from synthetic gravity to general relativity

â â Aalto University

G. Volovik

18.10.2019 Chernogolovka

Landau Institute

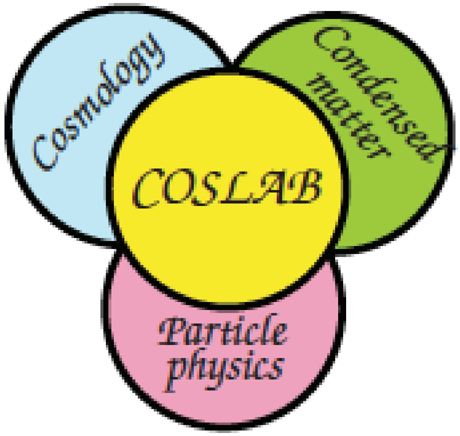
RUSSIAN ACADEMY OF SCIENCES
L.D Landau
INSTITUTE FOR
THEORETICAL
PHYSICS

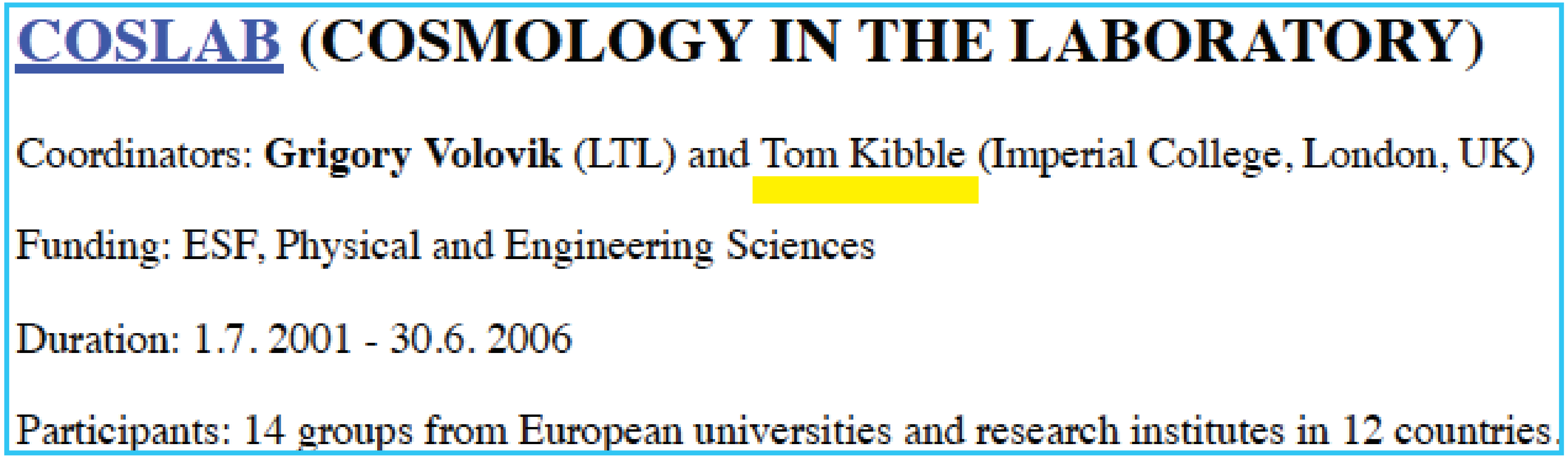
COSLAB (Cosmology in Laboratory) workshop, Finland, August 17-22, 2004

erc
European Research Council

European Commission

Horizon 2020
European Union funding
for Research & Innovation





COSLAB (COSMOLOGY IN THE LABORATORY)

Coordinators: **Grigory Volovik** (LTL) and Tom Kibble (Imperial College, London, UK)

Funding: ESF, Physical and Engineering Sciences

Duration: 1.7. 2001 - 30.6. 2006

Participants: 14 groups from European universities and research institutes in 12 countries.

Gravity as flow of vacuum: Tolman law in superfluid hydrodynamics

СОВРЕМЕННЫЕ
ПРОБЛЕМЫ
ФИЗИКИ

И. М. ХАЛАТНИКОВ

ВВЕДЕНИЕ В ТЕОРИЮ
СВЕРХТЕКУЧЕСТИ

Издательство «Наука».
Главная редакция физико-математической литературы, 1965

phonon free energy in moving superfluid

v -velocity of moving superfluid vacuum

c - speed of sound (analog of speed of light)

$$F_{\rm ph}(T,v)=\frac{F_{\rm ph}(T)}{\left(1-\frac{v^2}{c^2}\right)^2}\sim\left(\frac{T}{\sqrt{1-\frac{v^2}{c^2}}}\right)^4$$

$$F_{\rm ph}(T,v)\sim\left(\frac{T}{\sqrt{g_{00}}}\right)^4 \quad , \quad g_{00}(\mathbf{r})=1-\frac{v^2(\mathbf{r})}{c^2}$$

$$T(\mathbf{r})=T/\sqrt{g_{00}(\mathbf{r})}$$

Tolman law in GR

Black hole analog in moving liquid

W. G. Unruh (1981)
Experimental black-hole evaporation?
Phys. Rev. Lett. 46, 1351

Cosmology
Condensed matter
COSLAB
Particle physics

Painleve-Gulstrand metric - fluid metric

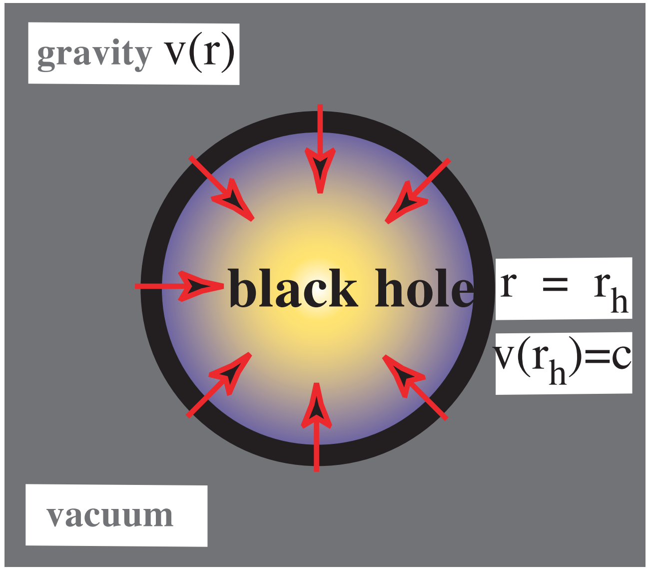
$$ds^2 = - dt^2 \underset{\uparrow \atop g_{00}}{(c^2 - v^2)} + \underset{\uparrow \atop g_{0r}}{2\, v}\, dr\, dt + dr^2 + r^2 d\Omega^2$$

$$v^2(r) = \frac{2GM}{r} = c^2 \frac{r_h}{r}$$

Painleve-Gulstrand metric - fluid metric

$$ds^2 = -\,dt^2\,\underset{\substack{\uparrow\\ g_{00}}}{(c^2 - v^2)} \;+\; \underset{\substack{\uparrow\\ g_{0r}}}{2\,v}\,dr\,dt \;+\; dr^2 \;+\; r^2 d\Omega^2$$

$$v^2(r) = \frac{2GM}{r} = c^2\,\frac{r_h}{r}$$



horizon at $g_{00} = 0$ (or $v(r_h) = c$)

СОВРЕМЕННЫЕ
ПРОБЛЕМЫ
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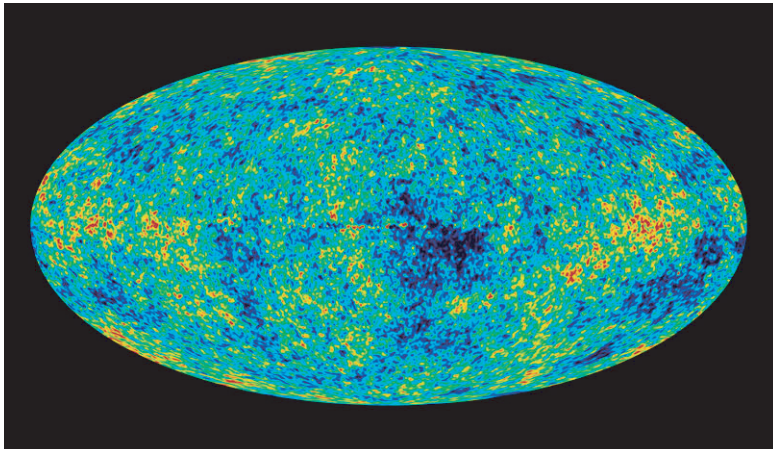
И. М. ХАЛАТНИКОВ

ВВЕДЕНИЕ В ТЕОРИЮ
СВЕРХТЕКУЧЕСТИ

Superfluid hydrodynamics as 1/2 of GR

Superfluids

Universe



acoustic gravity

metric theories of gravity

general relativity

geometry of effective space time
for matter (phonons)

$g_{\mu\upsilon}$

geometry of space time
for matter

geodesics for phonons

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = 0$$

geodesics for photons

equation
for phonons

$$\boxed{T^{\mu\nu}_{;\nu\ \text{Matter}} = 0}$$

equation
for matter

Khalatninkov
"Theory of superfluidity"
Nauka, 1971

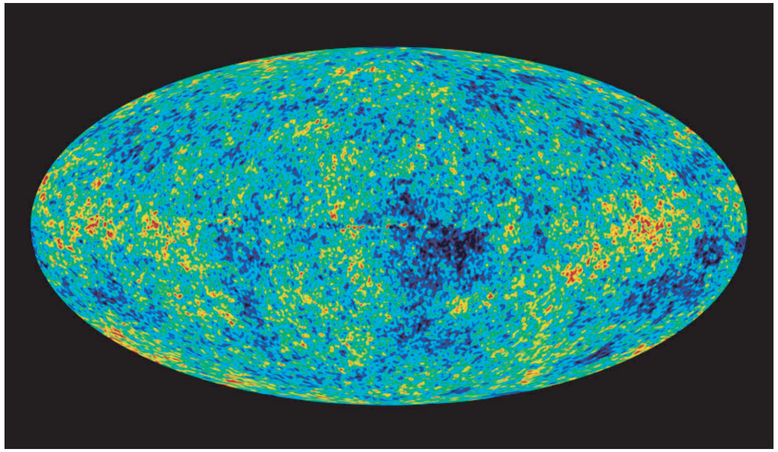
1/2 of GR

where is another 1/2 of GR ?

what about the other 1/2 of GR ?

superfluid component

quantum vacuum



equation
for phonons

$$\boxed{T^{\mu\nu}_{;\nu\ \mathrm{Matter}} = 0}$$

equation
for matter

at first glance no connections

equation for superfluid velocity

$$\boxed{\dot{\mathbf{v}}_s + \nabla\left(\mu + \mathbf{v}_s{}^2/2\right) = 0}$$

equation for mass density

$$\boxed{\dot{\rho} + \nabla\cdot\left(\rho\mathbf{v}_s + \mathbf{P}^{\mathrm{Matter}}\right) = 0}$$

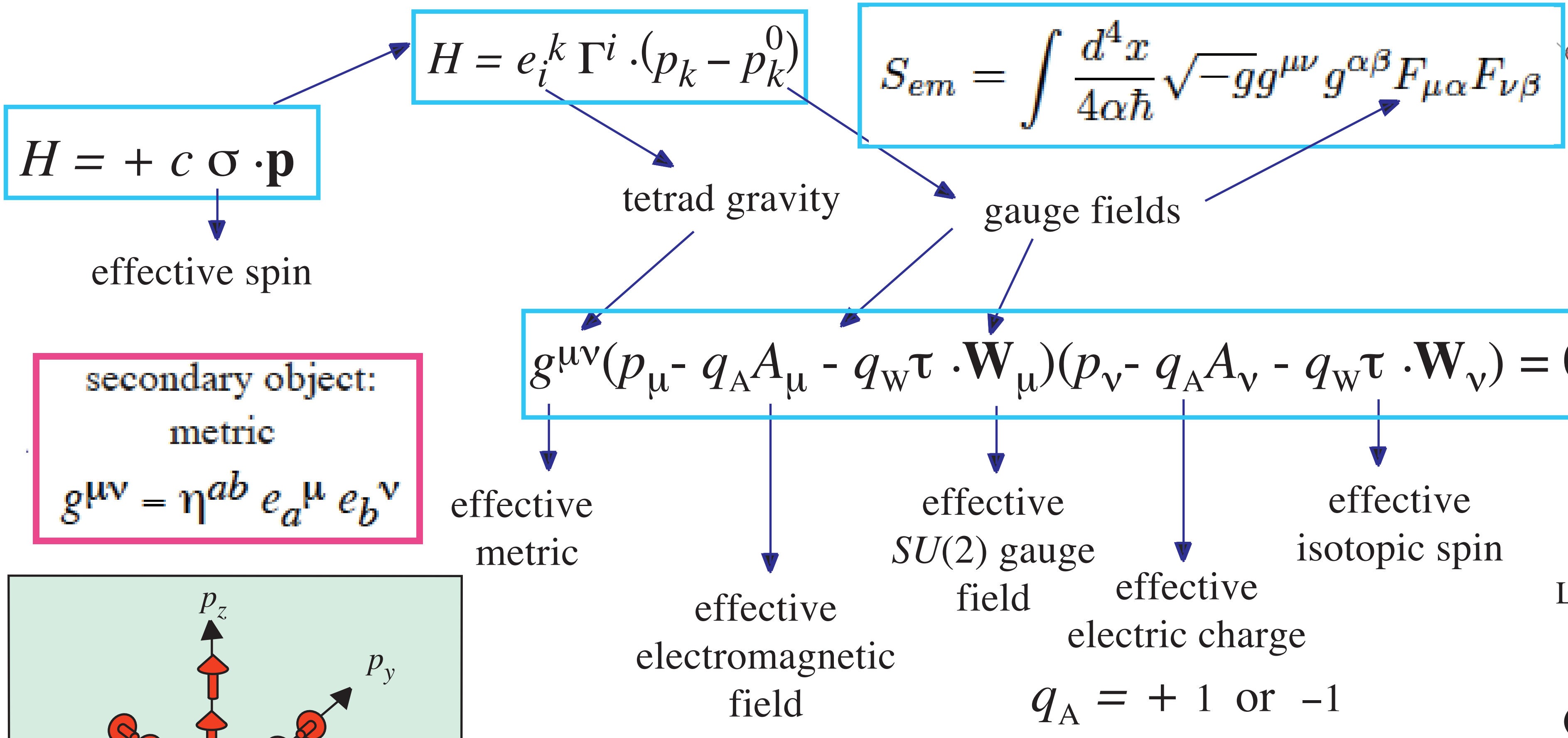
Einstein equations for metric field

$$\boxed{\frac{1}{8\pi G}\left(R_{\mu\nu} - g_{\mu\nu}R/2\right) = \Lambda g_{\mu\nu} + T^{\mathrm{Matter}}_{\mu\nu}}$$

Gravity emerging near Weyl points in Weyl semimetals & superfluids

topological origin of emergent relativity, gauge fields, tetrad gravity, chirality, spin ..

Atiyah-Bott-Shapiro construction: expansion of Hamiltonian near the node in terms of Dirac Γ-matrices



$$H = + c\,\sigma \cdot \mathbf{p}$$

effective spin

$$H = e_i^k\,\Gamma^i \cdot (p_k - p_k^0)$$

tetrad gravity

gauge fields

$$S_{em} = \int \frac{d^4x}{4\alpha\hbar}\sqrt{-g}\,g^{\mu\nu}g^{\alpha\beta}F_{\mu\alpha}F_{\nu\beta}$$

$$g^{\mu\nu}(p_\mu - q_A A_\mu - q_W \tau \cdot \mathbf{W}_\mu)(p_\nu - q_A A_\nu - q_W \tau \cdot \mathbf{W}_\nu) = 0$$

effective metric

effective electromagnetic field

effective $SU(2)$ gauge field

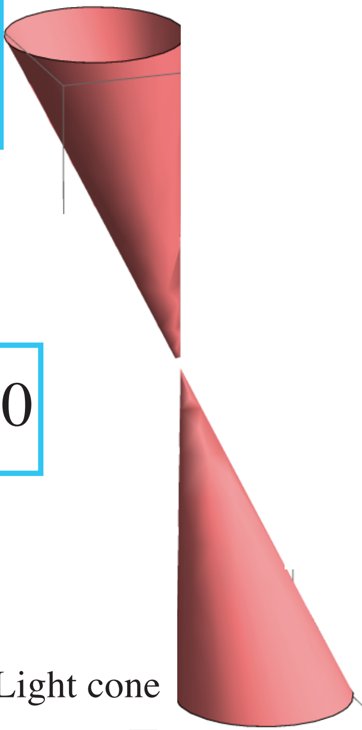
effective electric charge

$q_A = +1$ or -1

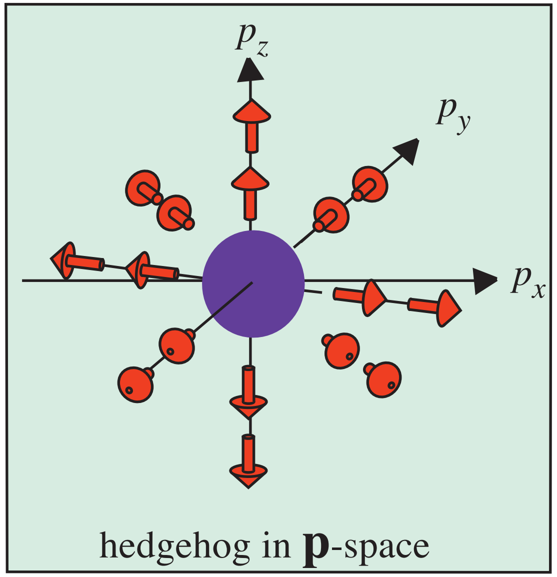
effective isotopic spin

secondary object:
metric

$$g^{\mu\nu} = \eta^{ab}\,e_a^{\mu}\,e_b^{\nu}$$

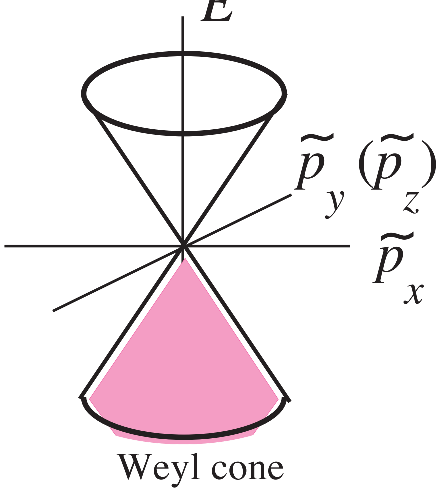


Light cone



hedgehog in **p**-space

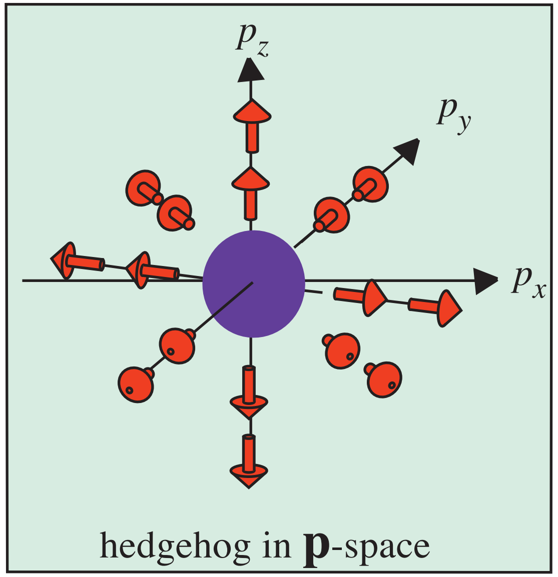
all ingredients of Standard Model:
chiral fermions & gauge fields emerge together with
geometry , spin , Dirac Γ−matrices, & physical laws:
Lorentz & gauge invariance, equivalence principle, etc



Weyl cone

General relativity vs hydrodynamics

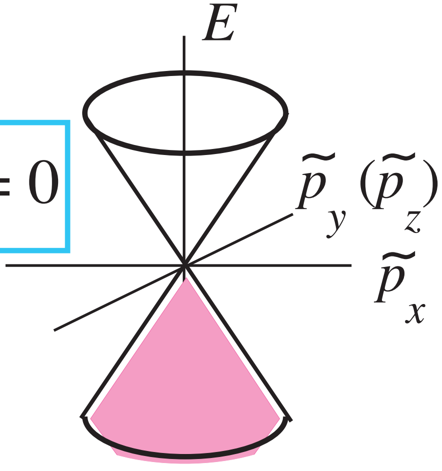
secondary object:
metric
$$g^{\mu\nu} = \eta^{ab} e_a^{\ \mu} e_b^{\ \nu}$$



hedgehog in **p**-space

$$g^{\mu\nu}(p_\mu - q_A A_\mu - q_W \tau \cdot \mathbf{W}_\mu)(p_\nu - q_A A_\nu - q_W \tau \cdot \mathbf{W}_\nu) = 0$$

$$S_{em} = \int \frac{d^4x}{4\alpha\hbar} \sqrt{-g} g^{\mu\nu} g^{\alpha\beta} F_{\mu\alpha} F_{\nu\beta}$$



Weyl cone

what are equations for metric (tetrad)?

UV cut-off
>>
Lorentz violating scale

$$\dot{\mathbf{v}}_s + \nabla(\mu + \mathbf{v}_s^2/2) = 0$$

$$\dot{\rho} + \nabla \cdot (\rho \mathbf{v}_s + \mathbf{P}^{\text{Matter}}) = 0$$

UV cut-off
<<
Lorentz violating scale

$$\frac{1}{8\pi G}(R_{\mu\nu} - g_{\mu\nu} R/2) = \Lambda g_{\mu\nu} + T^{\text{Matter}}_{\mu\nu}$$

Nieh-Yan anomaly in GR & hydrodynamics

Nieh & Yan
J. Math. Phys. 23, 373 (1982)
Ann. Phys. 138, 237 (1982)

Nieh-Yan gravitational anomaly in terms of tetrads, torsion, curvature & UV cut-off

$$\boxed{\partial_\mu J^\mu_5 = \frac{\Lambda^2}{4\pi^2} N(\mathbf{r},t)}$$

$$\boxed{N = \mathcal{T}\wedge\mathcal{T} + e\wedge e\wedge R}$$

Nissinen & GV, arXiv:1909.08936

thermal Nieh-Yan anomaly in terms of tetrads, torsion, curvature & temperature

$$\boxed{\partial_\mu J^\mu_5 = \gamma T^2 N(\mathbf{r},t)}$$

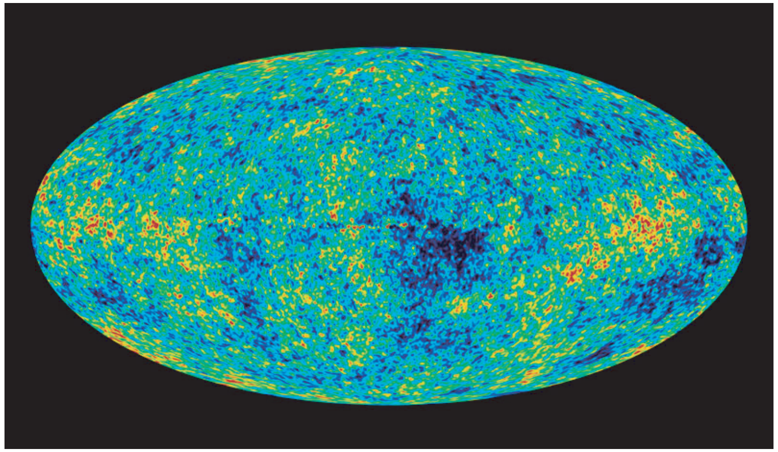
hydrodynamics

$$\boxed{\nabla_\mu j^\mu_5 = -\frac{T^2}{12}\left(1+\frac{m^*}{m}\right)N(\mathbf{r},t)}$$

GR

$$\boxed{\nabla_\mu j^\mu_5 = -\frac{T^2}{12}N(\mathbf{r},t)}$$

Gibbs-Duhem relation, self-sustained vacuum & cosmological constant



$$\boxed{\dot{\mathbf{v}}_s + \nabla(\mu + \mathbf{v}_s^2/2) = 0}$$

$$\boxed{\dot{\rho} + \nabla\cdot(\rho\mathbf{v}_s + \mathbf{P}^{\text{Matter}}) = 0}$$

$$\boxed{\frac{1}{8\pi G}(R_{\mu\nu} - g_{\mu\nu}R/2) = \Lambda g_{\mu\nu} + T_{\mu\nu}^{\text{Matter}}}$$

thermodynamic potential

$$\boxed{\varepsilon(\rho) - \mu\rho = -P}$$

plays the role of vacuum energy

$$\boxed{\varepsilon_{vac}(\rho) = \varepsilon(\rho) - \mu\rho = -P_{vac}}$$

in thermodynamic equilibrium the self-sustained liquid has:

$$\boxed{\varepsilon_{vac}(\rho) = 0}$$

vacuum energy & cosmological constant

$$\boxed{\varepsilon_{vac} = -P_{vac} = \Lambda}$$

if quantum vacuum is self-sustained, then due Gibbs-Duhem relation the cosmological constant Λ is zero in thermodynamic equilibrium

Vacuum energy in cond-mat Quantum Field Theory

$$H_{QFT} = H - \mu N$$

$$H_{QFT} = \int d^3x\, \psi^+\left(-\frac{\Delta}{2m} - \mu\right) \psi + \iint d^3x\, d^3y\, U(x\text{-}y)\, \psi^+(x)\psi^+(y)\psi(y)\psi(x)$$

Macroscopic (thermodynamic) approach for vacuum (ground state) energy

$$\epsilon_{vac} V = \langle H_{QFT} \rangle_{vac} = \langle H \rangle_{vac} - \mu \langle N \rangle_{vac}$$

$\langle H_{QFT} \rangle_{vac}$ ↓ thermodynamic infrared scale

$\langle H \rangle_{vac}$ ↓ “Planck” ultraviolet scale

$\mu \langle N \rangle_{vac}$ ↓ “Planck” ultraviolet scale

thermodynamic compensation of zero point vacuum energy

thermodynamic Gibbs-Duhem relation

$$\langle H \rangle - \mu \langle N \rangle - TS = -pV$$

$$\Big\downarrow \; T = 0$$

$$\epsilon_{vac} = -p_{vac}$$

all quantum vacua
(relativistic and non-relativistic)
obey the same equation of state

self-sustained vacuum can be described e.g. by variable suggested by Hawking - the 4-form field

action

$$\boxed{S = \int dV\, dt\, \varepsilon\, (q)}$$

$$\boxed{\begin{gathered} q^2 = -\frac{1}{24} F_{\kappa\lambda\mu\nu} F^{\kappa\lambda\mu\nu} \\ F_{\kappa\lambda\mu\nu} = \nabla_{[\kappa} A_{\lambda\mu\nu]} \\ F^{\kappa\lambda\mu\nu} = q\, e^{\kappa\lambda\mu\nu} \end{gathered}}$$

Maxwell equation

$$\begin{tikzcd}
\boxed{\nabla_\kappa \left(F^{\kappa\lambda\mu\nu}\; q^{-1} d\varepsilon/dq\right) = 0} \arrow[r] & \boxed{\nabla_\kappa \left(d\varepsilon/dq\right) = 0} \arrow[d] \\
 & \boxed{d\varepsilon/dq = \mu}
\end{tikzcd}$$

integration constant μ in dynamics
becomes chemical potential in thermodynamics

4-form field $F_{\kappa\lambda\mu\nu}$ is an example of conserved charge q in relativistic vacuum

lesson from cond-mat: cosmological constant vs vacuum energy

action in q-theory

$$S = \int d^4x\, (-g)^{1/2} \left[\, \varepsilon(q) + KR \,\right] + S_{\mathrm{matter}}$$

motion equation

$$d\varepsilon/dq = \mu$$

Einstein equations

$$K\left(Rg_{\mu\nu} - 2R_{\mu\nu}\right) + \left(\varepsilon - \mu q\right)g_{\mu\nu} = T_{\mu\nu}$$

Einstein tensor

cosmological term $\Lambda\, g_{\mu\nu} \neq \varepsilon\, g_{\mu\nu}$

matter

cosmological constant is not equal to vacuum energy: cosmological constant equals the thermodynamic potential

vacuum energy is determined by UV physics, it is large
thermodynamic potential is determined by IR physics,
it is small in equilibrium

dynamics of q in curved space: relaxation of Λ at fixed $\mu=\mu_0$

Maxwell equations

$$d\varepsilon/dq + R\, dK/dq = \mu_0$$

Einstein equations

$$K\big(Rg_{\mu\nu} - 2R_{\mu\nu}\big) + g_{\mu\nu}\,\Lambda(q) - 2\big(\nabla_\mu\nabla_\nu - g_{\mu\nu}\nabla^\lambda\nabla_\lambda\big)\,K = T^{\text{matter}}_{\mu\nu}$$

$$\Lambda(q) = \varepsilon(q) - \mu_0 q$$

dynamic solution

$$q(t) - q_0 \sim q_0\,\frac{\sin \omega t}{t}$$

$$\Lambda(t) \sim \omega^2\,\frac{\sin^2 \omega t}{t^2}$$

$$H(t) = \frac{\dot{a}(t)}{a(t)} = \frac{2}{3t}\big(1 - \cos \omega t\big)$$

$$\omega \sim E_{\text{Planck}}$$

similar to scalar field with mass $M \sim E_{\text{Planck}}$
A.A. Starobinsky, Phys. Lett. **B 91**, 99 (1980)

Relaxation of Λ (generic q-independent properties)

$$\Lambda(t) \sim \omega^2 \frac{\sin^2 \omega t}{t^2}$$

cosmological "constant"

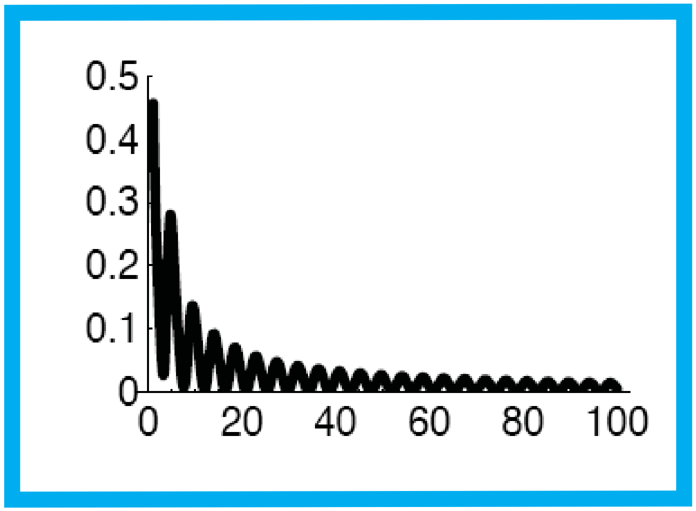
$$H(t) = \frac{\dot{a}(t)}{a(t)} = \frac{2}{3t}\left(1 - \cos \omega t\right)$$

Hubble parameter

$$G(t) = G_N \left(1 + \frac{\sin \omega t}{\omega t}\right)$$

Newton "constant"

$$\omega \sim E_{\text{Planck}}$$



$$<\Lambda(t_{\text{Planck}})> \sim E^4_{\text{Planck}}$$

$$\Lambda(t = \infty) = 0$$

natural solution of the main cosmological problem ?

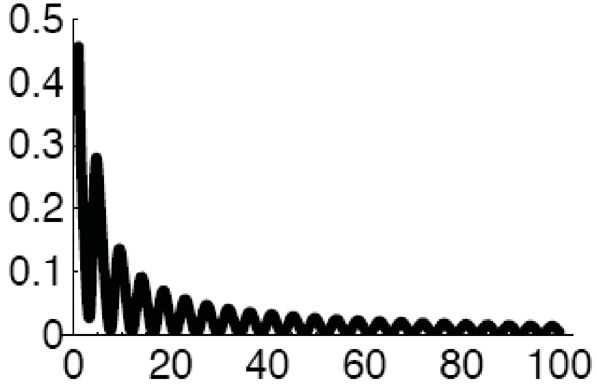
Λ relaxes from natural Planck scale value
to natural zero value

present value of Λ

$$\Lambda(t) \sim \omega^2 \frac{\sin^2 \omega t}{t^2}$$

$$\omega \sim E_{\text{Planck}}$$

dynamics of Λ:
from Planck to present value

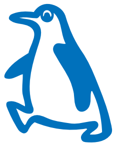


$$\langle\Lambda(t_{\text{Planck}})\rangle \sim E^4_{\text{Planck}}$$

$$\langle\Lambda(t_{\text{present}})\rangle \sim E^2_{\text{Planck}} \,/\, t^2_{\text{present}} \sim 10^{-120}\, E^4_{\text{Planck}}$$

coincides with present value of dark energy

something to do with coincidence problem ?



Dynamical evolution of Λ similar to that of gap Δ in superconductors after kick

dynamics of Λ in cosmology

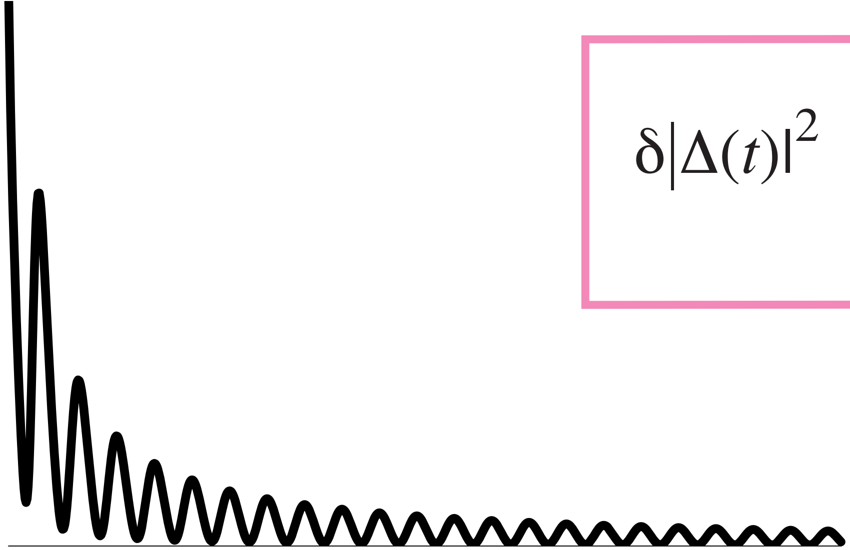
dynamics of Δ in superconductor

$$\boxed{\Lambda(t) \sim \omega^2 \frac{\sin^2 \omega t}{t^2}}$$

$$\boxed{\delta|\Delta(t)|^2 \sim \omega^{3/2} \frac{\sin \omega t}{t^{1/2}}}$$

$\omega \sim E_{\text{Planck}}$

$\omega = 2\Delta$



$t = 0$

$t = + \infty$

F.R. Klinkhamer & G.E. Volovik
Dynamic vacuum variable &
equilibrium approach in cosmology
PRD **78**, 063528 (2008)
Self-tuning vacuum variable &
cosmological constant,
PRD **77**, 085015 (2008)

intial states:

final states:

nonequilibrium vacuum with $\Lambda \sim E^4_{\text{Planck}}$

equilibrium vacuum with $\Lambda = 0$

superconductor with nonequilibrium gap Δ

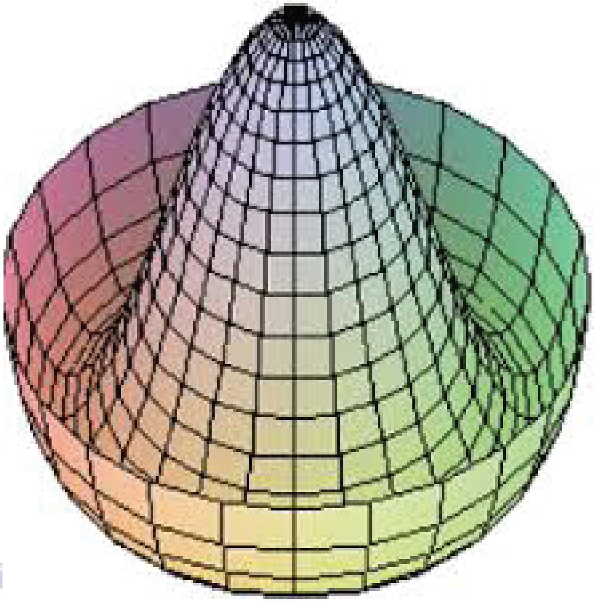
ground state of superconductor

$$\varepsilon(t) - \varepsilon_{\text{vac}} \sim \omega \frac{\sin^2 \omega t}{t}$$

V. Gurarie, Nonequilibrium dynamics of weakly and strongly paired superconductors: 0905.4498
A.F. Volkov & S.M. Kogan, JETP **38**, 1018 (1974)
Barankov & Levitov, ...

from super-fluid vacuum **to super-plastic vacuum**

I.E. Dzyaloshinskii & GV 1980
Poisson brackets
in condensed matter physics



Abrikosov's 70th Birthday Symposium, 6 Nov 1998 in Argonne

Klinkhamer & GV
Tetrads & q-theory
2019

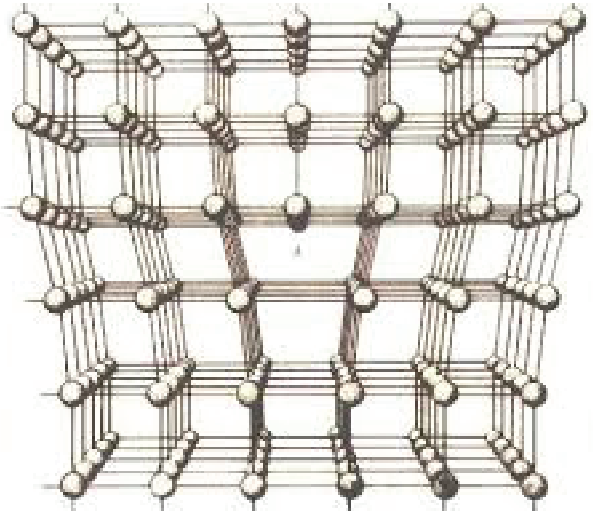
Nissinen & GV
Elasticity tetrads,
mixed anomalies,
(3+1)-d QHE
2019

tetrads of elasticity

Dzyaloshinskii & GV 1980
Poisson brackets in condensed matter physics
Andreev & Kagan, JETP **59**, 318 (1984)
GV & Dotsenko JETP Lett. **27**, 576 (1979)

system of 3 deformed crystallographic planes,
surfaces of constant phase

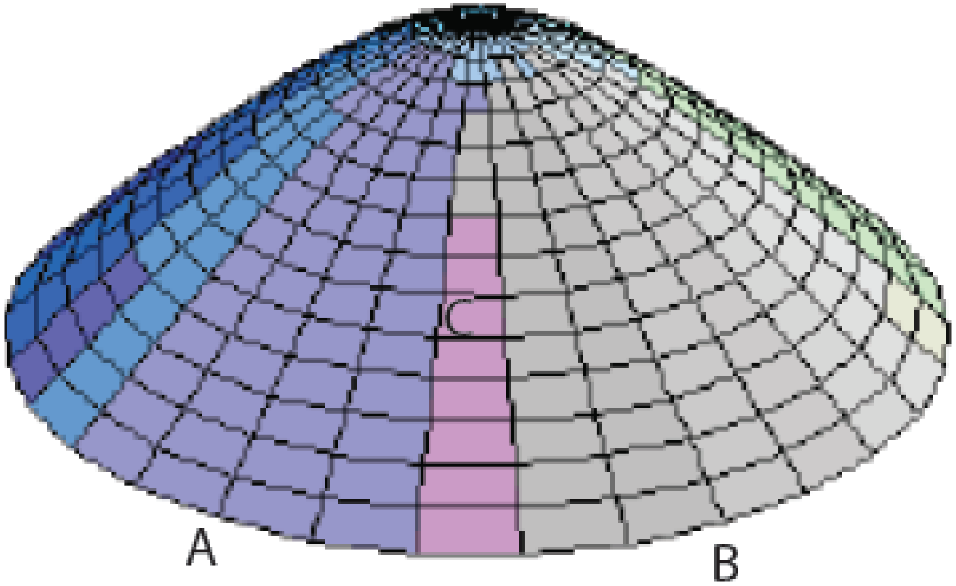
$$\boxed{X^a(x) = 2\pi n^a,\ n^a \in \mathbb{Z},\ \text{with } a = 1, 2, 3, \text{ in three dimensions}}$$



intersections of 3 surfaces
are points of deformed crystal lattice

The elasticity tetrads are gradients of the three U(1) fields X^a, $a = 1, 2, 3$,

$$\boxed{E^a_\mu(x) = \partial_\mu X^a(x)}$$



metric & distance (interval in GR)

$$\boxed{g_{\mu\nu} = E^a_\mu E^b_\nu \eta_{ab}} \qquad \boxed{dn^2 = g_{\mu\nu} dx^\mu dx^\nu}$$

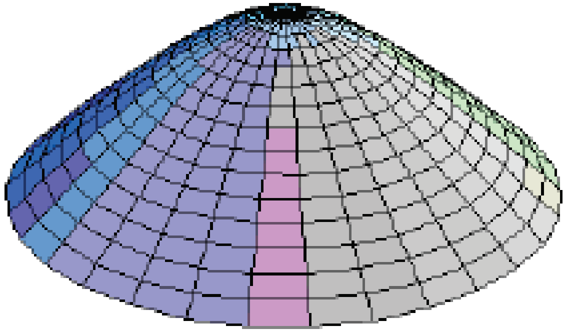
distance (interval) is dimensionless

expressed in terms of lattice points:

$$\boxed{AB = 5 \qquad (AC)^2 = 2^2 + 5^2}$$

elasticity gauge fields

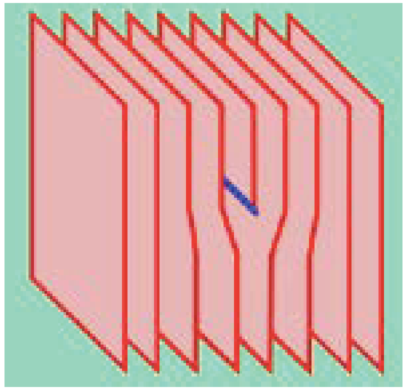
The elasticity tetrads are gradients of the three U(1) gauge fields X^a, $a = 1, 2, 3$,



$$\boxed{E^a_\mu(x) = \partial_\mu X^a(x)}$$

the field strength of these gauge fields -- torsion

$$\boxed{T^a_{kl} = (\partial_k E^a_l - \partial_l E^a_k)}$$



torsion describes dislocations

tetrads as gauge fields contribute to quantum anomalies

J. Nissinen & GV,
Elasticity tetrads,
mixed axial-gravitational anomalies,
& (3+1)-d quantum Hall effect,
PR Research **1**, 023007 (2019)

elasticity tetrads & (3+1)-d Quantum Hall Effect

Intrinsic (2+1)-d QHE

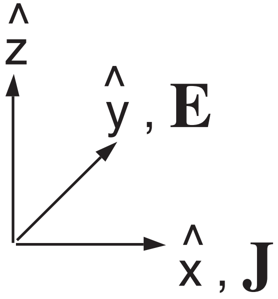
$$\boxed{S_{\mathrm{CS}} = \frac{\mathrm{e}^2}{16\pi} N \, \mathrm{e}^{\mu\nu\lambda} \int d^2x \, dt \, A_\mu \, F_{\nu\lambda}}$$

p-space invariant

r-space invariant

$$\boxed{\begin{aligned} N = \frac{1}{8\pi^2} \epsilon_{ij} \int_{-\infty}^{\infty} d\omega \int_{\mathrm{BZ}} dS \\ \times \operatorname{Tr}\left[(G\partial_\omega G^{-1})\left(G\partial_{k_i} G^{-1}\right)\left(G\partial_{k_j} G^{-1}\right)\right] \end{aligned}}$$

electric current $J_{\mathrm{x}} = \delta S_{\mathrm{CS}} / \delta A_{\mathrm{x}} = \dfrac{\mathrm{e}^2}{4\pi} N E_{\mathrm{y}}$



quantized intrinsic Hall conductivity
(without external magnetic field)

$$\boxed{\sigma_{\mathrm{xy}} = \frac{\mathrm{e}^2}{4\pi} N}$$

contribution of elasticity gauge to anomaly: (3+1)-d QHE & axial-gravitational anomaly

Intrinsic (3+1)-d QHE

elasticity tetrads have dimension of momentum,
no material parameters in action
only topological numbers
as in (2+1) QHE

$$S_{(3+1)\text{-d}}[A_\mu] = \frac{1}{8\pi^2}\sum_{a=1}^{3} N_a \int d^4x\, \underset{\uparrow}{E^a_\mu}\, \epsilon^{\mu\nu\alpha\beta} \underset{\uparrow}{A_\nu}\, \partial_\alpha A_\beta$$

elasticity gauge electromagnetic gauge

$$N_a = \frac{1}{8\pi^2}\epsilon_{ijk}\int_{-\infty}^{\infty} d\omega \int_{\mathrm{BZ}} dS^i_a \times \mathrm{Tr}\left[(G\partial_\omega G^{-1})\left(G\partial_{k_i}G^{-1}\right)\left(G\partial_{k_j}G^{-1}\right)\right]$$

momentum integral is over 2-D torus of the cross-section of BZ normal to a given tetrad

$$\frac{d\sigma_{ij}}{dE^a_k} = \frac{e^2}{2\pi h}\epsilon_{ijk}N_a$$

response of conductivity to deformation is quantized

J. Nissinen & GV,
Elasticity tetrads,
mixed axial-gravitational anomalies,
& (3+1)-d quantum Hall effect,
PR Research **1**, 023007 (2019)

some other anomalies with elasticity gauge fields

elasticity gauge electromagnetic gauge

$$\boxed{S_{3+1\mathrm{D}}[A] \propto N_1 \int d^3x dt\ \epsilon^{\mu\nu\alpha\beta} e_{abc} \overset{\downarrow}{E_\mu}{}^a \overset{\downarrow}{E_\nu}{}^b \overset{\downarrow}{E_\alpha}{}^c \overset{\downarrow}{A_\beta}}$$

$$\boxed{N_1 = \frac{1}{2\pi i} \int d\omega\ \mathrm{Tr}\ G(k_x,\omega)\partial_\omega G^{-1}(k_x,\omega)}$$

connection to Luttinger theorem

elasticity gauge electromagnetic gauge

$$\boxed{S_{3+1\mathrm{D}} = \frac{1}{2(2\pi)^2} N^c \int d^3x dt\ \epsilon^{\mu\nu\alpha\beta} e_{abc} \overset{\downarrow}{E_\mu}{}^a \overset{\downarrow}{E_\nu}{}^b \overset{\downarrow}{F_{\alpha\beta}}}$$

$$\boxed{P^a = E_i^a P^i \propto V_{BZ} N^a}$$

topological contribution to polarization
& to electric charge of dislocation

quantization of response of polarization
to deformation of Brillouin zone volume

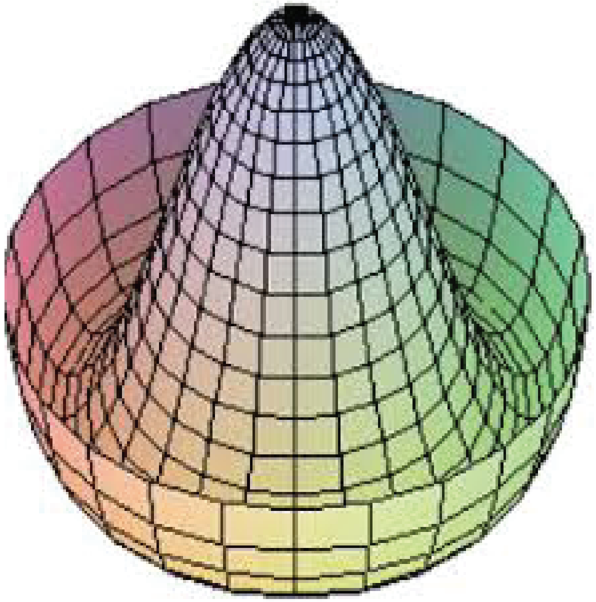
$$\boxed{\frac{dP^a}{dV_{BZ}} \propto N^a}$$

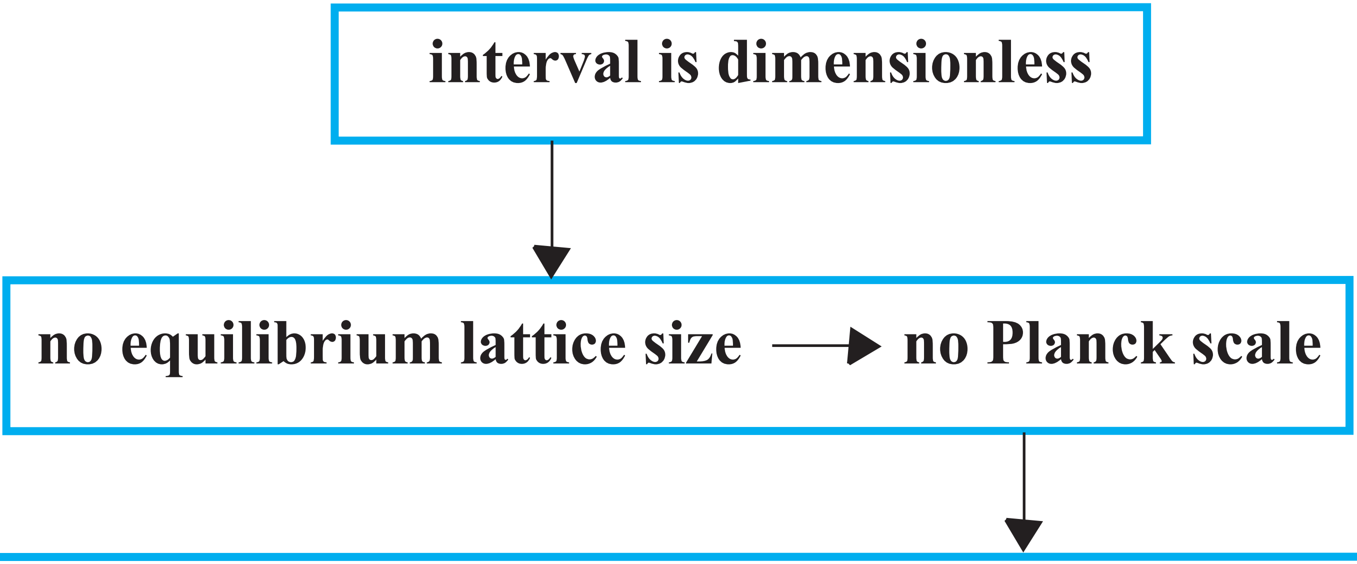
Song, He, Vishwanath & Wang,
Electric polarization as a nonquantized topological response & boundary Luttinger theorem,
arXiv:1909.08637

elasticity tetrads & gravity in superplastic vacuum

$$g_{\mu\nu} = E^a_\mu E^b_\nu \eta_{ab}$$

$$dn^2 = g_{\mu\nu} dx^\mu dx^\nu$$





interval is dimensionless

no equilibrium lattice size ⟶ no Planck scale

all physical parameters in SM and GR are dimensionless

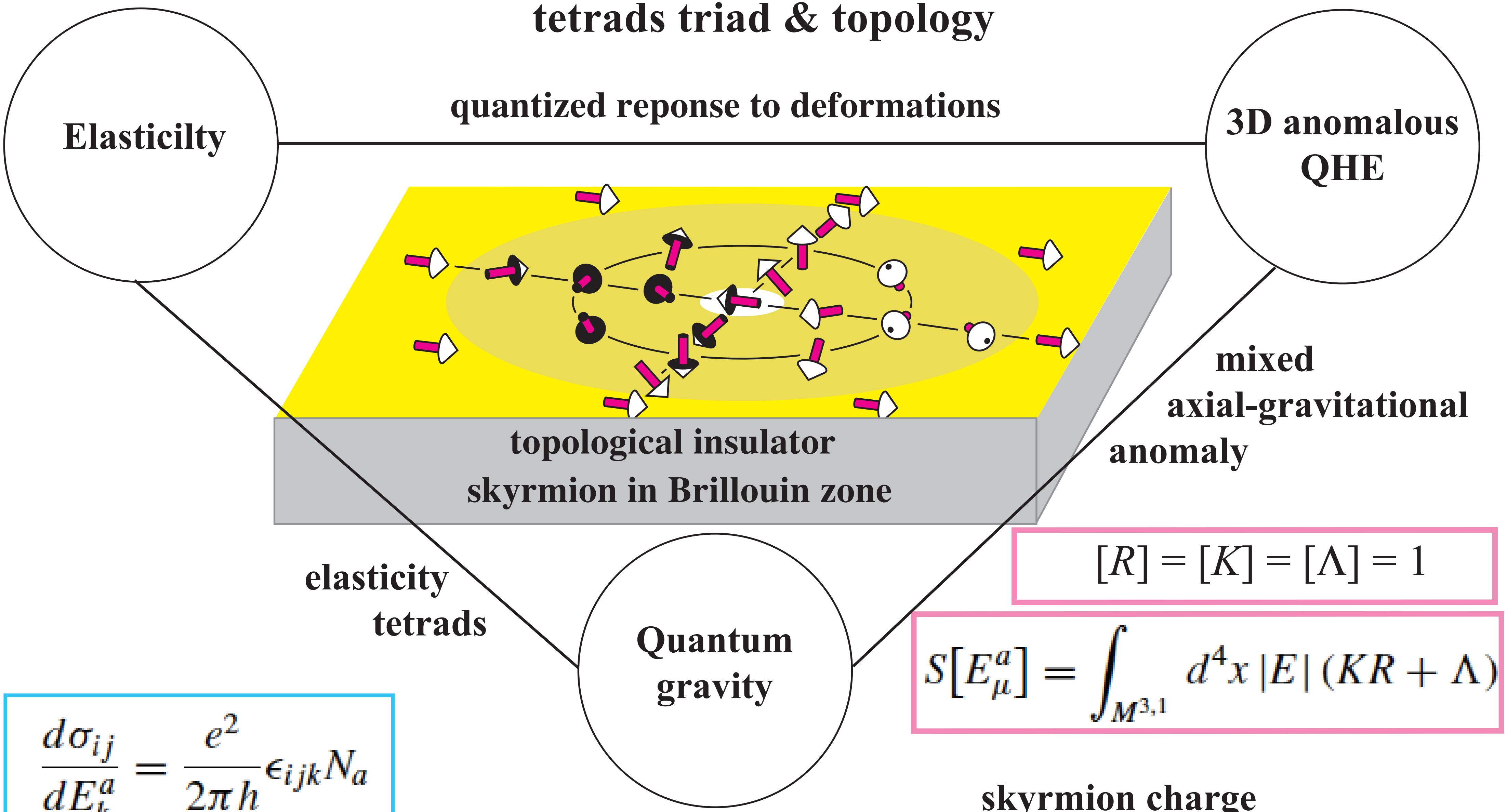
curvature, Newton constant § cosmological constant in Einstein action

$$S[E^a_\mu] = \int_{M^{3,1}} d^4x \, |E| \, (KR + \Lambda)$$

are dimensionless:

$$[R] = [K] = [\Lambda] = 1$$

tetrads triad & topology



Elasticilty

quantized reponse to deformations

3D anomalous QHE

topological insulator
skyrmion in Brillouin zone

mixed
axial-gravitational
anomaly

elasticity
tetrads

Quantum gravity

$$[R] = [K] = [\Lambda] = 1$$

$$S[E^a_\mu] = \int_{M^{3,1}} d^4x \, |E| \, (KR + \Lambda)$$

$$\frac{d\sigma_{ij}}{dE^a_k} = \frac{e^2}{2\pi h} \epsilon_{ijk} N_a$$

$$\frac{1}{8\pi^2} \sum_{a=1}^{3} N_a \int d^4x \, E^a_\mu \epsilon^{\mu\nu\alpha\beta} A_\nu \partial_\alpha A_\beta$$

skyrmion charge

$$N_a = \frac{1}{8\pi^2} \epsilon_{ijk} \int_{-\infty}^{\infty} d\omega \int_{\mathrm{BZ}} dS^i_a \times \mathrm{Tr}\left[(G\partial_\omega G^{-1})\left(G\partial_{k_i} G^{-1}\right)\left(G\partial_{k_j} G^{-1}\right)\right]$$

conclusion in the center of quantum triad

Gravity

Superfluids

QFT

sectors in
Landau Institute

plasma and lasers
quantum field theory
quantum mesoscopics
mathematical physics
nonequilibrium physics
gravitation and cosmology
problems of modern mathematics
electronic and optical properties of solids