The Abdus Salam
International Centre
for Theoretical Physics

Quantum Fluids, Quantum Field Theory and Gravity, Chernogolovka
18 October 2019

LOCALIZATION AND ERGODIC TRANSITIONS IN LOG-NORMAL ROSENZWEIG-PORTER RANDOM MATRIX ENSEMBLE.

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Collaboration:

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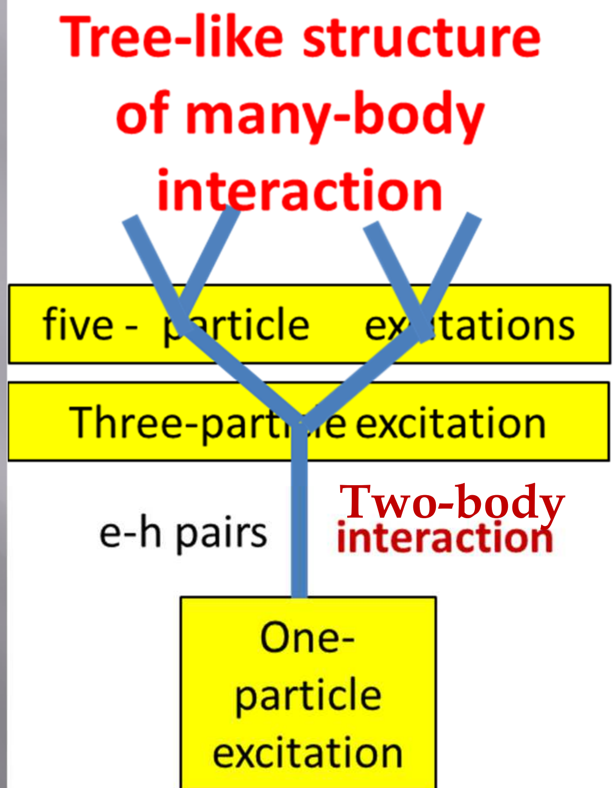
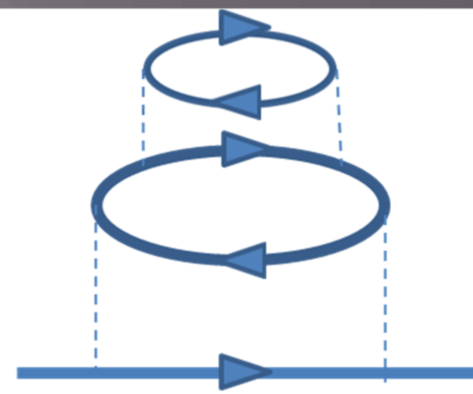
MBL and sing-particle localization on hierarchical graphs



Altshuler, Gefen, Kamenev, Levitov , 1997

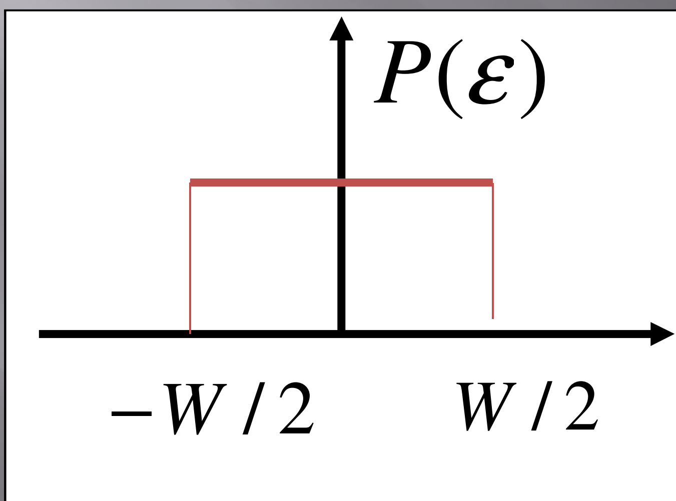


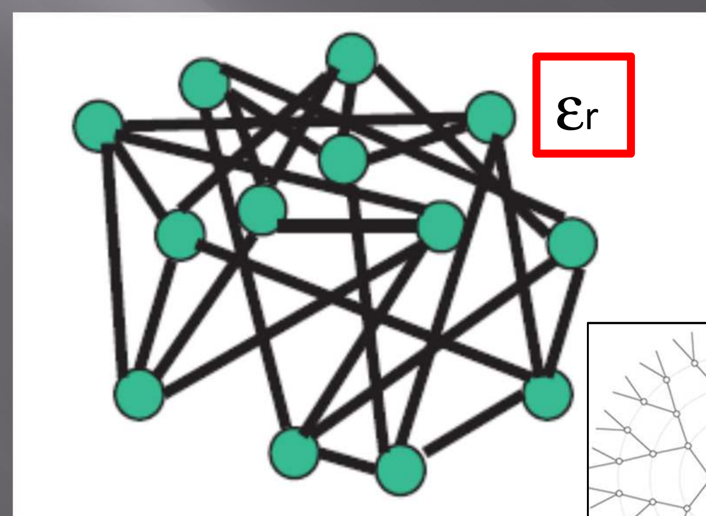
Basko, Aleiner, Altshuler, 2005

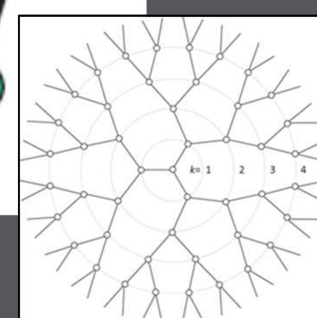
Anderson model on hierarchical boundary-less graphs

$$H = -I \sum_{<r,r'>} c_r^+ c_{r'} + \sum_r \varepsilon_r n_r$$

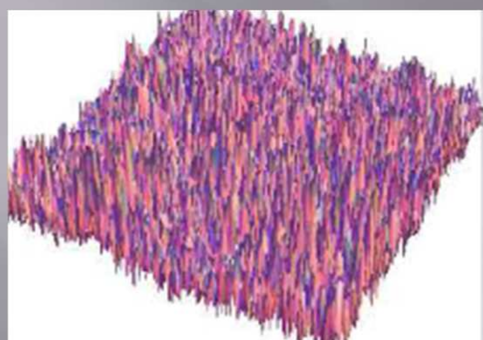
Disorder strength W



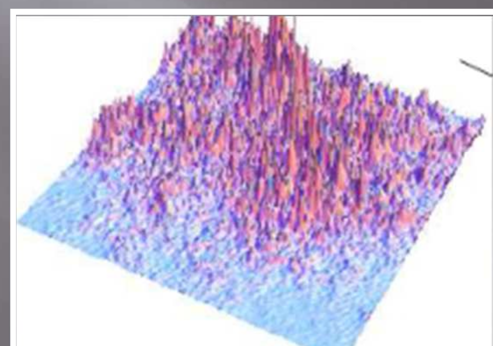




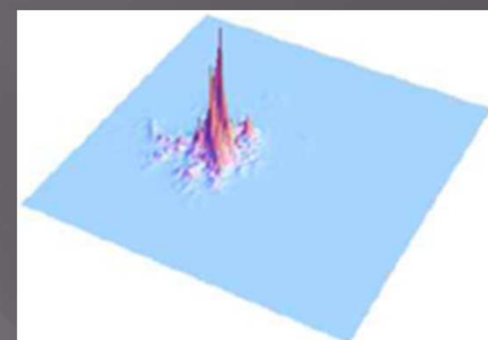
Localization and ergodic transitions



Ergodic extended (EE)



Extended but not ergodic (NEE)

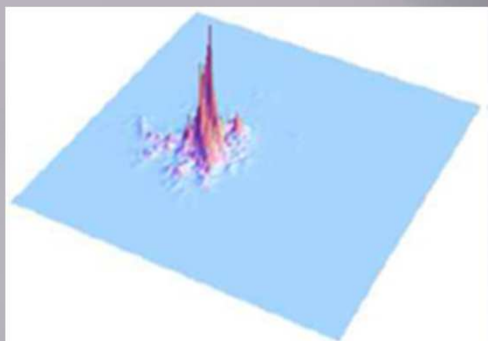


Localized (LOC)

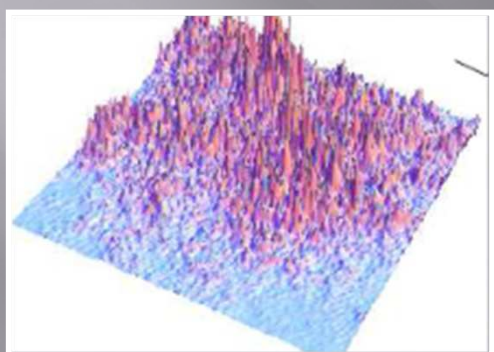
in a ... ters.

3D Anderson model: NEE states only at the AT point.

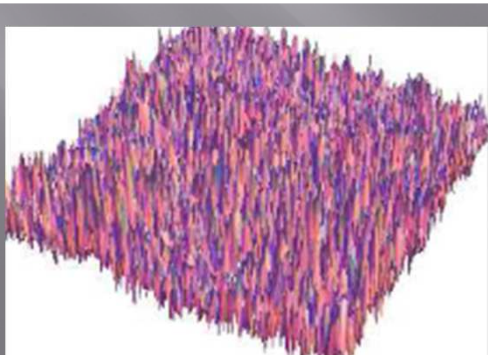
LOCALIZED, EXTENDED ERGODIC AND EXTENDED NON-ERGODIC PHASES



Finite number of occupied sites in thermodynamic limit

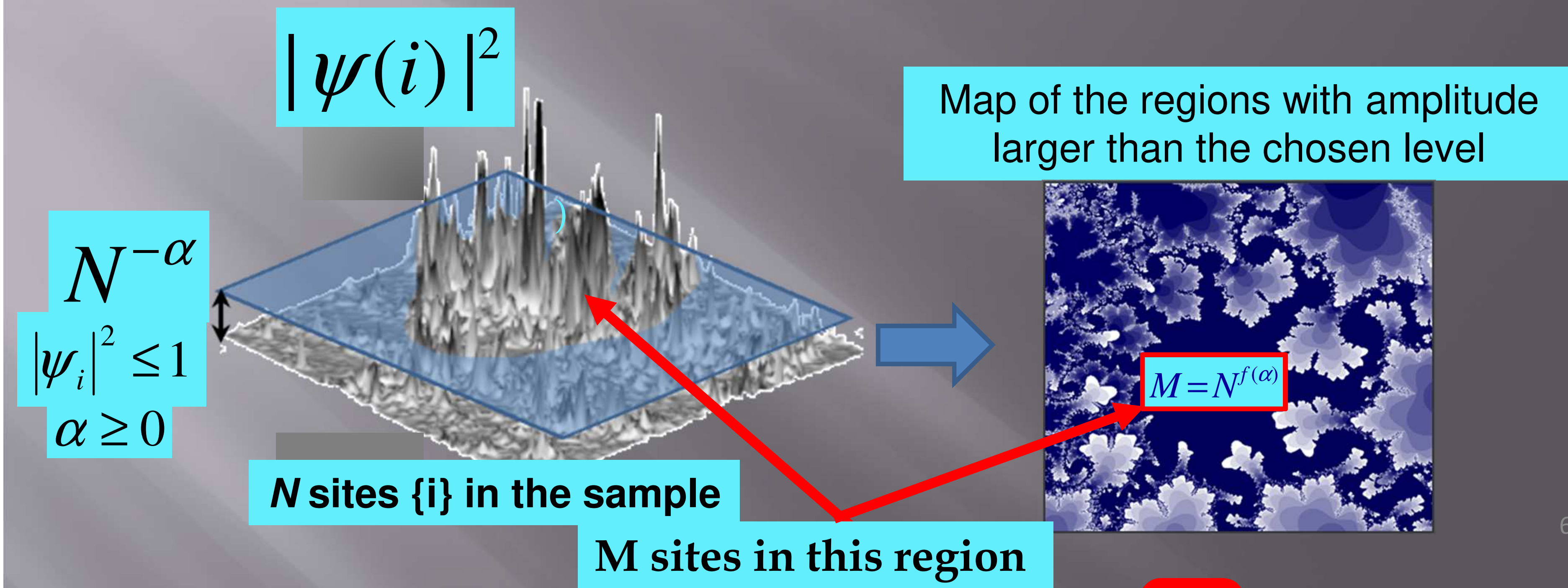


Infinite number of occupied sites but zero fraction of all sites in the thermodynamic limit



Finite fraction of occupied sites

Multifractal NEE

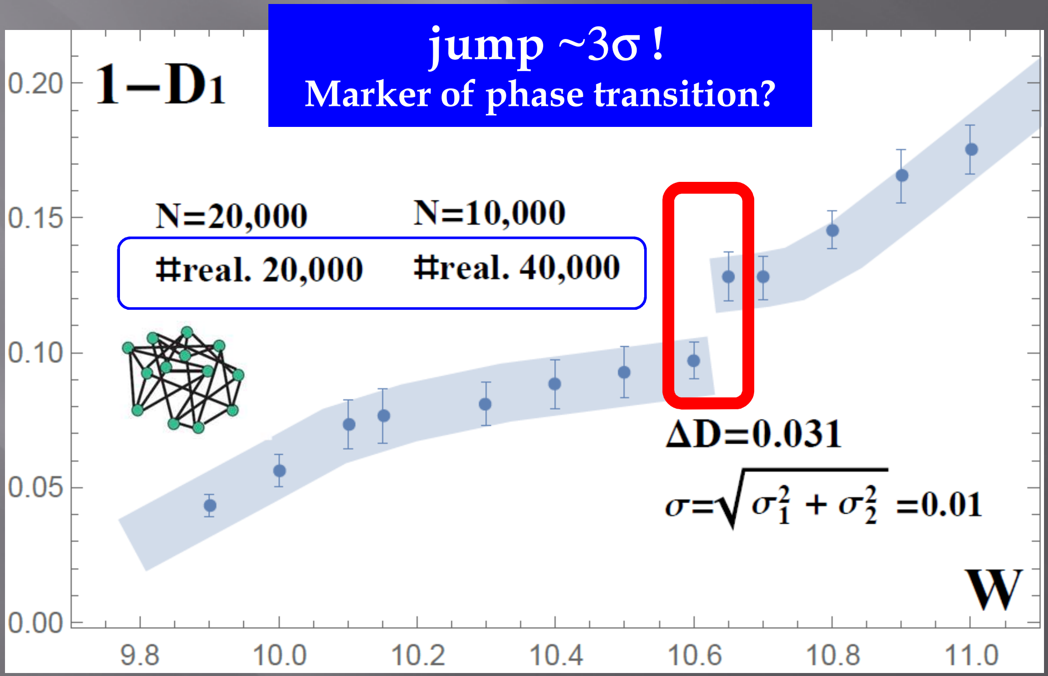


$$\sum_i |\psi(i)|^{2q} = \frac{c_q}{N^{D_q(q-1)}}$$

Previous talk: no multifractal phase on granulated RRG (NL Sigma model)

But…

Cuevas,VEK,Ioffe, Altshuler (unpublished)



Gaussian Rosenzweig-Porter RMT: rigorously proven multifractal phase

$$\langle H_{nm} \rangle = 0$$

NxN matrix, uncorrelated random entries

Scaling with matrix size

$$\sigma = \frac{\lambda^2}{N^{\gamma}} << 1$$

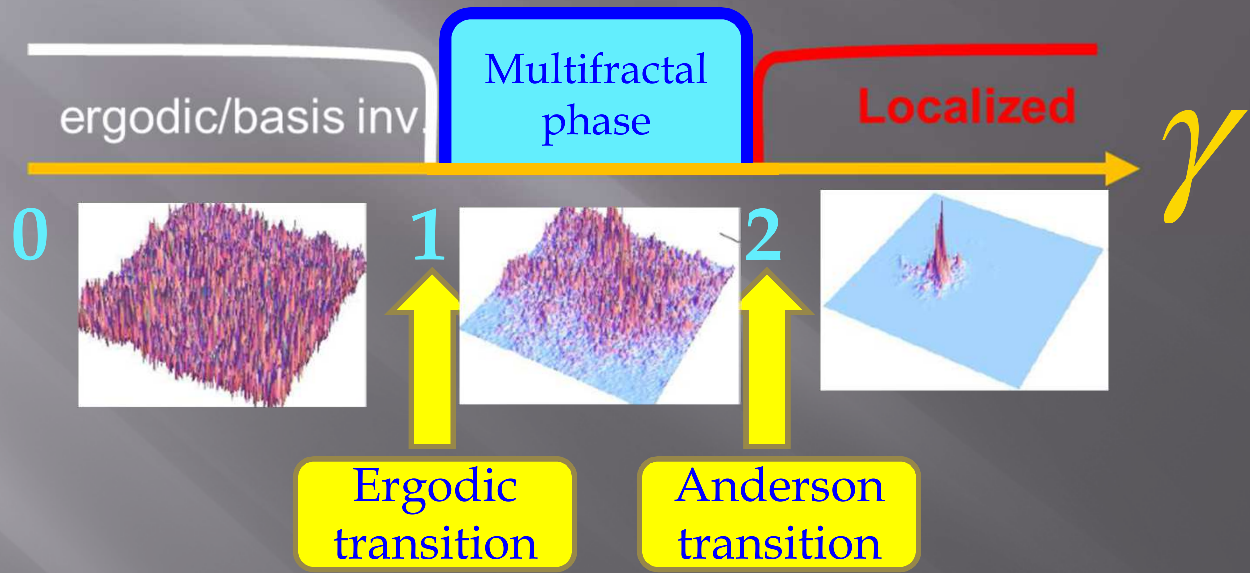
Special diagonal:

$$\langle H_{nm}^2 \rangle = \begin{pmatrix} 1 & \sigma & \sigma & \sigma \\ \sigma & 1 & \sigma & \sigma \\ \sigma & \sigma & 1 & \sigma \\ \sigma & \sigma & \sigma & 1 \end{pmatrix}$$

Simplest non-invariant RMT

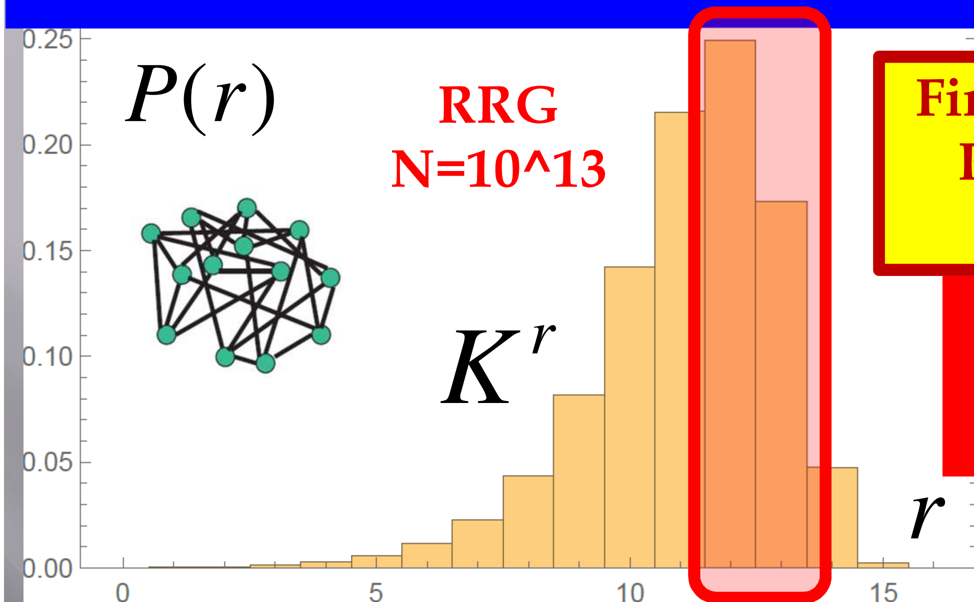
Multifractal phase in Gaussian RP

V.E.K., I.M. Khaymvich, E. Cuevas, M. Amini,
New J. Phys., v.17, 12202 (2015)



Anderson model on RRG and Rosenzweig-Porter ensemble

Two-point Green's function $G_{0,r} \sim \exp[-\lambda\, r]$ on a tree is an effective transmission matrix element



Finite fraction of all distances r on RRG are at a maximal distance d=lnN/lnK

Approximation: neglect all other distances!

Only two distribution functions: P(ε) for diagonal and P($G_{0,d}$) for off-diagonal matrix elements =Rosenzweig-Porter RMT

$$H_{nn} = \varepsilon_n$$

$$H_{nm} = G_{0,d} \equiv G$$

Log-normal distribution of off-diagonal elements

$$P(G) = \frac{A}{G}\exp\left[-\frac{\ln^2(G/G_{typ})}{2\ln G_{typ}^{-1}}\right]$$

$$G_{typ} \sim N^{-\gamma/2}$$

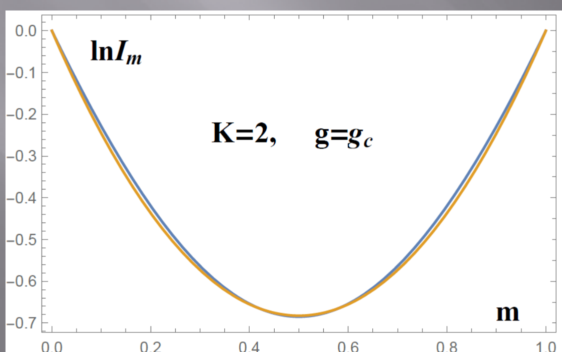
$$\gamma = \frac{2\lambda(W)}{\ln K}$$

Exact for NLσM, g>>1

λ(W) = Lyapunov exponent on a tree with branching K

p-extension:

$$\tilde{P}(\ln G) \sim \exp\left[-\ln N \frac{\left(\frac{\ln G}{\ln N}+\frac{\gamma}{2}\right)^2}{p\,\gamma}\right]$$



Good for K=2 up to the AT point

RRG --> "multifractal" log-normal distribution with p=1

Role of tails of the lognormal distribution

$$G_{typ} = \exp[\langle \ln G \rangle] \sim N^{\gamma/2}$$

$$\langle G \rangle \sim N^{\gamma_{av}/2}$$

$$\gamma_{av} = (1 - p/2)\gamma$$

Averaged off-diagonal matrix elements have different scaling in N compared to the typical ones:

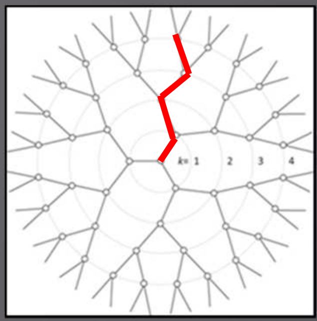
effect of tails in the distribution

Imbedded symmetry

For a tree (or 1D system):

$$G_{0,r} = \prod_{n \in path} G_{nn}$$

$$y = \prod_{n \in path} G_{nn}^{-1} = G_{0r}^{-1}$$



Basic symmetry on a tree

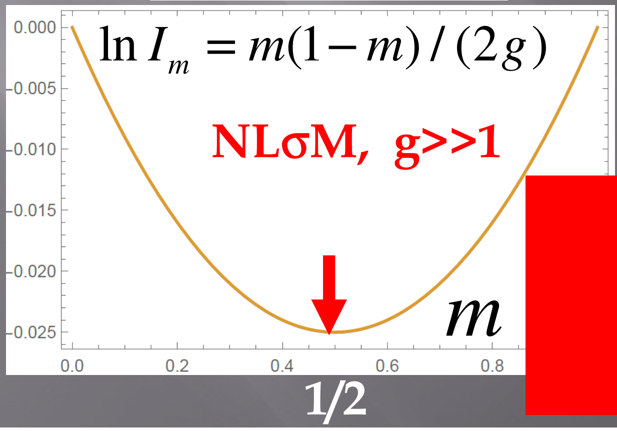
F(y)=F(1/y)

P(1/G)=G^4 P(G)

$$(I_m)^r = \int \frac{dy}{y^{2m}} F(y)$$

$$I_m = I_{1-m}$$

$$F(y) = \frac{2}{y} \int_B \frac{dm}{2\pi i} y^{2m} (I_m)^r$$



$$\tilde{P}(\ln G) \sim \exp\left[-\ln N \frac{\left(\frac{\ln G}{\ln N} + \frac{\gamma}{2} \right)^2}{p\,\gamma} \right]$$

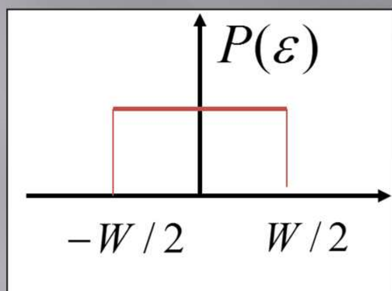
p=1

is a symmetry parameter

Criterion of localization transition

Localization transition: few sites resonant with a given one (Anderson's criterion):

$$P_{res} = W^{-1}\int_0^W d\omega \int_{\omega=|\varepsilon_n-\varepsilon_m|}^{\infty} dG\, P(G) \sim 1/N$$



$$N\langle G\rangle_W \sim N^0$$

Truncation of lognormal at G~W~O(1)

$$\gamma^{(AT)} = \begin{cases} \frac{4}{2-p}, & \text{if } p<1 \\ 4p, & \text{otherwise} \end{cases}$$

Criterion of ergodic transition

Breit-Wigner width Γ is of the order of disorder strength W (Mott's criterion)

$$\Gamma \equiv N\delta_{typ} \sim \sum_j G_{ij}^2 \rho_j \sim \left\langle G^2 \right\rangle \delta_{typ}^{-1} \sim W$$

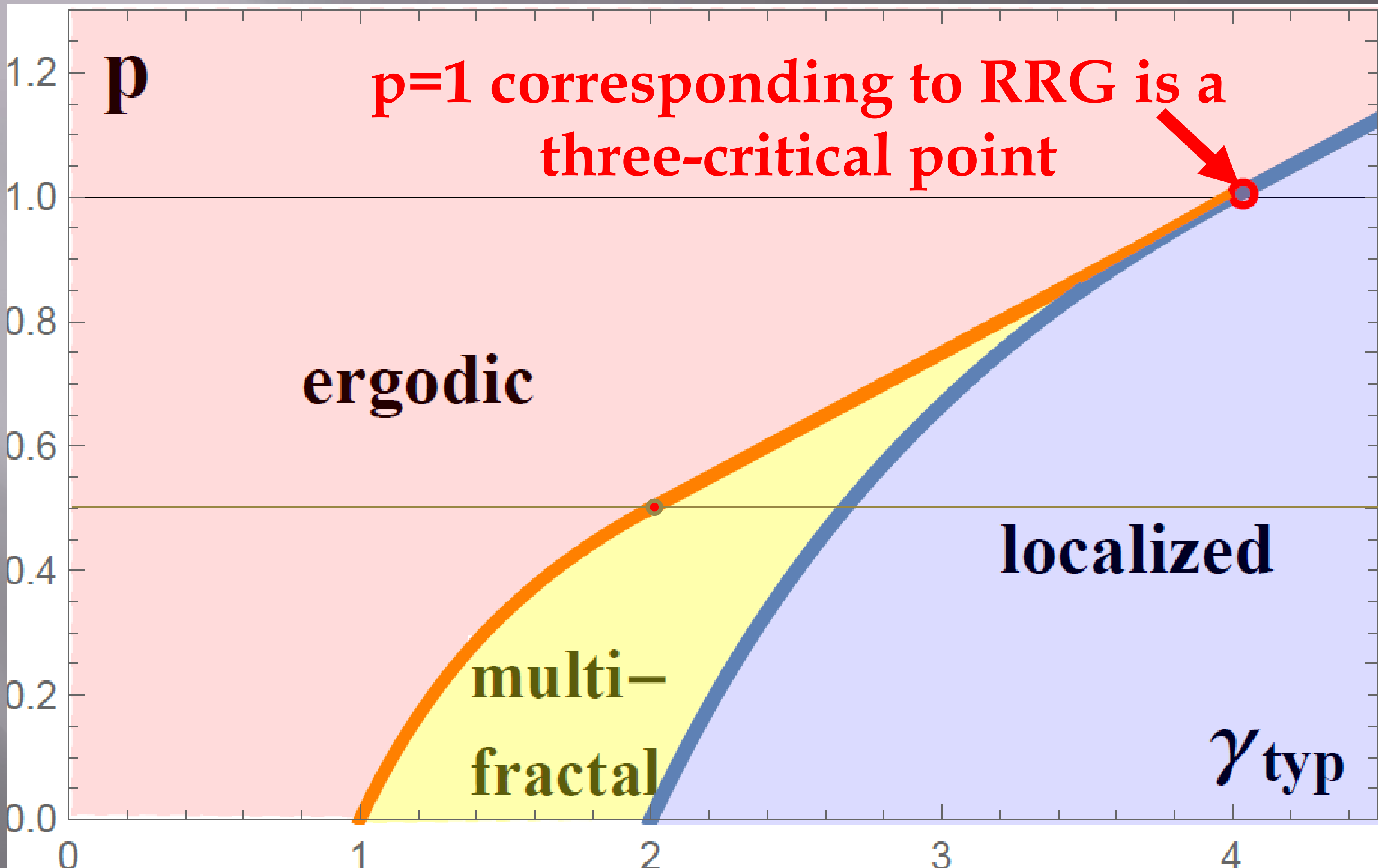
$$N \left\langle G^2 \right\rangle_W \sim W^2 \sim N^0$$

Truncation of lognormal at G~W~O(1)

G>W lead to states at the Lifshits tail whch do not contribute to δ_{typ}

$$\gamma^{(ET)} = \begin{cases} \frac{1}{1-p}, & \text{if } p < \frac{1}{2}, \\ 4p, & \text{if } p > \frac{1}{2} \end{cases}$$

Phase diagram



KL (Kullback-Leibler) statistics

$$KL1 = \left\langle \sum_r |\psi_n(r)|^2 \ln\left(\frac{|\psi_n(r)|^2}{|\psi_{n+1}(r)|^2}\right) \right\rangle$$

Sensitive to localization transition

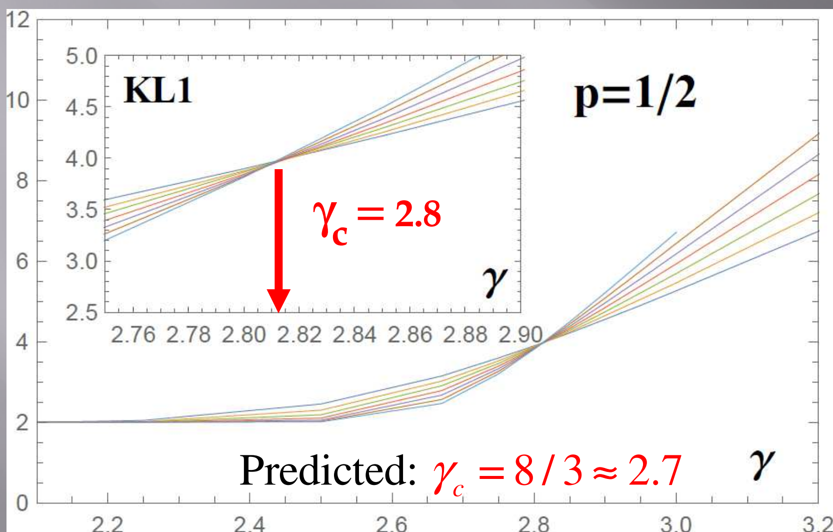
One and the same disorder realization. Neighboring states strongly correlated in extended phase and uncorrelated in localized phase

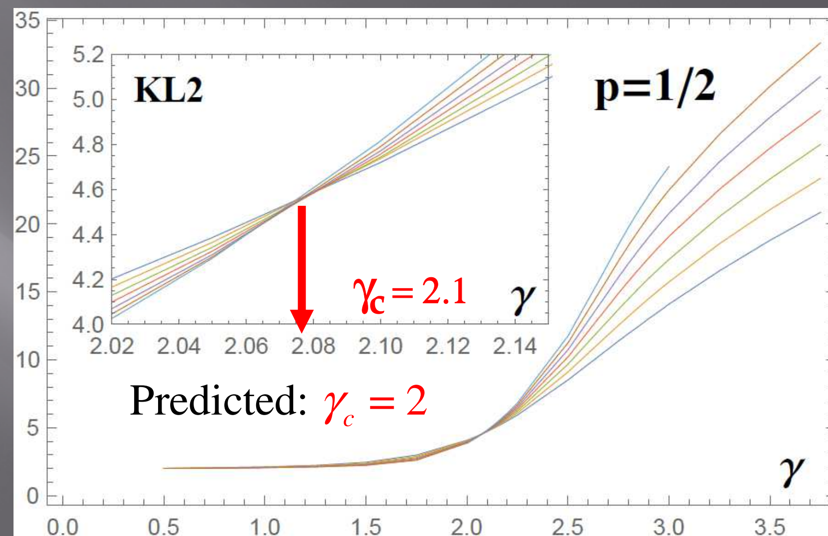
$$KL2 = \left\langle \sum_r |\psi(r)|^2 \ln\left(\frac{|\psi(r)|^2}{|\varphi(r)|^2}\right) \right\rangle$$

Sensitive to ergodic transition

Different disorder realizations for φ and ψ. Correlated only in the ergodic phase

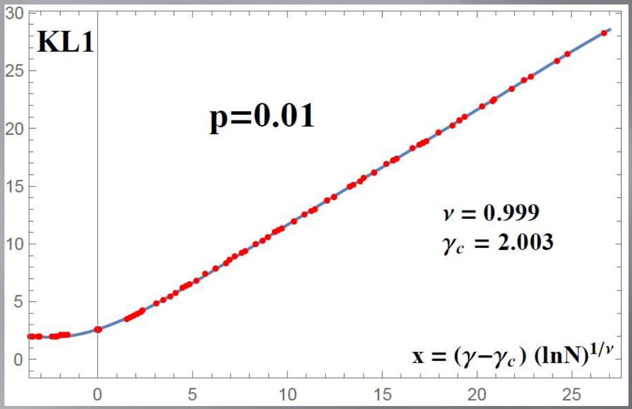
KL statistics for p=1/2

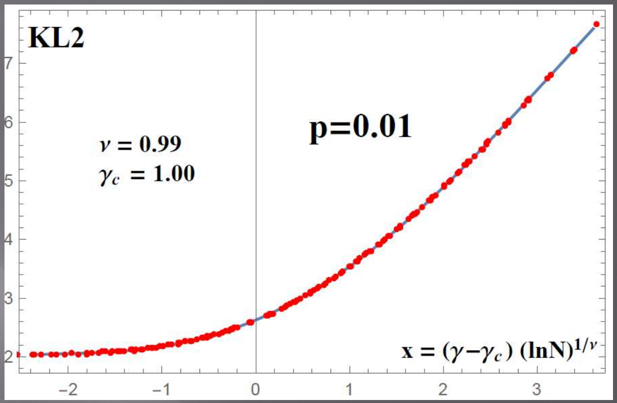


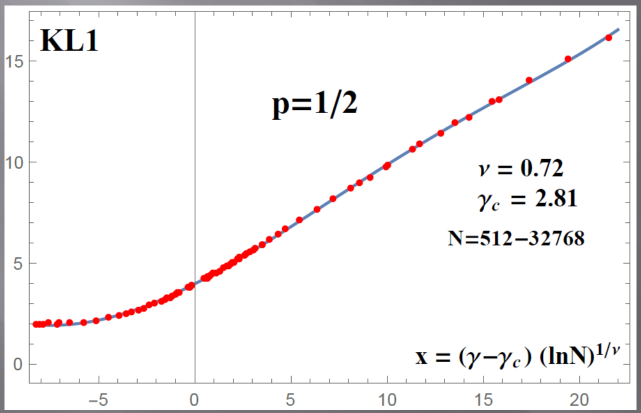


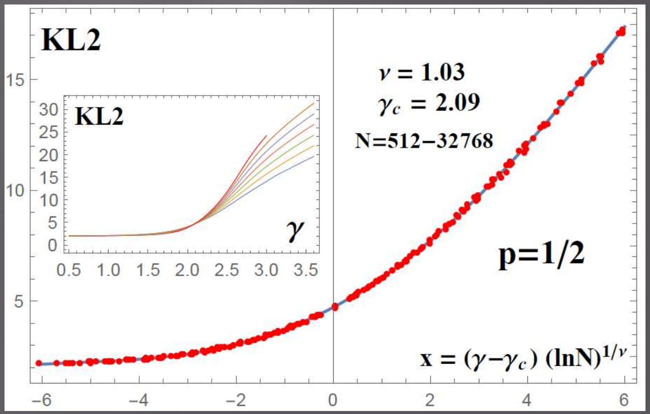
Data collapse and exponents ν

$$\xi \sim |\gamma - \gamma_c|^{-\nu}$$

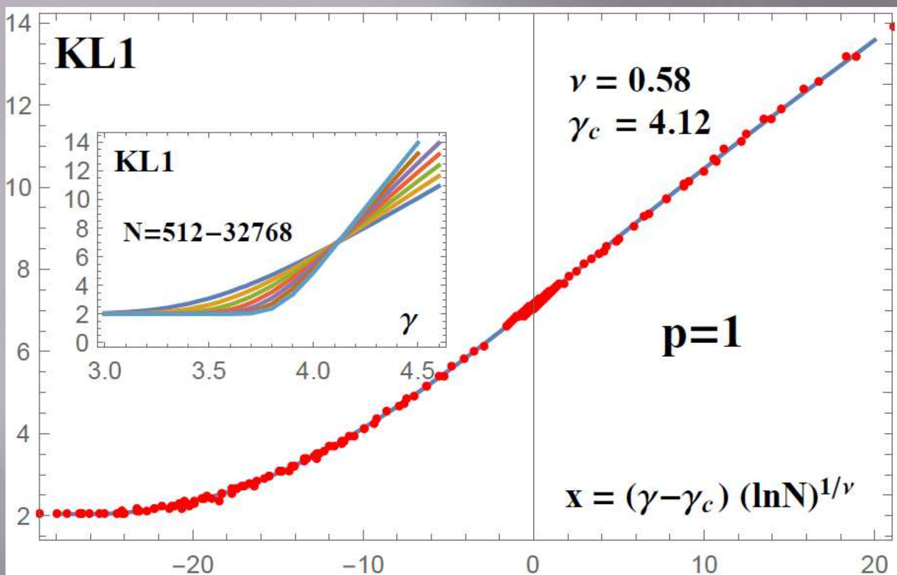


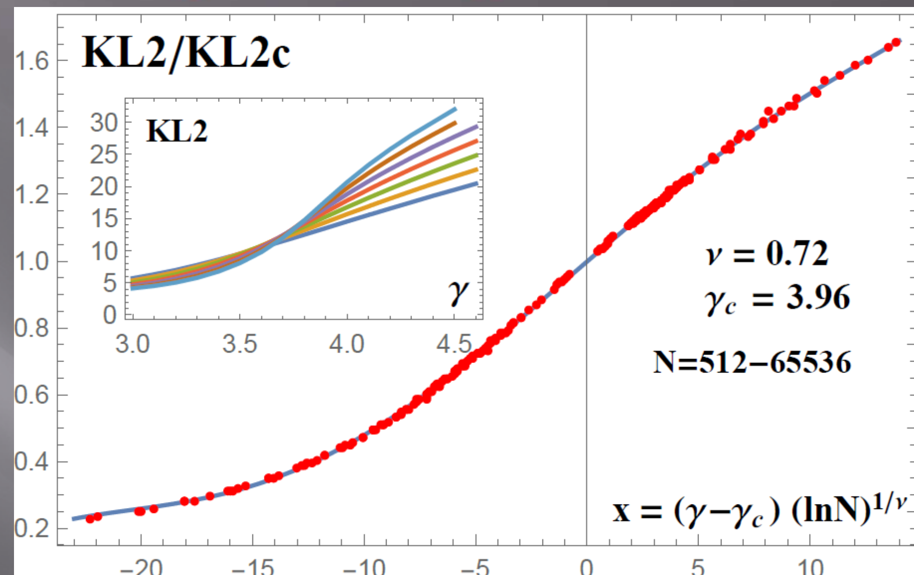






Data collapse and exponents ν at the three-critical point p=1

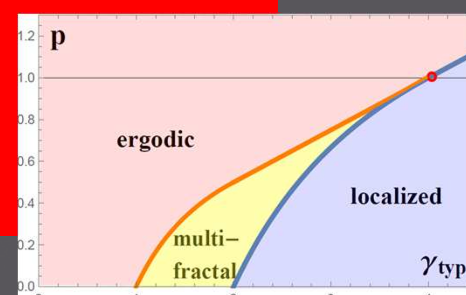




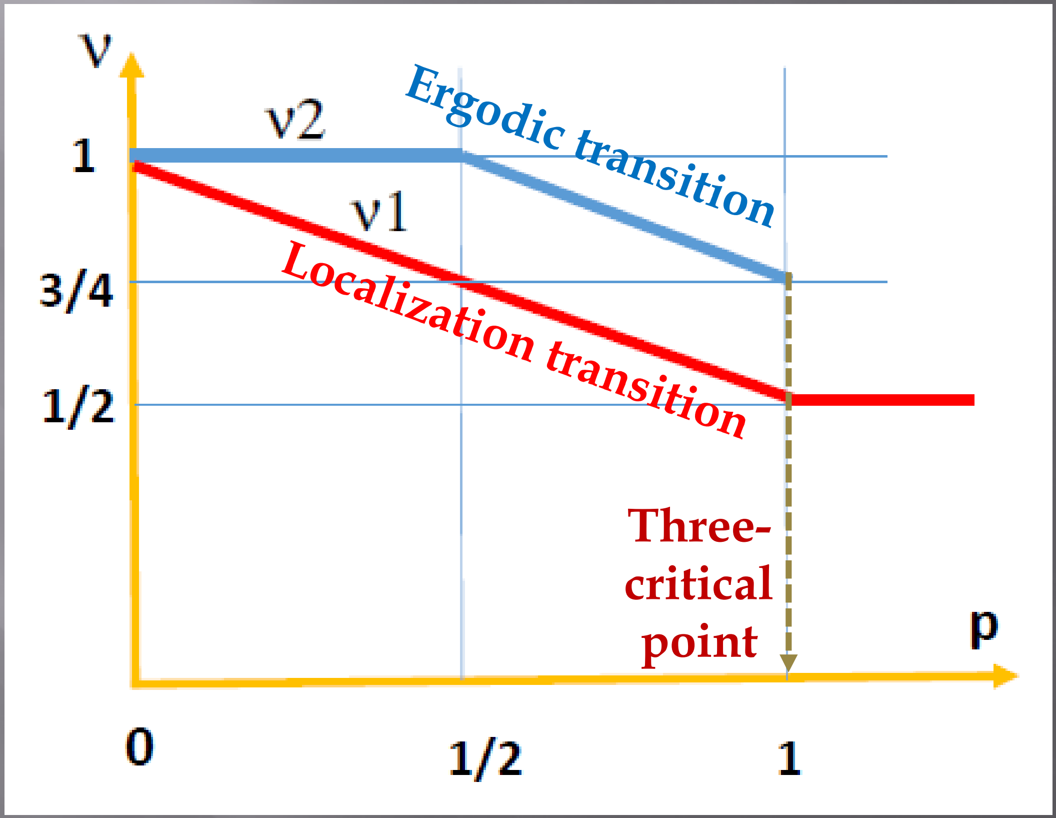
Merging of two transition points at γ=4

Two different critical exponents:
ν=1/2 and ν=3/4
(two critical lengths)

For a BL case see Efetov's "Supersymmetry" book, 1990



Conjecture about critical exponents ν

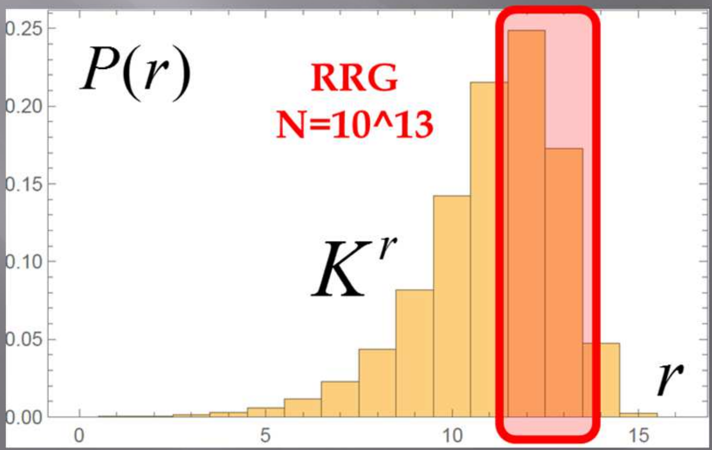


Self consistency eq. and RP

Mirlin and Fyodorov 1992
Tikhonov and Mirlin, 2018

$$g_0(Q) = \int e^{-\mathrm{Str}\left[-2g\,(Q-Q')^2 + \frac{\pi\eta}{\delta_0}\Lambda Q'\right]} g_0^K(Q')DQ'$$

Zero-dimensional Efetov's supermatrices: non-zero space modes are neglected



$H_{nn} = \varepsilon_n$

$H_{nm} = G_{0,d} \equiv G$

Zero-dimensional RP random matrices

i.i.d. off-diagonal matrix elements

Equivalent APPROXIMATIONS?

Conclusion

- Rosenzweig-Porter RMT corresponding to RRG and its p-extension
- "Multifractal", tailed distribution of off-diagonal matrix elements
- RRG symmetry requirement p=1
- Criteria of localization and ergodicity
- Phase diagram for log-normal RP RMT; p=1 is a three-critical point
- Kullback-Leibler statistics of eigenvectors
- Numerical characterization of Anderson and Ergodic transitions in logarithmically-normal RP RMT, critical length and exponents ν