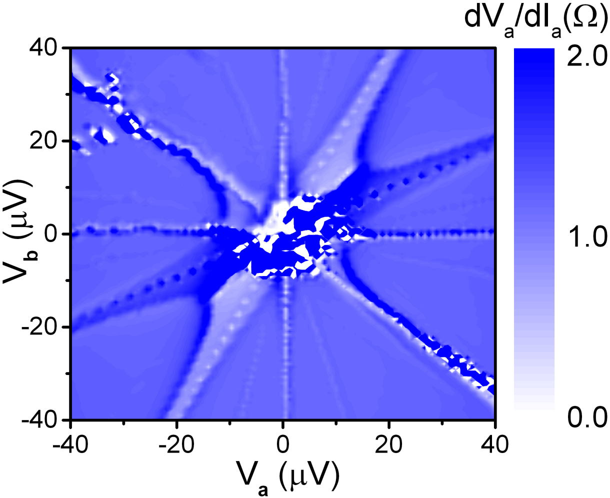


Resonances for a Bijunction ($T = 200$ mK)

A.H. Pfeffer, J.E. Duvauchelle, H. Courtois, R. Mélin, D. Feinberg, F. Lefloch, PRB '14

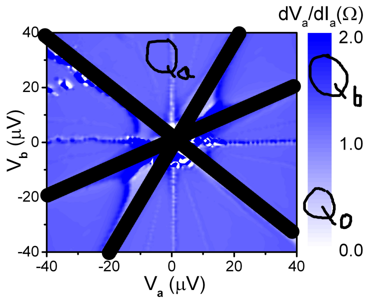






Resonances for a Bijunction

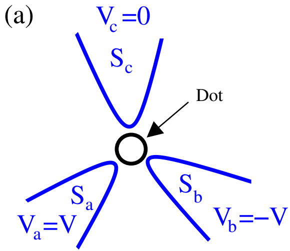
A.H. Pfeffer, J.E. Duvauchelle, H. Courtois, R. Mélin, D. Feinberg, F. Lefloch, PRB '14



Three additional resonance lines

- $2V_0 = V_a + V_b$; $2V_a = V_0 + V_b$; $2V_b = V_0 + V_a$
- Just permutation of the 3 terminals → equivalent resonances
- Are they due to quartets or to classical synchronization by an external impedance ?

Main questions addressed in this work



- Properties of out of equilibrium steady state at finite dc voltage bias ?
- Manifestations of quartet physics ?
- Main outcome: out of equilibrium generalizations of Andreev bound-states: Floquet-Wannier-Stark resonances.

General remarks

- dc voltages generate time-dependent phases: $\varphi_j(t) = \varphi_j(0) + \frac{2eV_j}{\hbar} t$
- Possibility to get time-periodic hamiltonians $H(\varphi)$:
  - Two terminal case
  - Three terminal case in "quartet" configuration: $V_a = V$, $V_b = -V$, $V_c = 0$
- Analogy with band-structure theory: $\varphi \leftrightarrow k$
  - Time dependent view-point: Bloch oscillations
  - Static view-point: Wannier-Stark ladders

Floquet theory for time periodic Hamiltonians

- $H(\varphi)$ 2π-periodic in φ, $H(\varphi) = \sum_m e^{-im\varphi} H_m$
- $\varphi = \omega_0 t$, $\omega_0 = 2\pi/T$
- Quasi-periodic solutions of the Schrödinger equation:

$$|\chi(t)\rangle = e^{-iEt} \sum_{m} e^{-im\omega_0 t} |\chi_m\rangle$$

- Maps to a steady state problem in $\mathcal{H}_{\mathrm{Large}} = \mathcal{H}_{\mathrm{Phys}} \otimes l^2(\mathbb{Z})$

$$(E + m\omega_0)|\chi_m\rangle = \sum_n H_n |\chi_{m-n}\rangle$$

- Translational symmetry in m is broken by a linear potential $-m\omega_0$

Wannier-Stark ladders in $\mathcal{H}_{\rm Large}$

If $\{|\chi_m\rangle\}$ gives an eigenstate with energy E, the translated family $\{|\tilde{\chi}_m\rangle\}$ with $|\tilde{\chi}_m\rangle = |\chi_{m+n}\rangle$ is also an eigenstate with energy $\tilde{E} = E + n\omega_0$.

Redundancy in $\mathcal{H}_{\rm Large}$: $(\{|\chi_m\rangle\}, E)$ and $\left(\{|\tilde{\chi}_m\rangle\}, \tilde{E}\right)$ generate the same Floquet state $|\chi(t)\rangle$ in $\mathcal{H}_{\rm Phys}$.

Quantum dynamics in $\mathcal{H}_{\mathrm{Large}}$: Bloch oscillations

- At $\omega_0 = 0$, eigenstates in $\mathcal{H}_{\mathrm{Large}}$ are plane waves, of the form $|\chi(\varphi)\rangle \otimes |\varphi\rangle$, with $|\varphi\rangle = \sum_m e^{-im\varphi}|m\rangle$. Then:

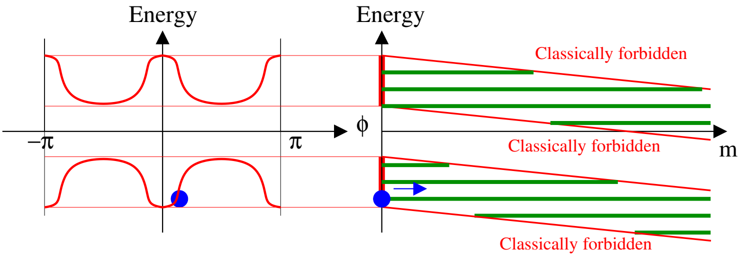
$$H(\varphi)|\chi(\varphi)\rangle = E(\varphi)|\chi(\varphi)\rangle$$

- Dynamics: If $|\Psi(t=0)\rangle = |\chi(0)\rangle \otimes |\varphi_0\rangle$, then

$$|\Psi(t)\rangle = |\chi(t)\rangle \otimes |\varphi(t)\rangle \qquad \begin{aligned} \varphi(t) &= \varphi_0 + \omega_0 t \\ i\frac{d|\chi(t)\rangle}{dt} &= H(\varphi(t))|\chi(t)\rangle \end{aligned}$$

- If $|\chi(t)\rangle$ is a Floquet state, we get a periodic evolution in $\mathcal{H}_{\mathrm{Large}}$, with frequency ω_0.

Connection between Bloch oscillations and Wannier-Stark ladders



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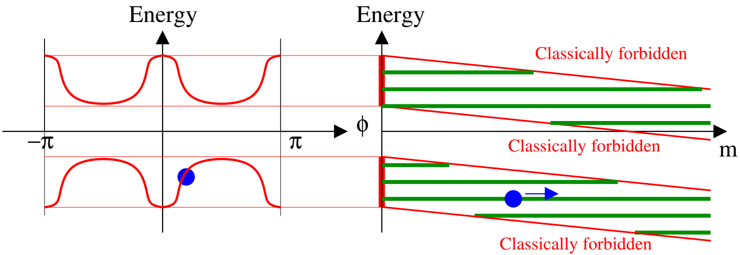
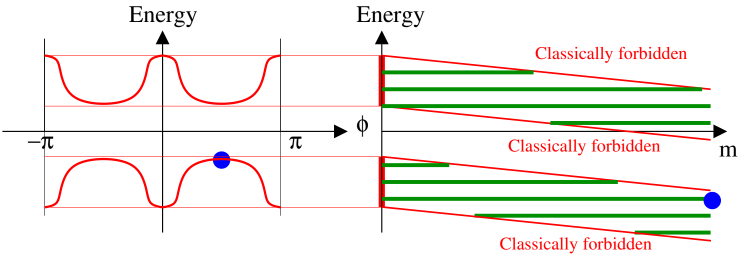
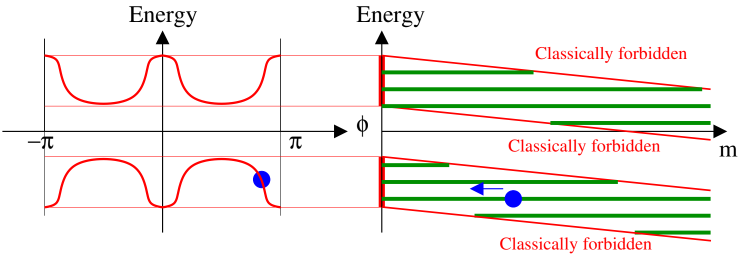


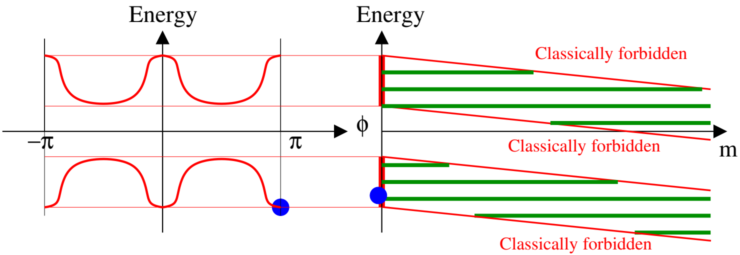
Connection between Bloch oscillations and Wannier-Stark ladders



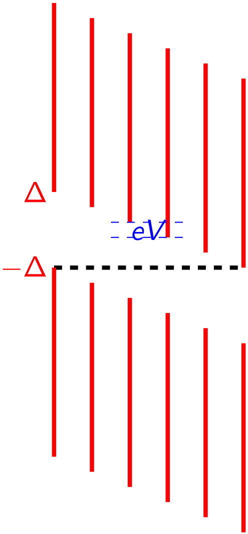
Connection between Bloch oscillations and Wannier-Stark ladders

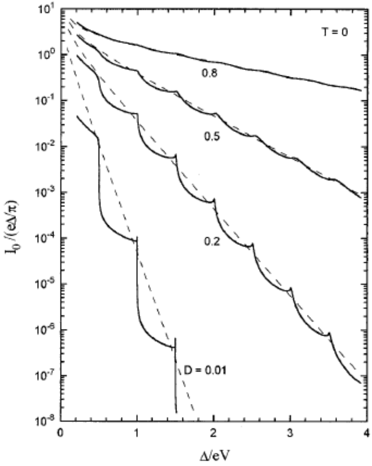


Connection between Bloch oscillations and Wannier-Stark ladders



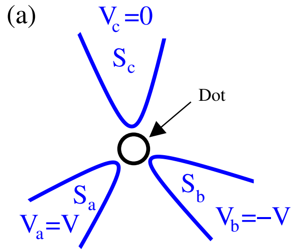
Multiple Andreev reflections





Bratus et al. Phys. Rev. B **55**, 12666 (1997)

Model Hamiltonian



$$H(t) = \sum_{jk\sigma} \epsilon_k c^\dagger_{jk\sigma} c_{jk\sigma} + \Delta_j c^\dagger_{jk\uparrow} c^\dagger_{j-k\downarrow} + \Delta^*_j c_{j-k\downarrow} c_{jk\uparrow}$$
$$+ J_{jk}\left(e^{-i\frac{e}{\hbar}V_j t} c^\dagger_{jk\sigma} d_\sigma + e^{i\frac{e}{\hbar}V_j t} d^\dagger_\sigma c_{jk\sigma}\right)$$

Floquet quasi-particle operators

$$i\frac{d}{dt}\gamma^\dagger(t) = [H(t), \gamma^\dagger(t)]$$

$$\gamma^\dagger(t) = u(t)d_\uparrow^\dagger + v(t)d_\downarrow + \sum_{jk}(u_{jk}(t)c_{jk\uparrow}^\dagger + v_{jk}(t)c_{j-k\downarrow})$$

Periodic case: $\omega_j = \frac{e}{\hbar}V_j = s_j\omega_0$, s_j integer. Then:

$$\begin{aligned} u(t) &= e^{-iEt}\sum_m e^{-im\omega_0 t}u(m) \\ u_{jk}(t) &= e^{-iEt}\sum_m e^{-im\omega_0 t}u_{jk}(m) \end{aligned}$$

Elimination of the reservoir amplitudes

$$\begin{aligned}
\{E + m\omega_0 - \sum_j G_j(E+(m+s_j)\omega_0)\}u(m) + & \\
\sum_j F_j(E+(m+s_j)\omega_0)v(m+2s_j) &= 0 \\
\sum_j F_j^*(E+(m-s_j)\omega_0)u(m-2s_j) + & \\
\{E + m\omega_0 - \sum_j G_j(E+(m-s_j)\omega_0)\}v(m) &= 0
\end{aligned}$$

Here, $G_j(E)$ and $F_j(E)$ are ordinary and anomalous Green's functions in the leads. We get a problem of two coupled Wannier-Stark ladders. Notation: $\Psi_m = (u(m), v(m))^T$.

A few properties of $G_j(E)$ and $F_j(E)$

$$\begin{aligned} G_j(E) &= \sum_k J_{jk}^2 \frac{E + \epsilon_k}{E^2 - \epsilon_k^2 - |\Delta_j|^2} \\ F_j(E) &= \sum_k J_{jk}^2 \frac{\Delta_j}{E^2 - \epsilon_k^2 - |\Delta_j|^2} \end{aligned}$$

- $G_j(E)$ and $F_j(E)$ are *real* as long as E lies inside the BCS gap, i.e. $|E| < |\Delta|$.
- The imaginary part of $G_j(E)$ and $F_j(E)$ has a singular threshold behavior proportional to $(E - |\Delta|)^{-1/2}$, reflecting the BCS singularity in the quasi-particle continua at the gap.

Difference equations: $|E+\xi| < \Delta$

$$M_0(m)\Psi_m - M_+(m+1)\Psi_{m+2} - M_-(m-1)\Psi_{m-2} = 0$$

$$M_0(m) = \begin{pmatrix} (E+\xi)\left(1+\frac{\sum_j \Gamma_j}{\sqrt{\Delta^2-(E+\xi)^2)}}\right) & -\frac{\Gamma_c\Delta}{\sqrt{\Delta^2-(E+\xi)^2)}} \\ -\frac{\Gamma_c\Delta}{\sqrt{\Delta^2-(E+\xi)^2)}} & (E+\xi)\left(1+\frac{\sum_j \Gamma_j}{\sqrt{\Delta^2-(E+\xi)^2)}}\right) \end{pmatrix}$$

$$M_+(m) = \begin{pmatrix} 0 & \frac{\Gamma_b\Delta e^{i\varphi_b}}{\sqrt{\Delta^2-(E+\xi)^2)}} \\ \frac{\Gamma_a\Delta e^{-i\varphi_a}}{\sqrt{\Delta^2-(E+\xi)^2)}} & 0 \end{pmatrix}$$

$$M_-(m) = \begin{pmatrix} 0 & \frac{\Gamma_a\Delta e^{i\varphi_a}}{\sqrt{\Delta^2-(E+\xi)^2)}} \\ \frac{\Gamma_b\Delta e^{-i\varphi_b}}{\sqrt{\Delta^2-(E+\xi)^2)}} & 0 \end{pmatrix}$$

Difference equations: $|E+\xi| > \Delta$

$$M_0(m)\Psi_m - M_+(m+1)\Psi_{m+2} - M_-(m-1)\Psi_{m-2} = 0$$

$$M_0(m) = \begin{pmatrix} (E+\xi)\left(1 + i\frac{\sum_j \Gamma_j}{\sqrt{(E+\xi)^2-\Delta^2}}\right) & -\frac{i\Gamma_c\Delta}{\sqrt{(E+\xi)^2-\Delta^2}} \\ -\frac{i\Gamma_c\Delta}{\sqrt{(E+\xi)^2-\Delta^2}} & (E+\xi)\left(1 + i\frac{\sum_j \Gamma_j}{\sqrt{(E+\xi)^2-\Delta^2}}\right) \end{pmatrix}$$

$$M_+(m) = \begin{pmatrix} 0 & \frac{i\Gamma_b\Delta e^{i\varphi_b}}{\sqrt{(E+\xi)^2-\Delta^2}} \\ \frac{i\Gamma_a\Delta e^{-i\varphi_a}}{\sqrt{(E+\xi)^2-\Delta^2}} & 0 \end{pmatrix}$$

$$M_-(m) = \begin{pmatrix} 0 & \frac{i\Gamma_a\Delta e^{i\varphi_a}}{\sqrt{(E+\xi)^2-\Delta^2}} \\ \frac{i\Gamma_b\Delta e^{-i\varphi_b}}{\sqrt{(E+\xi)^2-\Delta^2}} & 0 \end{pmatrix}$$

First transform m into a continuous variable ξ:
$\epsilon = 2\omega_0, \quad m\omega_0 = \xi$, the difference equation becomes:

$$M_0(\xi)\Psi(\xi) - M_+(\xi + \frac{\epsilon}{2})\Psi(\xi + \epsilon) - M_-(\xi - \frac{\epsilon}{2})\Psi(\xi - \epsilon) = 0.$$

Semi-classical Ansatz:

$$\Psi(\xi) = e^{i\frac{\theta(\xi)}{\epsilon}}\chi(\xi),$$

where $\chi(\xi)$ can be expanded as a series in ϵ:

$$\chi(\xi) = \sum_{n=0}^{\infty} \epsilon^n \chi_n(\xi)$$

Classical limit

Consider $\epsilon \to 0$. Then:

$$L_0(\xi, \theta'(\xi))\chi_0(\xi) = 0,$$

where:

$$L_0(\xi, \theta'(\xi)) = M_0(\xi) - e^{i\theta'(\xi)} M_+(\xi) - e^{-i\theta'(\xi)} M_-(\xi).$$

Setting $k(\xi) = \theta'(\xi)$, this defines a curve in classical phase-space (ξ, k), obtained by imposing:

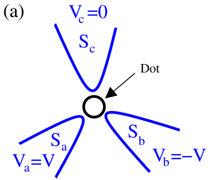
$$\det\left(M_0(\xi) - e^{ik} M_+(\xi) - e^{-ik} M_-(\xi)\right) = 0$$

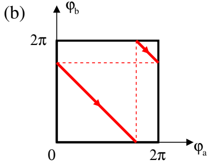
More explicitely:

$$E + \xi = \pm E_A(k)$$

Basis for tilted band picture.

Static limit: plane-wave solutions $\Psi_m = \exp(ikm/2)\,\Psi$, which correspond to quasiparticle operators for static Bogoliubov-De Gennes Hamiltonians with superconducting order-parameter phases: $\varphi_j(k) = \varphi_j + s_j k$ where $V_j = s_j V$ on lead S_j, $s_j \in \{\pm 1, 0\}$.

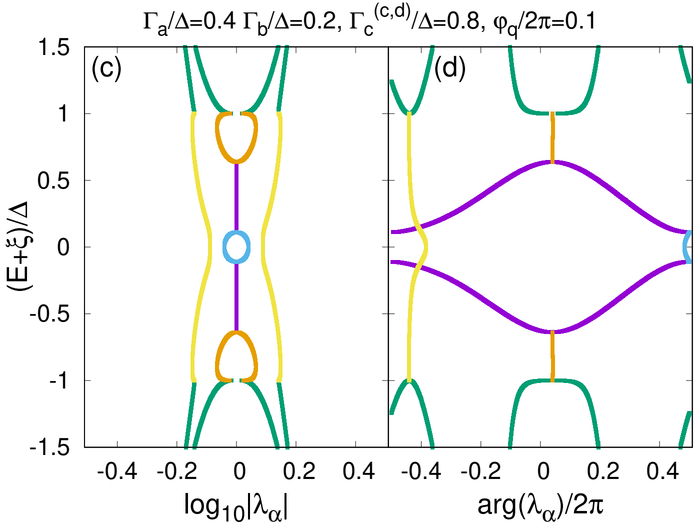




Adiabatic approximation: $k = 2\omega_0 t$, and at each time t, the system is in an eigenstate of $H(t)$.

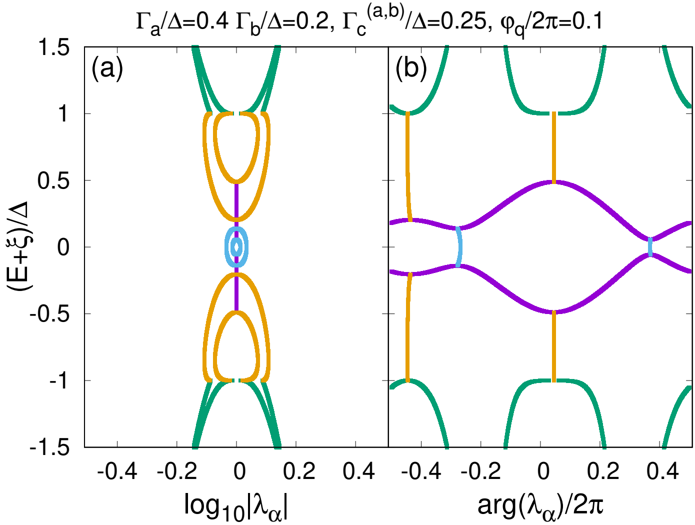
$E = \pm E_A(k)$: energy dispersion relation of the doublet of Andreev bound state bands.

Example of Andreev band structure (one local minimum)



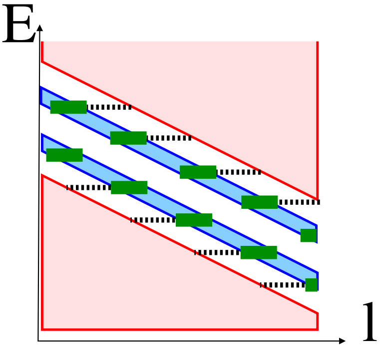
$$\lambda_\alpha = e^{ik_\alpha}$$

Example of Andreev band structure (two local minima)

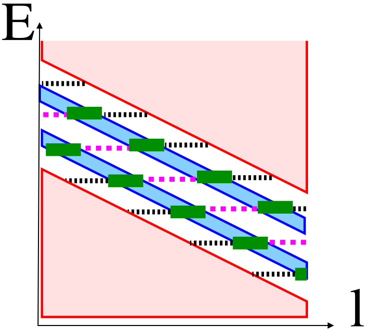


$$\lambda_\alpha = e^{ik_\alpha}$$

Non-coinciding resonances



Coinciding resonances



- Tunneling between ladders and continua

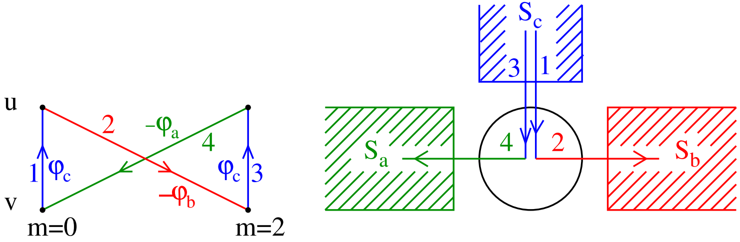
⇒ Finite width of FWS-Andreev resonances

- Tunneling between ladders and continua
- Inter-ladder tunneling

⇒ Landau-Zener-Stückelberg transitions

Differences Between 2 and 3 Terminals

- Ladders parameterized by the quartet phase
  $\varphi_Q = \varphi_a + \varphi_b - 2\varphi_c$
  $\Rightarrow$ Level crossings as a function of φ_Q
- Phase-sensitive Multiple Andreev reflections



- 2 Cooper pairs from S_c are transferred, one to S_a, one to S_b.
- Process involves an amplitude $\exp\{i(2\varphi_c - \varphi_a - \varphi_b)\}$.

Berry phase signature on the Floquet spectrum I

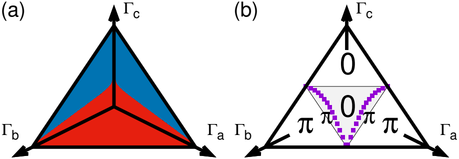
In the limit of very small voltage, one can neglect inter-band tunneling. This leads to Bohr-Sommerfeld quantization condition:

$$E = \sigma\langle E_A\rangle - (2n + W)\omega_0, \quad \sigma = \pm 1$$

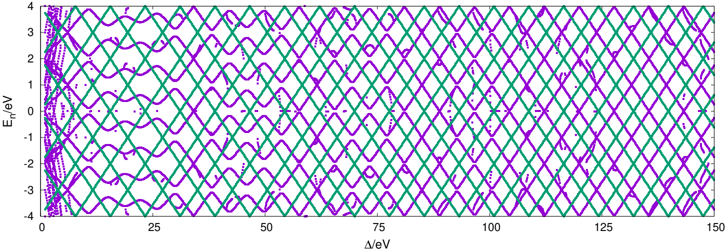
- Suggests to plot E/ω_0 versus $1/\omega_0$.
- Slope measures $\langle E_A\rangle$, and intercept is sensitive to W.
- W is the Berry phase accumulated by the Nambu spinor $\Psi(k)$ as k runs from $-\pi$ to π. It is the winding number of the parametrized curve in $\mathbb{C}$ defined by: $k \to \Gamma(k) = \Gamma_a e^{i(\varphi_a - k)} + \Gamma_b e^{i(\varphi_b + k)} + \Gamma_c e^{i\varphi_c}$.
- $L_0(\xi, k) \simeq \begin{pmatrix} E + \xi & -\Gamma(k) \\ -\Gamma(k)^* & E + \xi \end{pmatrix}$
- W jumps when $\Gamma(k) = 0$ for some k: gap closing condition.

Berry phase signature on the Floquet spectrum II

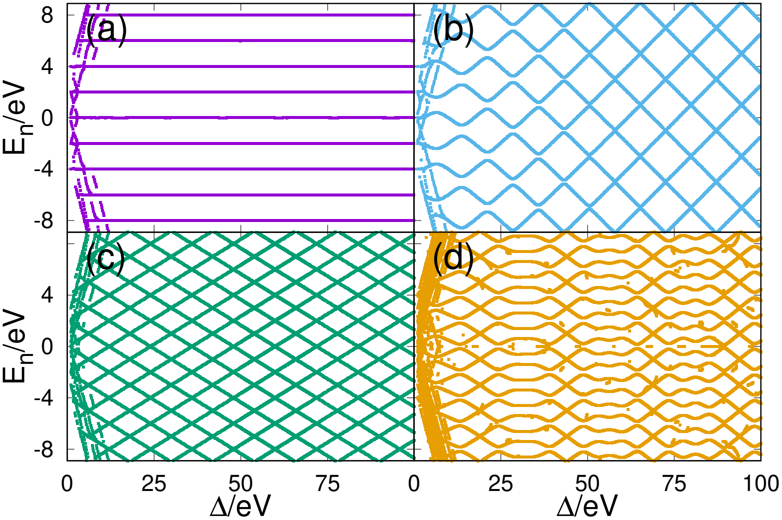
Phase diagrams: (a) number of minima in $E_A(k)$, (b) W.



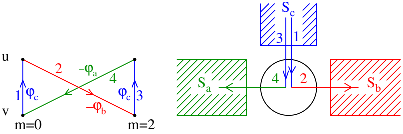
Floquet spectrum on a case with $W = 1$.

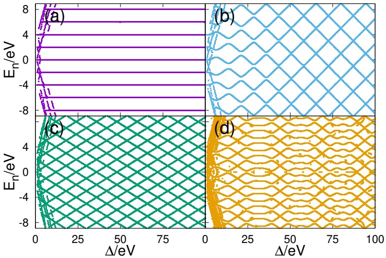


Other examples of Floquet spectrum



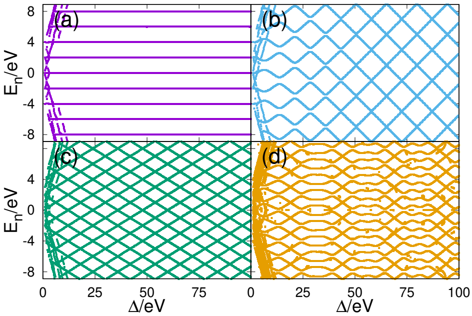
Cases (a) and (b) $\rightarrow 4\omega_0$-periodicity! Because of the absence of "vertical rungs" (Andreev processes involving reservoir c), there exist two decoupled blocks in $\mathcal{H}_{\mathrm{Large}}$.





Floquet spectrum: Cases (a) and (c): decoupled ladders

Symmetric configurations: $\Gamma_a = \Gamma_b$ and $\varphi_Q = 0$. Then σ^x commutes with $M_0(m)$ and $M_\pm(m)$, which leads to two decoupled ladders.



Single particle properties

Dressed quasi-particle operators take the form

$$\gamma^{\dagger}_{jk\sigma}(t) = \gamma^{\dagger(0)}_{jk\sigma}(t) + e^{-iE_{jk}t} \sum_m e^{-im\omega_0 t} \left(u_{jk}(m) d^{\dagger}_{\sigma} + \sigma v_{jk}(m) d_{-\sigma} \right) + \cdots$$

The stationary state $|\mathcal{S}\rangle$ is defined by:

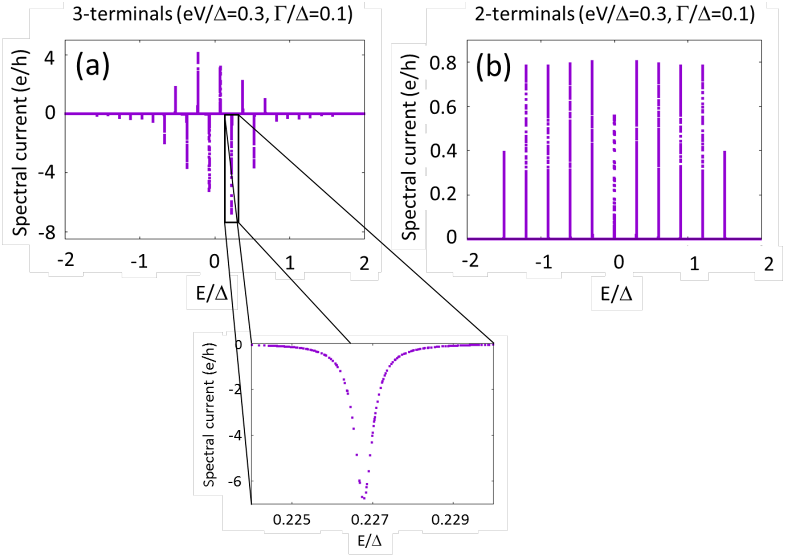
$$\gamma_{jk\sigma}|\mathcal{S}\rangle = 0$$

Then:

$$\langle S|d^{\dagger}_{\sigma H}(t) d_{\sigma H}(t)|S\rangle = \sum_{m,n} e^{-i(m-n)\omega_0 t} \sum_{j,k} v_{jk}(m) v_{jk}(n)^*$$

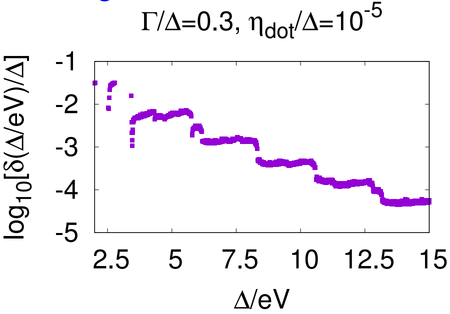
- dc average involves a sum over k which exhibits resonances when $E_{jk} = \pm E_R + p\omega_0$, p integer.
- Harmonic content related to shape of Floquet-Wannier-Stark wave-functions.

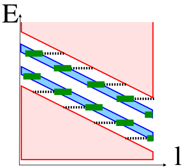
Spectral current



Width of Floquet-Wannier-Stark-Andreev Resonances

Width of the resonances due to
Tunneling between ladders and continua





- Envelope $\delta(\Delta/eV) \sim \exp(-\Delta/eV)$ because of tunneling through classically forbidden region of length $\sim \Delta/eV$

- Steps related to thresholds of multiple Andreev reflections coupling quantum dot to quasiparticle continua (discreteness of auxiliary variable l)

$\Rightarrow$ Sensitivity to other relaxation mechanism at low-voltage (i.e. at large Δ/eV)

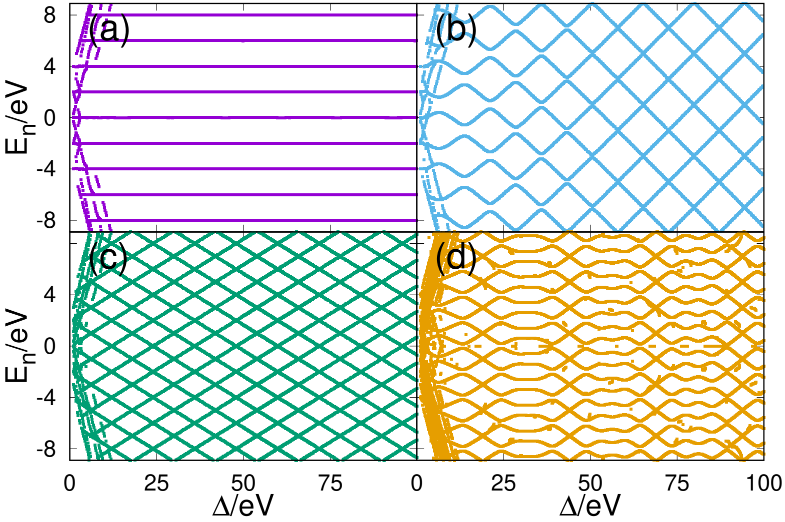
Two particle properties

Dressed quasi-particle operators take the form

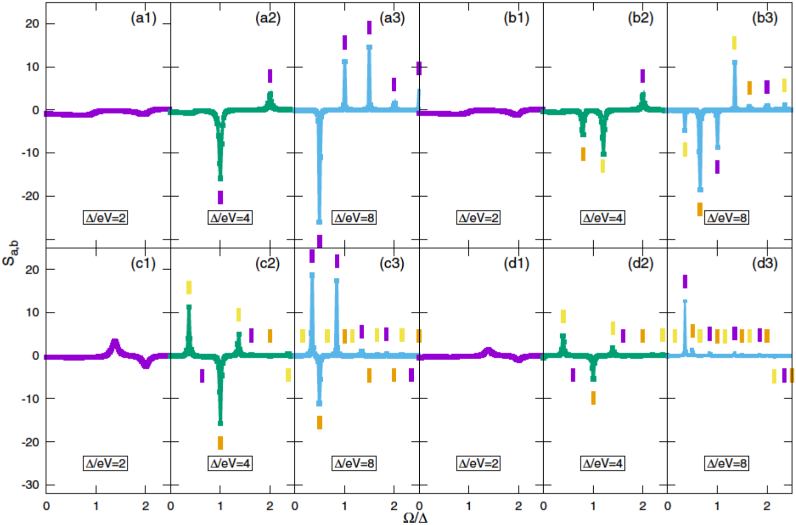
$$\gamma^{\dagger}_{jk\sigma}(t) = \gamma^{\dagger(0)}_{jk\sigma}(t) + e^{-iE_{jk}t} \sum_{m} e^{-im\omega_0 t} \left(u_{jk}(m) d^{\dagger}_{\sigma} + \sigma v_{jk}(m) d_{-\sigma} \right) + \cdots$$

- $S(\omega, \omega)$ has narrow peaks at $\omega = p\omega_0$ and $\omega = \pm 2E_R + p\omega_0$, p integer.
- Some peaks merge near avoided crossings between the two Wannier Stark ladders, i.e. $E_R = 0$ or $E_R = \pm\omega_0/2$. This is likely to enhance zero frequency noise.

Finite frequency noise

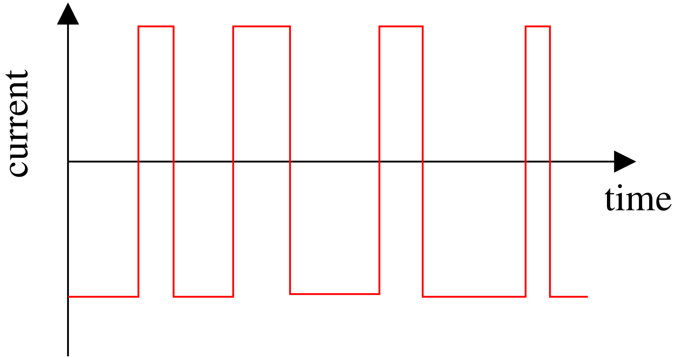


Finite frequency noise



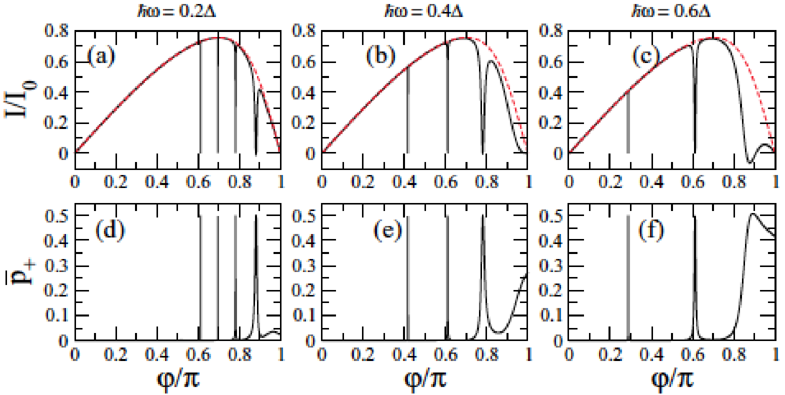
Noise for two coupled Wannier-Stark ladders

Thermal noise in a two-terminal point contact at equilibrium:



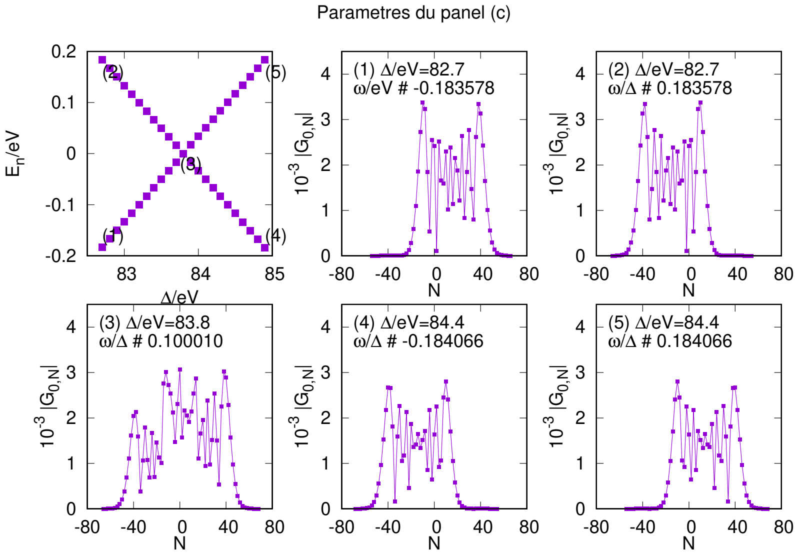
Can we generalize such picture for two coupled Wannier-Stark ladders ?

Example: ABS dynamics in an irradiated QPC

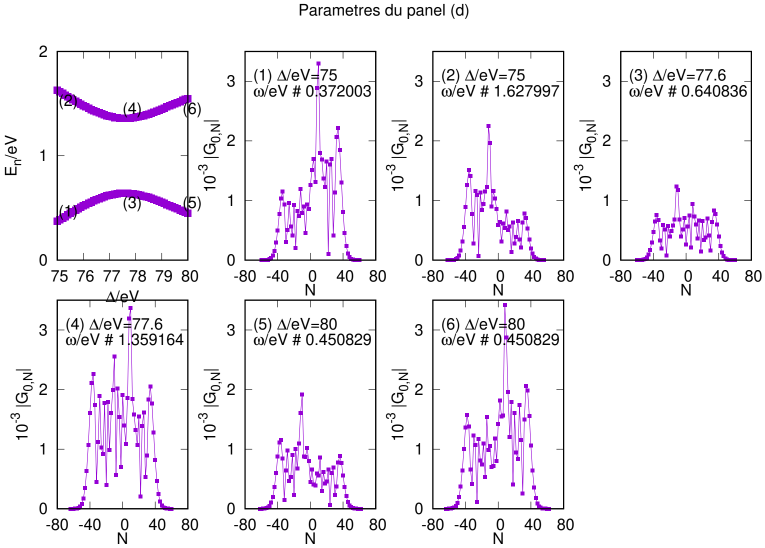


F. S. Bergeret et al, Phys. Rev. **B** 84, 054504 (2011)

Resolvent near a level crossings

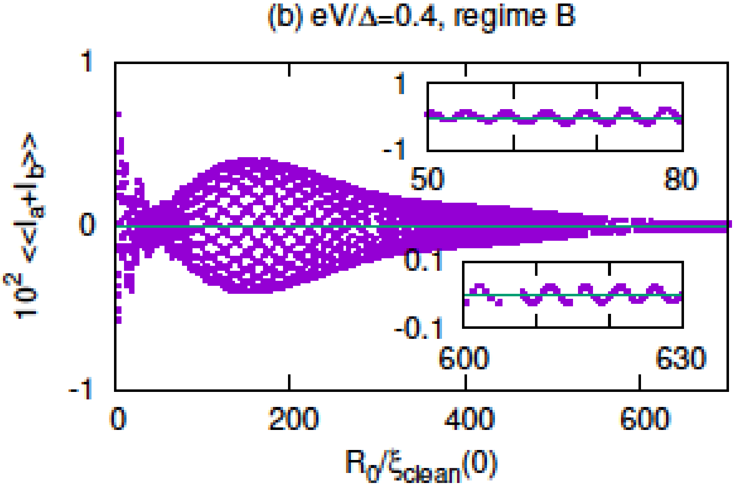


Resolvent near an avoided level crossings



Questions and Perspectives

- More quantitative description of single particle and two particle properties near avoided crossings. Relative weight of the two Wannier-Stark ladders? Related to the choice of a stationary state (here: Keldysh prescription).
- Physical relaxation mechanisms between these ladders?
- Role of Coulomb interactions on the dot?
- Two quantum dots $\rightarrow$ possible long range correlation through Floquet-Tomasch mechanism?
- Manipulations with NMR pulses $\rightarrow$ towards a Floquet-Andreev qubit?


(b) eV/Δ=0.4, regime B
1
0
-1
10^2 <<I_a+I_b>>
0
200
400
600
R_0/ξ_clean(0)
1
-1
50
80
0.1
-0.1
600
630

Periodic modulation of a Wannier-Stark ladder

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Quantum phase-slip junction under microwave irradiation

A. Di Marco,[1,2] F. W. J. Hekking,[1,2] and G. Rastelli[2,3]
[1]*Université Grenoble Alpes, LPMMC, F-38000 Grenoble, France*
[2]*CNRS, LPMMC, 25 Avenue des Martyrs B.P. 166, F-38042 Grenoble Cedex, France*
[3]*Zukunftskolleg, Fachbereich Physik, Universität Konstanz, D-78457 Konstanz, Germany*
(Received 25 February 2015; revised manuscript received 24 April 2015; published 19 May 2015)

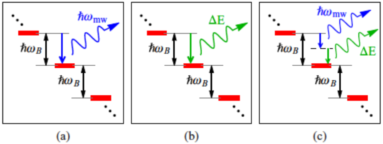
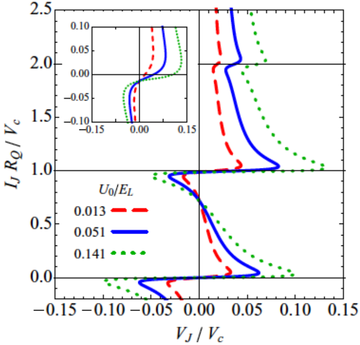
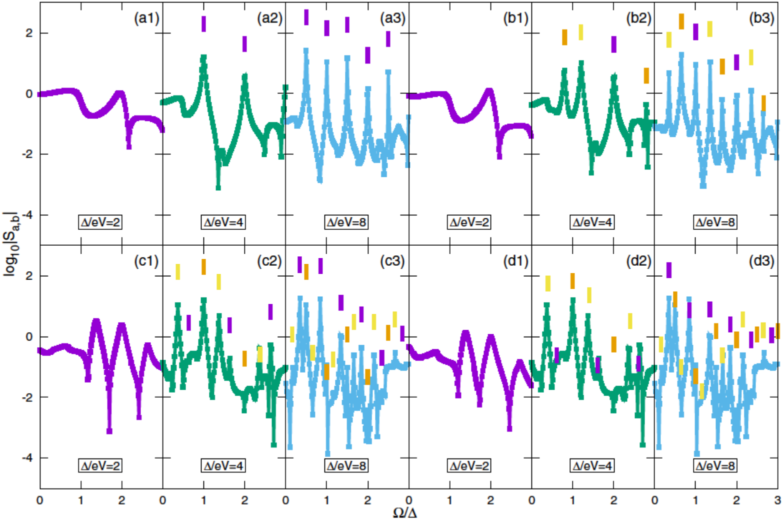


FIG. 3. (Color online) Wannier-Stark ladder. The tilt provided by the bias current I_0 induces an energy separation $\hbar\omega_B$ between adjacent phase states indicated by red horizontal bars. (a) Phase locking occurs when the resonant condition $\omega_B = m\omega_{\rm mw}$ is satisfied. For $m = 1$, a photon with energy $\hbar\omega_{\rm mw}$ is exchanged with the microwave source. (b) Environment-assisted transitions between adjacent states in the Wannier-Stark ladder lead also to the appearance of a finite voltage across the QPSJ element. (c) Wannier-Stark ladder in the presence of both microwave and environmental photons with energies $\hbar\omega_{\rm mw}$ and ΔE, respectively.



Finite frequency noise



Analogy between superconductivity and solid state physics

Band Theory	Superconductivity
Wave-vectors	Superconducting phases
Position on the lattice in real space	Number of transmitted Cooper pairs N
Wannier functions labelled by sites on a periodic lattice	Periodicity in phases implies N integer
Plane waves in Bloch theory $\|k\rangle = \sum_x \exp(ikx)\|x\rangle$	States with fixed superconducting phase $\|\varphi\rangle = \sum_N \exp(iN\varphi)\|N\rangle$
Hopping between neighboring tight-binding sites	Transferring pairs between leads by Andreev reflection
External potential	Charging energy
Electric field $dk/dt = -eE$	Josephson relation $d\varphi_n/dt = 2eV_n/\hbar$
Wannier-Stark ladder	**Floquet-Wannier-Stark ladders**

VOLUME 60, NUMBER 23 PHYSICAL REVIEW LETTERS 6 JUNE 1988

Stark Localization in GaAs-GaAlAs Superlattices under an Electric Field

E. E. Mendez, F. Agulló-Rueda, and J. M. Hong

IBM T. J. Watson Research Center, Yorktown Heights, New York 10598

(Received 21 January 1988)

We have observed that a strong electric field $\mathcal{E}$ shifts to higher energies the photoluminescence and photocurrent peaks of a GaAs-$Ga_{0.65}Al_{0.35}As$ superlattice of period D (=65 Å), which we explain by a field-induced localization of carriers to isolated quantum wells. Good agreement is found between observed and calculated shifts when the large field-induced increase of the exciton binding energy is taken into account. At moderate fields $[\simeq(2-3)\times10^4$ V/cm], the coupling between adjacent wells is manifested by four additional peaks that shift at the rates $\pm e\mathcal{E}D$ and $\pm 2e\mathcal{E}D$ and correspond to transitions that involve different levels of the Stark ladder.

PACS numbers: 73.60.Br, 73.40.Lq, 78.55.Cr

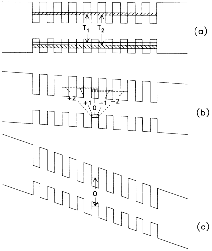


FIG. 1. Sketches of the conduction- and valence-band potential profiles for GaAs-$Ga_{1-x}Al_xAs$ superlattice under a (a) small, (b) moderate, and (c) high electric field, clad by thick $Ga_{1-x}Al_xAs$ regions. The diagrams are approximately scaled for a 30–35-Å superlattice with $x=0.35$, and fields of 2×10^3, 2×10^4, and 1×10^5 V/cm, respectively.

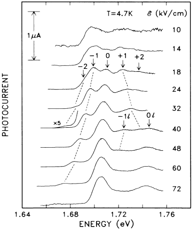


FIG. 3. Photocurrent (PC) spectra for the same superlat of Fig. 2, at representative electric fields. The peaks labele ±1, and ±2 are for transitions involving heavy-hole st and electrons weakly delocalized, as illustrated in Fig. 1 Analogous transitions for light holes are denotes by $0l$ $-1l$.

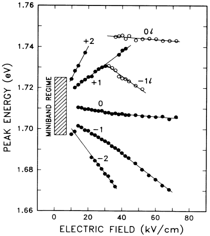


FIG. 4. Transition energies for the PC structures of Fig. 1(a) vs electric field. The filled circles correspond to heavy-hole transitions, whereas the open circles refer to light holes.

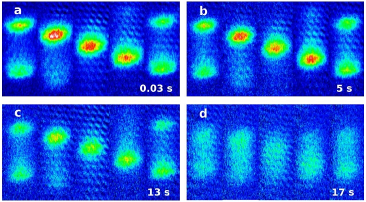


FIGURE 5.9. *Oscillations de Bloch d'atomes de* ^{88}*Sr (bosons) sous l'effet de la gravité dans un réseau de période* $a = 266\,$*nm et de profondeur* $V_0 \approx 3\,E_{\rm r}$ *[figure extraite de Poli et al. (2011)]. La période de Bloch est* $\omega_{\rm B}/2\pi = 574\,$*Hz et les oscillations de Bloch peuvent être observées pendant près de 20 secondes. Les images correspondent à l'oscillation n° 1, 2900, 7500 et 9800. La valeur extrêmement basse de la longueur de diffusion pour les atomes de* ^{88}*Sr permet de minimiser le déphasage des oscillations dû aux interactions. On peut déduire de ces oscillations la valeur de* g *à* 6×10^{-6} *près. La précision de cette mesure de* g *est notablement amélioré si on utilise plutôt – sur le même montage expérimental – la spectroscopie des états de Wannier–Stark (voir § 5).*

Spectroscopy of Bloch oscillations: Wannier-Stark ladders

PHYSICAL REVIEW VOLUME 117, NUMBER 2 JANUARY 15, 1960

Wave Functions and Effective Hamiltonian for Bloch Electrons in an Electric Field

GREGORY H. WANNIER
Bell Telephone Laboratories, Murray Hill, New Jersey

(Received August 3, 1959)

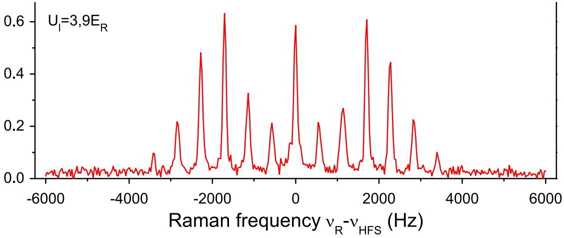


FIGURE 5.15. *Spectroscopie Raman des états de Wannier–Stark d'atomes de rubidium dans un réseau optique en présence de gravité. On observe des transitions* $|\Phi_j\rangle \rightarrow |\Phi_{j'}\rangle$ *allant jusqu'à* $|j' - j| = 6$ *pour cette valeur de la profondeur du réseau. La fréquence des oscillations de Bloch est* $\omega_{\mathrm{B}}/(2\pi) = 569\,$*Hz pour la longueur d'onde de la lumière choisie pour le réseau (532 nm) [figure extraite de Beaufils et al. (2011)].*