bla

E_{10} , $K(E_{10})$ and the Standard Model

Hermann Nicolai
MPI für Gravitationsphysik, Potsdam
(Albert Einstein Institut)

Based on joint work with (in various combinations):

T. Damour, M. Henneaux, A. Kleinschmidt and K. Meissner

В честь Исаака Марковича Халатникова

по случаю столетнего юбилея

Questions we would like to answer

- General Relativity (GR) and Quantum Theory?
- Resolution of singularities in GR?
- Structure of space-time: discrete, emergent,...?
- Resolution of singularities in QFT?
- UV completion of Standard Model (SM)?
- Why $SU(3)_c \times SU(2)_w \times U(1)_Y$?
- Why three generations of spin-$\frac{1}{2}$ fermions?
- Dark Matter and Dark Energy?

... and most important: is there any way to validate/falsify any of the existing proposals?

This Talk

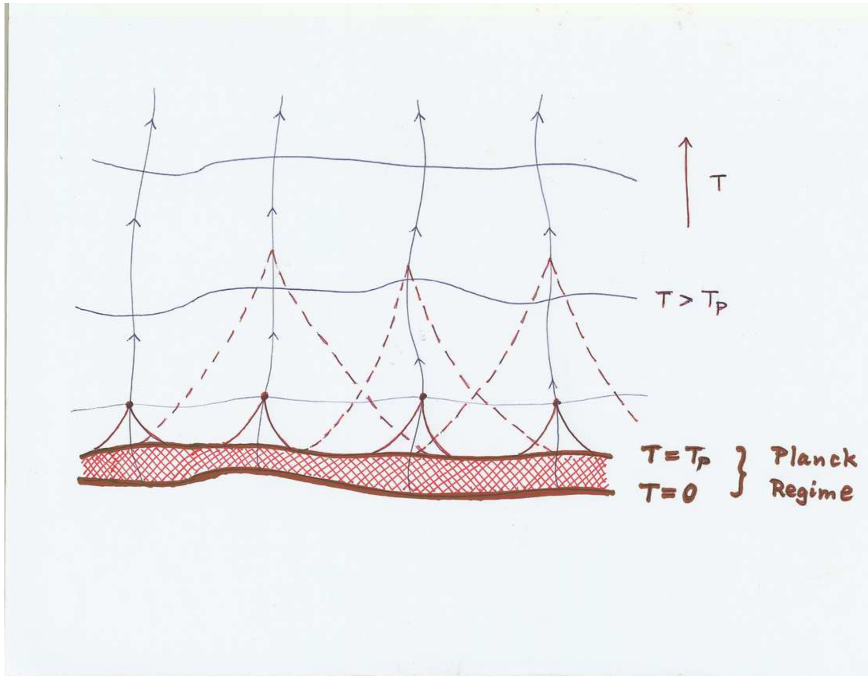
Far from a finalized proposal, but to point out the possibility that the way the SM gets linked to a Planck scale unified theory of quantum gravity may work in ways completely different from currently popular ideas.

Proposal makes use of several ingredients:

- **BKL-type analysis of cosmological singularities**
- **The $E_{10}/K(E_{10})$ σ-model: an attempt to formulate M theory with *emergent* space-time and matter degrees of freedom**
- **Beyond, but staying close to, maximal ($N=8$) supergravity**

Main concern: finding *some* way to link these abstract considerations to 'real physics', that is, the SM or a minimal extension thereof, and thereby understand or derive it from a more fundamental theory.

BKL and Spacelike Singularities



For $T \to 0$ spatial points decouple and the system is effectively described by a continuous *superposition of one-dimensional systems* $\to$ effective dimensional reduction to $D = 1$! [Belinski,Khalatnikov,Lifshitz (1972)]

Habitat of Quantum Gravity

- **Cosmological evolution as one-dimensional motion in the moduli space of d-geometries** [Wheeler,DeWitt,...]

$$\mathcal{M} \equiv \mathcal{G}^{(d)} = \frac{\mathrm{Riem}(\Sigma)}{\mathrm{Diff}(\Sigma)} = \frac{\{\text{spatial metrics } \mathbf{g}_{mn}(\mathbf{x})\}}{\{\text{spatial diffeomorphisms}\}}$$

- **Formal canonical quantization $\rightarrow$ WDW equation.**
- **Unification of space-time, matter and gravitation: $\mathcal{M}$ should incorporate matter degrees of freedom in a natural manner (not simply $\mathcal{M} = \mathcal{G}^{(3)} \times \mathcal{M}_{matter}$).**
- **Can we understand and 'simplify' $\mathcal{M}$ by means of embedding into a group theoretical coset $G/K(G)$?**
- **Main conjecture: $G = \mathrm{E}_{10}$ and $K(G) = \mathrm{K}(\mathrm{E}_{10})$**
- **Fits with conjectured emergence of E_{10} in reduction of maximal supergravity to $D = 1$.** [Julia(1983)]

Hamiltonian Constraint

Hamiltonian constraint ($\to$ WDW operator)

$$\mathcal{H} = \kappa\, G_{mn\,pq}(\mathrm{g})\Pi^{mn}\Pi^{pq} - \frac{1}{2\kappa}\sqrt{\mathrm{g}}R^{(d)}(\mathrm{g}) + \cdots$$

with DeWitt metric $G_{mn\,pq} = \mathrm{g}^{-1/2}(\mathrm{g}_{mp}\mathrm{g}_{nq}+\mathrm{g}_{mq}\mathrm{g}_{np}-\mathrm{g}_{mn}\mathrm{g}_{pq})$**.**

BKL limit: reduce to one spatial point and diagonal metric degrees of freedom $\mathrm{g}_{mn}(t) = \delta_{mn}\exp(\beta^m(t))$

$$\mathcal{H}_{\mathrm{red}} = G_{mn}\pi^m\pi^n + V_{\mathrm{eff}}(\beta)$$

with *Lorentzian* (indefinite) metric G_{mn} **on** $\mathbb{R}^d$

Effective potential V_{eff} **simplifies in near singularity limit**

$$V_{\mathrm{eff}}(\beta) = \sum_A \Theta_\infty(w_A(\beta))$$

'Sharp wall potentials' $\leftrightarrow$ wall forms $w_A(\beta) \equiv G_{mn}w_A^m\beta^n$ **constrain motion in DeWitt mini-superspace**

[Damour,Henneaux,HN: "Cosmological Billiards", CQG20(2003)R145]

The Group Theory Connection

- **Identify space of diagonal degrees of freedom with Cartan subalgebra (CSA) of some Lie algebra.**
- **DeWitt metric on $\{\beta^m\} \equiv$ Cartan Killing metric**
- **Leading wall forms associated with *simple roots* of some indefinite Kac Moody algebra (KMA)**

  $$A_{ij} = G^{mn}(w_i)_m(w_j)_n \qquad (G_{mn} = \text{DeWitt metric!})$$

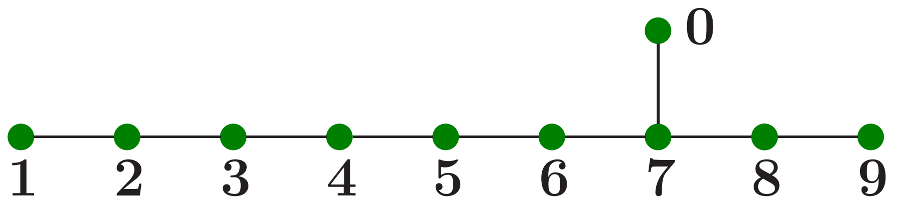
  ***e.g.* KMA = AE_3 for Einstein gravity ($D = 4$) and KMA = E_{10} for maximal supergravity ($D = 11$).**
  [Damour,Henneaux, PRL86(2001)4749]

- **'Cosmobilliards' take place in Weyl chamber of KMA $\Rightarrow$ chaotic oscillations if KMA is *hyperbolic*.**
  [Damour,Henneaux,Julia,HN:PLB509(2001)323]

- **E_{10} is maximally extended hyperbolic KMA: contains *all* simply laced hyperbolic KMAs.** [S.Viswanath,0801.2586]

What is E_{10}?

E_{10} is the 'group' associated with the Kac-Moody Lie algebra $\mathfrak{g} \equiv \mathfrak{e}_{10}$ defined via the Dynkin diagram [e.g. Kac]

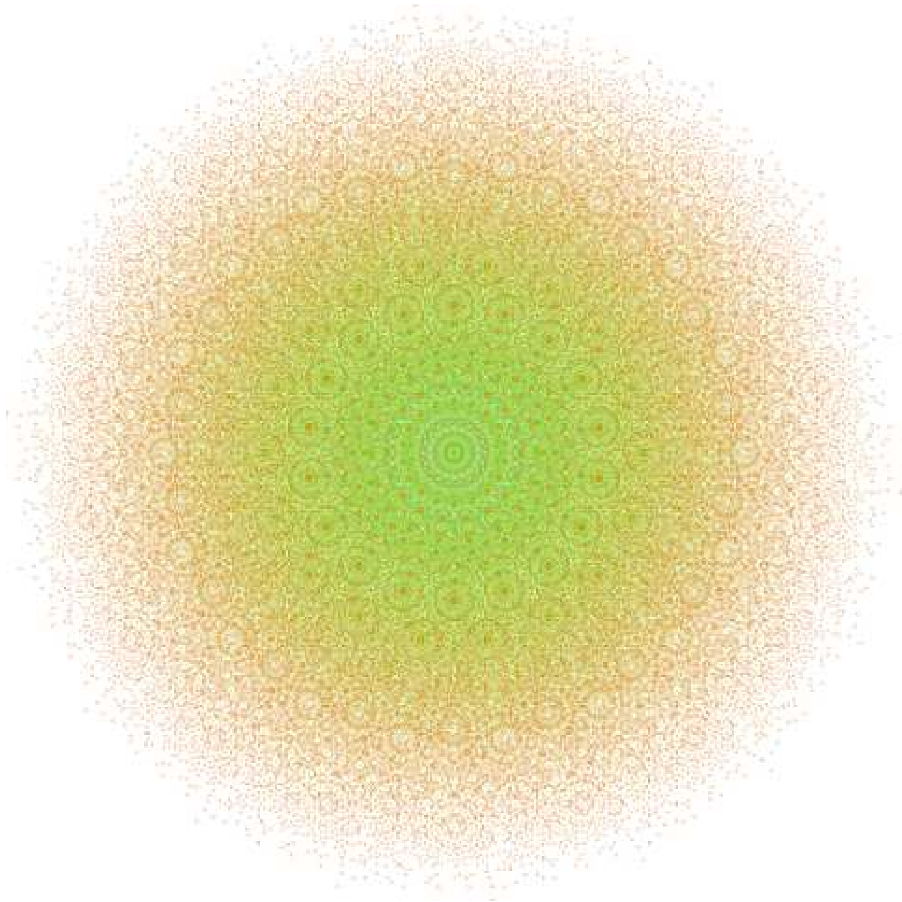


Defined by **generators** $\{e_i, f_i, h_i\}$ and **relations** via **Cartan matrix** A_{ij} **('Chevalley-Serre presentation')**

$$
\begin{aligned}
[h_i, h_j] &= 0, \qquad & [e_i, f_j] = \delta_{ij} h_i, \\
[h_i, e_j] &= A_{ij} e_j, \qquad & [h_i, f_j] = -A_{ij} f_j, \\
(\mathrm{ad}\, e_i)^{1-A_{ij}} e_j &= 0 \qquad & (\mathrm{ad}\, f_i)^{1-A_{ij}} f_j = 0.
\end{aligned}
$$

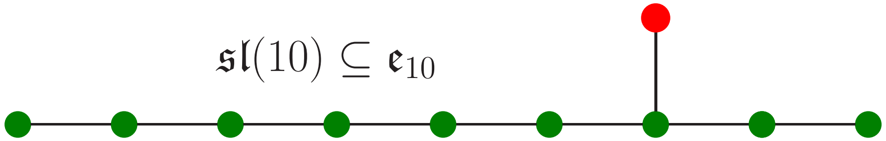
$\mathfrak{e}_{10}$ is the free Lie algebra generated by $\{e_i, f_i, h_i\}$ modulo these relations → infinite dimensional as A_{ij} is ***indefinite*** → Lie algebra of ***exponential growth*** !

A planar slice through the E_{10} root system



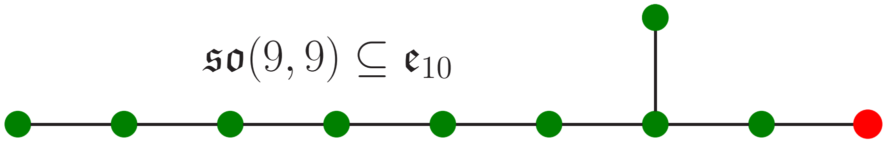
Duality symmetries: all in one?

$\mathfrak{sl}(10) \subseteq \mathfrak{e}_{10}$



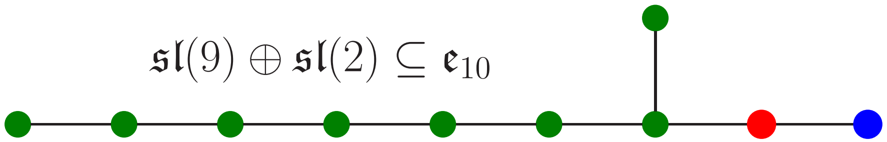
$D = 11$ **SUGRA**

$\mathfrak{so}(9,9) \subseteq \mathfrak{e}_{10}$



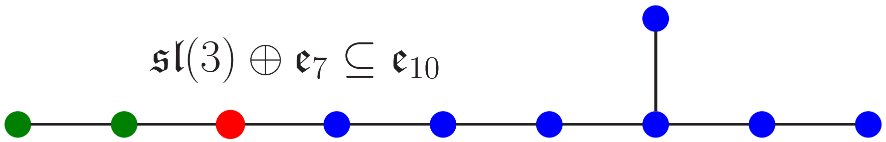
mIIA $D = 10$ **SUGRA**

$\mathfrak{sl}(9) \oplus \mathfrak{sl}(2) \subseteq \mathfrak{e}_{10}$



IIB $D = 10$ **SUGRA**

$\mathfrak{sl}(3) \oplus \mathfrak{e}_7 \subseteq \mathfrak{e}_{10}$



$\mathcal{N} = 8,\ D = 4$ **SUGRA**

Example: $SL(10)$ level decomposition

- **Decomposition w.r.t. $SL(10)$ subgroup in terms of $SL(10)$ tensors → *level expansion*** [Damour,Hennaux,HN(2002)]

$$\alpha = \ell\alpha_0 + \sum_{j=1}^{9} m^j \alpha_j \quad \Rightarrow \quad \mathrm{E}_{10} = \bigoplus_{\ell \in \mathbb{Z}} \mathrm{E}_{10}^{(\ell)}$$

- **Up to $\ell \leq 3$ basic fields of $D = 11$ SUGRA together with their magnetic duals (spatial components)**

$\ell = 0$	G_{mn}	**Graviton**
$\ell = 1$	A_{mnp}	**3-form**
$\ell = 2$	$A_{m_1 \dots m_6}$	**dual 6-form**
$\ell = 3$	$h_{m_1 \dots m_8 \vert n}$	**dual graviton**

- **Analysis up to level $\ell \leq 28$ yields 4 400 752 653 representations (Young tableaux) of $SL(10)$** [Fischbacher,HN:0301017]
- **Lie algebra structure (structure constants, etc.) understood only up to $\ell \leq 4$. Also: no matter where you stop it will get even more complicated beyond!**

Tantalizing Hints, Persistent Questions

- **Recover bosonic multiplets and dynamics of maximal supergravities by appropriately 'slicing' E_{10}.**
- E_{10} **'knows all' about supersymmetry $\rightarrow$ may well *supersede supersymmetry* as a unifying principle!**
- **Quantum Gravity: old problems in a new guise!**

BUT

- **No concrete realization of KMA (after 50 years!)**
- **Physical significance of higher level representations?**
- **How is (de-)emergence of space-time realized?**
- **How is UV completion of SM achieved?**

While it may take a long time to resolve these questions there is some progress on another front....

Fermions and $\mathrm{K}(\mathrm{E}_{10})$

... probably a key issue for further progress...

Important point: maximally supersymmetric theories *not* based on (hypothetical) superextensions of E_n:

- **There is no proper superextension of E_n for any n.**
- **For $D \geq 3$ supergravity fermions transform in *maximal compact subgroup* $K(E_n) \subset E_{n(n)}$, e.g.**

$K(E_7) \equiv SU(8)$	**fermions $\in$ 8 and 56**
$K(E_8) \equiv Spin(16)/Z_2$	**fermions $\in$ $\mathbf{16}_v$ and $\mathbf{128}_c$**

- **The associated (double-valued) fermion representations are not 'liftable' to E_n representations**
- **Expect all of this to remain true for $\mathrm{K}(\mathrm{E}_{10}) \subset \mathrm{E}_{10}$.**

What is $\mathrm{K}(\mathrm{E}_{10})$**?**

For E_{10} **the 'maximal compact' subalgebra is defined as fixed point algebra of the Chevalley involution**

$$\omega(e_j) = -f_j\ , \quad \omega(f_j) = -e_j\ , \quad \omega(h_j) = -h_j$$

together with invariance property $[\omega(x), \omega(y)] = \omega([x, y])$

$$\Rightarrow \quad \mathrm{E}_{10} = \mathrm{K}(\mathrm{E}_{10}) \oplus \mathrm{K}(\mathrm{E}_{10})^{\perp}\ , \qquad x = \omega(x) \textbf{ for } x \in \mathrm{K}(\mathrm{E}_{10})$$

This definition is analogous to the corresponding one for the finite-dimensional case, e.g. $x = \omega(x) \in \mathfrak{so}(n) \subset \mathfrak{sl}(n)$ **for** $\omega(x) = -x^T$**, with corresponding decomposition** $\mathfrak{sl}(n) = \mathfrak{so}(n) \oplus \mathfrak{so}(n)^{\perp}$

Consequently, $\mathrm{K}(\mathrm{E}_{10})$ **is generated by** $x_i := e_i - f_i = \omega(x_i)$ **with Berman-Serre relations**

$$\begin{aligned} \big[x_i\,,\, x_j\big] &= 0 && \textit{if } i \textit{ and } j \textit{ are non-adjacent} \\ \big[x_i, [x_i, x_j]\big] + x_j &= 0 && \textit{if } i \textit{ and } j \textit{ are adjacent} \end{aligned}$$

Theorem: ***each set of*** $\{x_i\}$ ***satisfying the above relations provides a realization of*** $\mathrm{K(E_{10})}$***.*** [S.Berman(1989)]

But: $\mathrm{K(E_{10})}$ **is** ∞**-dimensional and a very strange beast!**

- $\mathrm{K(E_{10})}$ **has finite-dimensional (unfaithful) representations**
- $\Rightarrow$ $\mathrm{K(E_{10})}$ **is *not* simple (**$\equiv$ **has non-trivial ideals)**
- **No faithful fermionic (double-valued) representations are known!**

More specifically: ***Rarita-Schwinger*** **(RS) representation** $\rightarrow$ **8 gravitinos and 56 spin-**$\frac{1}{2}$ **fermions of maximal** $N = 8$ **supergravity** ***at one spatial point*** **form an unfaithful irreducible spinorial representation of** $\mathrm{K(E_{10})}$**.**

Complete breaking of $N = 8$ **supersymmetry: absorb eight Goldstinos to get eight massive gravitinos** $\Rightarrow$

Idem for 8 ***massive*** **gravitinos and 48 spin-**$\frac{1}{2}$ **fermions** $\cong$ **3** $\times$ **16 quarks and leptons?!?**

$N = 8$ Supergravity: a strange coincidence?

$SO(8) \to SU(3)\times U(1)$ **breaking and 'family-color locking'**

$$
\begin{array}{llll}
(u\,,\,c\,,\,t)_L & : & \mathbf{3}_c \times \bar{\mathbf{3}}_f \to \mathbf{8} \oplus \mathbf{1}\,, & +\frac{1}{2} = \frac{2}{3} - q \\
(\bar{u}\,,\,\bar{c}\,,\,\bar{t})_L & : & \bar{\mathbf{3}}_c \times \mathbf{3}_f \to \mathbf{8} \oplus \mathbf{1}\,, & -\frac{1}{2} = -\frac{2}{3} + q \\
(d\,,\,s\,,\,b)_L & : & \mathbf{3}_c \times \mathbf{3}_f \to \mathbf{6} \oplus \bar{\mathbf{3}}\,, & -\frac{1}{6} = -\frac{1}{3} + q \\
(\bar{d}\,,\,\bar{s}\,,\,\bar{b})_L & : & \bar{\mathbf{3}}_c \times \bar{\mathbf{3}}_f \to \bar{\mathbf{6}} \oplus \mathbf{3}\,, & +\frac{1}{6} = \frac{1}{3} - q \\
(e^-, \mu^-, \tau^-)_L & : & \mathbf{1}_c \times \mathbf{3}_f \to \mathbf{3}\,, & -\frac{5}{6} = -1 + q \\
(e^+, \mu^+, \tau^+)_L & : & \mathbf{1}_c \times \bar{\mathbf{3}}_f \to \bar{\mathbf{3}}\,, & +\frac{5}{6} = 1 - q \\
(\nu_e\,,\,\nu_\mu\,,\,\nu_\tau)_L & : & \mathbf{1}_c \times \bar{\mathbf{3}}_f \to \bar{\mathbf{3}}\,, & -\frac{1}{6} = 0 - q \\
(\bar{\nu}_e\,,\,\bar{\nu}_\mu\,,\,\bar{\nu}_\tau)_L & : & \mathbf{1}_c \times \mathbf{3}_f \to \mathbf{3}\,, & +\frac{1}{6} = 0 + q
\end{array}
$$

Supergravity and Standard Model assignments agree if spurion charge is chosen as $q = \frac{1}{6}$ [Gell-Mann (1983)]

Realized at $SU(3)\times U(1)$ **stationary point!** [Warner,HN, NPB259(1985)412]

Embedding SM Symmetries into $\mathrm{K}(\mathrm{E}_{10})$

[Meissner,HN: Phys.Rev.D91(2015)065029]

Spurion charge shift can be realised as $\exp(\frac{1}{6}\omega\mathcal{I})$

$$\mathcal{I} = \frac{1}{2}\big(T \wedge \mathbf{1} \wedge \mathbf{1} + \mathbf{1} \wedge T \wedge \mathbf{1} + \mathbf{1} \wedge \mathbf{1} \wedge T + T \wedge T \wedge T\big) \;\; \Rightarrow \;\; \mathcal{I}^2 = -\mathbf{1}$$

acting on 56 fermions χ^{ijk} **in** $\mathbf{8} \wedge \mathbf{8} \wedge \mathbf{8}$ **of SU(8), with**

$$T = \begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \big[= \textbf{imaginary unit for SU(3)} \times \textbf{U(1)}\big]$$

$\mathcal{I}$ **is *not* in SU(8)** $\equiv K(\mathrm{E}_7)$ **... but it is in** $K(\mathrm{E}_{10})$**!**

Also need to extend action of $\mathcal{I}$ **to gravitinos.**

Why $\mathcal{I}$ belongs to $\mathrm{K(E_{10})}$

[Kleinschmidt,HN:Phys.Lett.B747 (2015)]

D=11 fermions in Coulomb gauge split as ($\hat{a} = 1,2,3\,;\ \bar{a} = 4,...,10$)

$$\Psi_A^a = (\Psi_{\alpha i}^{\hat{a}}\,,\ \Psi_{\alpha i}^{\bar{a}}) \quad \textbf{with } i,j = 1,...,8 \textbf{ and } \alpha = 1,2,3,4$$

$N{=}8$ supergravity fermions from $D{=}11$ gravitino [Cremmer,Julia(1979)]

$$\psi_{\hat{a}\alpha}^i \propto \Psi_{\alpha i}^{\hat{a}} - \frac{1}{2}\sum_{\bar{c}=4}^{10} \Gamma_{ij}^{\bar{c}}(\gamma^5\gamma_{\hat{a}}\Psi_j^{\bar{c}})_\alpha \quad , \qquad \chi^{ijk} \propto \sum_{\bar{a}=4}^{10} \Gamma_{[ij}^{\bar{a}}\Psi_{k]\alpha}^{\bar{a}}$$

With redefined variables $\Phi_A^{\mathsf{a}} = \Gamma_{AB}^a \Psi_B^a$ (no summation!) [Damour,Hillmann]

$$\delta\chi_{ijk} = (T\wedge T\wedge T)_{ijk}{}^{lmn}\chi_{lmn} \quad \leftrightarrow \quad \delta\Phi_{i\alpha}^{\mathsf{a}} = T_{ij}\Phi_{j\alpha}^{\mathsf{a}} \qquad (*)$$

Latter formula provides a realization of $\mathcal{I}$ on *all* fermions.

For any *real* E_{10} root α we have (with $\alpha^{\mathsf{a}} \equiv G^{\mathsf{ab}}\alpha_{\mathsf{b}}$) [Kleinschmidt,HN]

$$\delta(\alpha)\Phi_A^{\mathsf{a}} = \left(-\frac{1}{2}\alpha^{\mathsf{a}}\alpha_{\mathsf{b}} + \frac{1}{4}\delta_{\mathsf{b}}^{\mathsf{a}}\right)\Gamma(\alpha)_{AB}\Phi_B^{\mathsf{b}}$$

Thus need only find linear combination to reproduce $(*)$, which is possible because there are *infinitely many* real roots in E_{10}.

The proof requires over-extended root of $\mathrm{E}_{10} \Rightarrow$ no way to realise q-shift with finite-dimensional R symmetries!

More properly, this representation is acted on by

$$\mathcal{Q}_{RS} = \mathrm{K}(\mathrm{E}_{10})/\mathcal{N}_{RS} = \mathrm{SO}(32, 288)$$

where $\mathcal{N}_{RS}$ is the 'normal subgroup' generated by the RS ideal in $\mathrm{K}(\mathrm{E}_{10})$ – but $\mathcal{Q}_{RS}$ is *not* a subgroup of $\mathrm{K}(\mathrm{E}_{10})$.

In recent work we have been able to embed full SM group $\mathbf{SU(3)}_c \times \mathbf{SU(2)}_w \times \mathbf{U(1)}_Y$ into $\mathcal{Q}_{RS}$ together with a family symmetry $\mathbf{SU(3)}_f$ which does *not* commute with electroweak symmetries. [Meissner,HN, PRL121(2018)091601]

Big open questions: how does $\mathrm{K}(\mathrm{E}_{10})$ 'unfold' to give rise to spatial dependence and space-time symmetries via infinite chain of finite groups $\mathcal{Q}_{RS} < \cdots < \mathrm{K}(\mathrm{E}_{10})$?? And how is $\mathrm{K}(\mathrm{E}_{10})$ broken to SM symmetries??

Curious Gravitinos

[K.Meissner,HN: PRD100(2019)035001]

Under $\mathbf{SU(3)}_c \times \mathbf{U(1)}_{em}$ gravitinos transform as

$$\left(\mathbf{3}_c\,,\,\frac{1}{3}\right) \oplus \left(\bar{\mathbf{3}}_c\,,\,-\frac{1}{3}\right) \oplus \left(\mathbf{1}_c\,,\,\frac{2}{3}\right) \oplus \left(\mathbf{1}_c\,,\,-\frac{2}{3}\right)$$

Unusual features:

- **strong and electromagnetic interactions $\Rightarrow$**
- **would have been seen *unless* mass is very high, and cosmological abundance *extremely low***
- **would be stable against decay into SM matter because of peculiar quantum numbers**

$\Big[$ → **very different from gravitinos in $N=1$ SUGRA models, which are uncharged under SM symmetries, and interact only weakly**$\Big]$

Not the usual Dark Matter Candidate

- **No SUSY: all gravitinos have masses** $\sim M_{\rm PL}$
- **Split as $\mathbf{3}\oplus\bar{\mathbf{3}}\oplus\mathbf{1}\oplus\mathbf{1}$ under SU(3) $\rightarrow$ could form color singlet bound states with ordinary quarks.**
- **Fractionally charged $\Rightarrow$ stable despite large mass!**
- **Despite strong and electromagnetic interactions can easily pass through Earth because of large mass.**
- **Non-relativistic $\Rightarrow$ time of flight measurements?**
- **DM mass density in solar system $\sim 10^6$ GeV/m^3 $\Rightarrow$ 10^{-13} gravitinos/m^3 $\Rightarrow$ flux $L \lesssim 10^{-9}$ m^{-2}s^{-1} $\rightarrow$ DM detector would get hit only extremely rarely.**
- **Idea: look for long ionized tracks in *ultrastable* material (rock, diamond,...?) $\rightarrow$ need a *'paleo-detector'***

  [see e.g.:J.Bramante et al.,1803.08044[hep-ph];S.Baum et al., 1806.05991[astro-ph.CO]]

Explaining UHECRs?

[K.Meissner, HN: JCAP1909(2019)041]

New mechanism: **color triplet gravitinos could explain observed UHECR events via gravitino-antigravitino annihilation in the 'skin' of neutron stars, provided**

- **Gravitinos get absorbed into stars ...**
- **... and get 'condensed' in neutron stars so as to enable them to annihilate in appreciable rates**

New features:

- **could explain dominant appearance of ions (rather than protons) towards very highest energies**
- **with some 'reasonable' assumptions calculated event rates come close to the ones observed at Pierre Auger Observatory (in Argentina)**
- **Hints of E_{10} and $K(E_{10})$ 'in the sky'?**

Summary and Outlook

- E_{10} **and** $K(E_{10})$ **unify and generalize known duality symmetries of supergravity and string theory.**
- **All results obtained so far indicate that** E_{10} **requires a setting beyond known concepts of space and time.**
- $\Rightarrow$ **quantum field theory, general covariance and local supersymmetry would have to be *emergent*.**
- **However: explaining how this emergence works in detail remains an outstanding challenge!**
- **Intriguing links between** $K(E_{10})$ **and SM fermions:** $\rightarrow$ **can** E_{10} **and** $K(E_{10})$ ***supersede supersymmetry*** **as a guiding principle towards unification?**
- **Ultimate hope: no multiverse, but an actual explanation why low energy world is the way it is...**