Anomalous Hall effect in Weyl semimetals

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Chernogolovka, 20 October 2019

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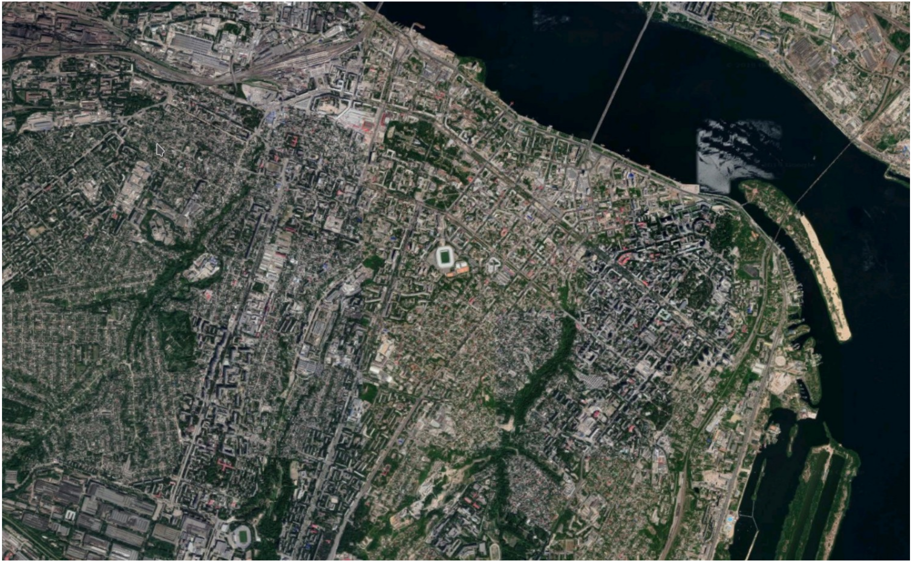


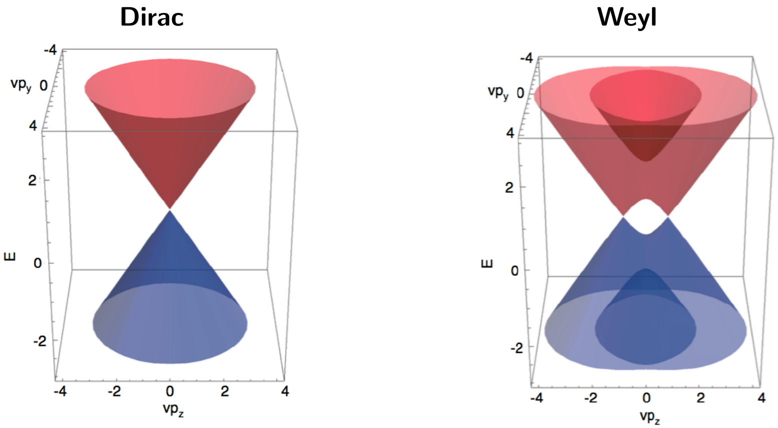
This is where it all begins...

Voskresenskaya (Lenina) str. 2

Outline

Dirac and Weyl semimetals

Dirac Hamiltonian: $H = \begin{pmatrix} m + m'\sigma_z & v\boldsymbol{\sigma}\mathbf{p} \\ v\boldsymbol{\sigma}\mathbf{p} & -m + m'\sigma_z \end{pmatrix}$

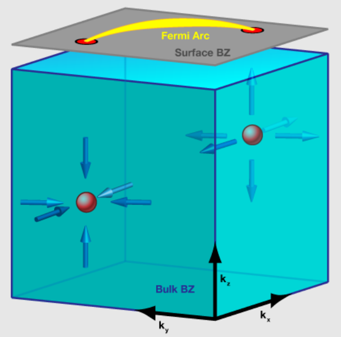


$m = m' = 0$
Double degenerate

$0 < m < m'$
Broken time-reversal

Weyl semimetal

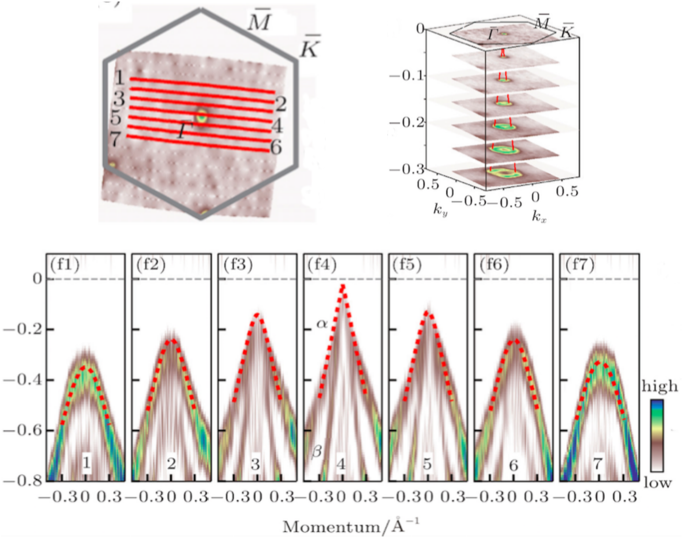
- Berry flux: $\mathbf{A} = -i\langle u(\mathbf{k})|\nabla_{\mathbf{k}}|u(\mathbf{k})\rangle$, $\mathbf{B} = \mathrm{rot}_{\mathbf{k}}\,\mathbf{A}$
- Weyl nodes $\Longrightarrow$ monopoles emitting Berry flux



- Effective Hamiltonian near Weyl node: $H = v\boldsymbol{\sigma}\mathbf{p}$
- Fermi arcs on the surface connecting Weyl nodes
- Time-reversal symmetry **preserved** $\Longrightarrow$ at least **four** Weyl points
- Time-reversal symmetry **broken** $\Longrightarrow$ at least **two** Weyl points

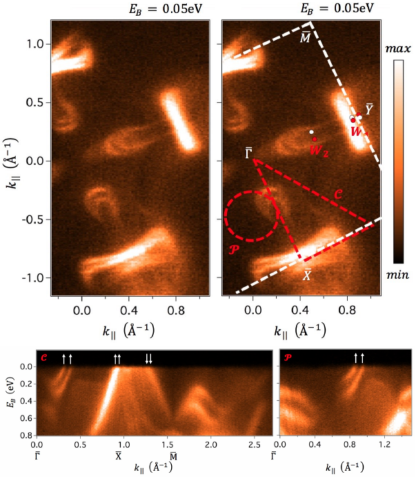
Dirac semimetal

ARPES spectra of Na_3Bi Dirac semimetal [Liang et al. '16]



Weyl semimetal with TR symmetry

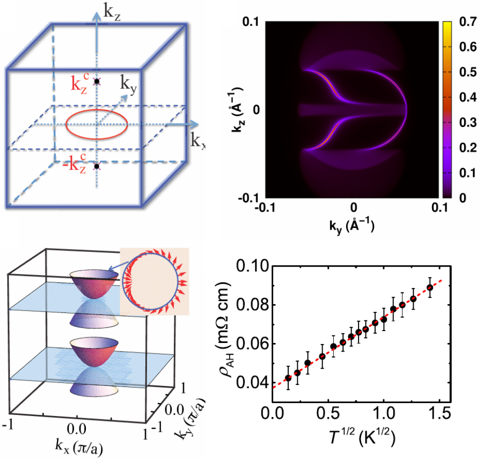
Surface Fermi arcs in TaAs Weyl semimetal [Belopolski et al. '16]



24 Weyl points

Weyl semimetal without TR symmetry

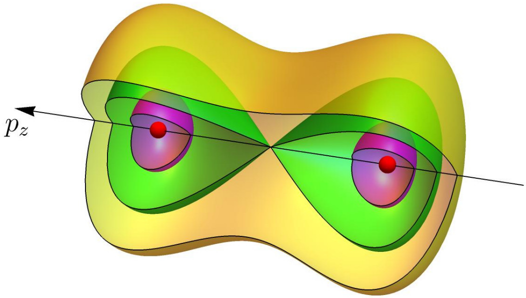
- Candidate material $\mathrm{HgCr_2Se_4}$



- Two parabolic Weyl points and a nodal ring [Xu et al. '11]
- Anomalous Hall resistance $\propto \sqrt{T}$ [Yang et al '19]

Minimal model of Weyl semimetal

- Hamiltonian $H = \sigma_x p_x + \sigma_y p_y + \sigma_z M(q) + V(\mathbf{r})$, $\qquad M(q) = \left(\frac{p^2}{2m} - \lambda\right)$
- Fermi surface (Lifshits transition at $\epsilon = \lambda$)



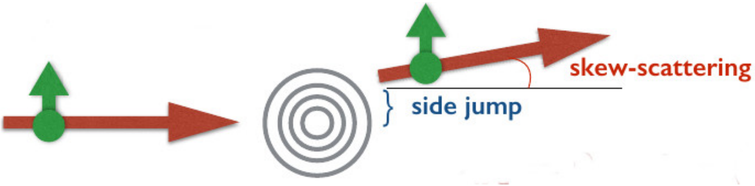
- Posible types of disorder:
  - Clean: $V = 0$
  - Gaussian: $\langle V \rangle = 0, \quad \langle V^2 \rangle = \alpha$
  - Poisson: randomly distributed impurities with density n_{imp}

Anomalous Hall effect

- Broken TR symmetry $\Longrightarrow$ anomalous Hall effect
- Intrinsic mechanism (clean limit) [Karplus, Luttinger '54]:
  - Spin-orbit coupling $\Longrightarrow$ Berry curvature $\mathbf{B} \sim \boldsymbol{\sigma}$
  - Magnetization $\Longrightarrow$ breaks spin symmetry

$$\mathbf{v} = \frac{\partial \epsilon}{\partial \mathbf{p}} + e\mathbf{E} \times \mathbf{B}$$

- Extrinsic mechanism (impurities involved):
  - Skew scattering [Smit '55, '58]
  - Side jump [Berger '70]



Kubo formalism

- Conductivity [Kubo, Štreda]
  - Lateral:

$$\sigma_{xx} = -\frac{1}{4\pi} \mathrm{Tr} \left\langle j_x (G^R - G^A) j_x (G^R - G^A) \right\rangle$$

  - Hall I (Fermi surface, kinetic):

$$\sigma^{\mathrm{I}}_{xy} = \frac{1}{2\pi} \mathrm{Tr} \left\langle j_x G^R j_y G^A \right\rangle$$

  - Hall II (edge states, thermodynamic):

$$\sigma^{\mathrm{II}}_{xy} = \frac{ie}{4\pi} \mathrm{Tr} \left\langle (x j_y - y j_x)(G^R - G^A) \right\rangle = ec \frac{\partial n}{\partial B}$$

- Clean limit $\Longrightarrow$ momentum p_z conserved

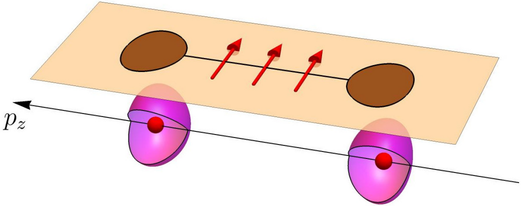
$$\sigma_{xy} = \int \frac{dp_z}{2\pi} \sigma_{xy}(p_z)$$

- Problem reduces to 2D massive Dirac model at each p_z

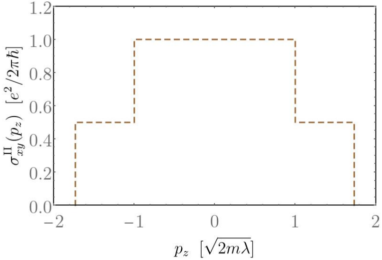
$$H(p_z) = \sigma_x p_x + \sigma_y p_y + M(p_z)\sigma_z$$

Contribution of surface states

- Surface states: Fermi arc

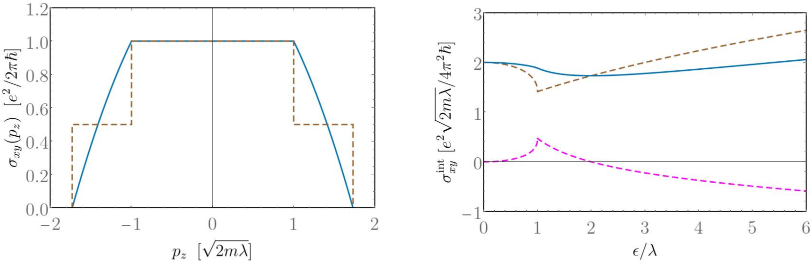


- Contributes to thermodynamic part $\sigma^{\mathrm{II}}_{xy}(p_z)$



Intrinsic contribution

- Total σ_{xy} with kinetic (σ^{I}_{xy}) and thermodynamic ($\sigma^{\mathrm{II}}_{xy}$) parts



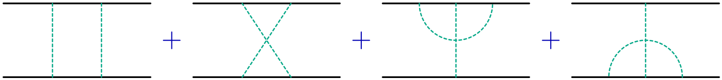
- At $\epsilon \gg \lambda$ square root growth $\sigma_{xy} \propto \sqrt{\epsilon}$
- At $\epsilon \ll \lambda$ dominated by Fermi arc
  $\Longrightarrow$ distance between Weyl points [Burkov '14]

Extrinsic contribution

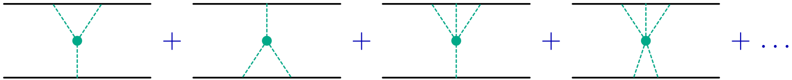
- Skew scattering: $W(\mathbf{p},\mathbf{p}') \neq W(\mathbf{p}',\mathbf{p})$
- Born approximation = Fermi golden rule

$$W(\mathbf{p}',\mathbf{p}) \propto \left|\langle \mathbf{p}'|V|\mathbf{p}\rangle\right|^2$$

- **No skew scattering in the Born approximation!**
- Weak disorder (Gaussian parameter α)
  $\Longrightarrow$ two-impurity skew scattering [Ado et al '15]

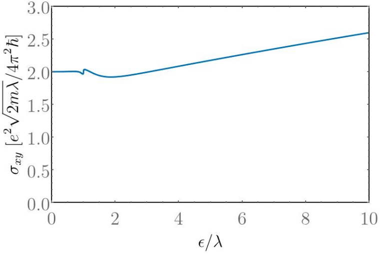


- Strong impurities (concentration n_{imp}, scattering length a)
  $\Longrightarrow$ go beyond Born approximation



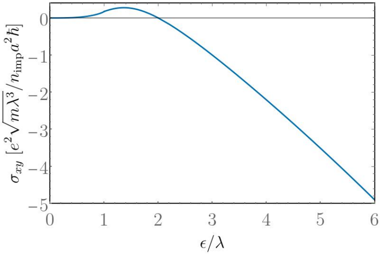
Anomalous Hall conductivity

Weak



- independent of α
- positive
- slow growth $\sigma_{xy} \propto \sqrt{\epsilon}$
- wins at low energy

Strong

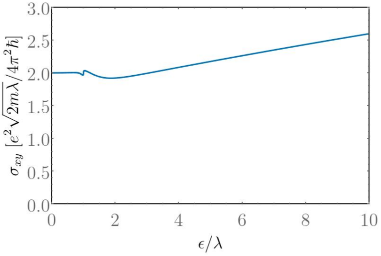


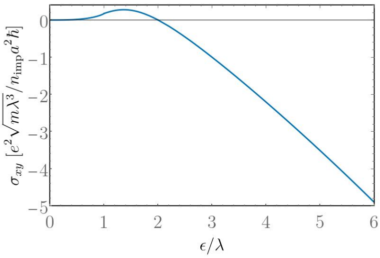
- $\sigma_{xy} \propto n_{\text{imp}}^{-1} a^{-2}$
- negative
- fast growth $\sigma_{xy} \propto \epsilon^{3/2}$
- wins at high energy

Crossover at $n_{\text{imp}} a^2 \sim \min\{\epsilon, \epsilon^3/\lambda^2\}$

Summary

1. Weyl semimetals with broken TR symmetry exhibit anomalous Hall effect
2. At low energies AHE is dominated by surface Fermi arc states
3. At high energy strongly depends (even the sign!) on the nature of disorder
4. Simple Born approximation is insufficient
   - For weak disorder two-impurity scattering important
   - For strong impurities at least third order required





Appendix: Scattering of slow electrons

- Point-like impurity $a \ll \lambda_F$
- Isotropic case [Landau&Lifshits III §132] $\implies$ s-wave scattering
- In general (anisotropic, strong SOC) case at $r > a$

$$\psi(\mathbf{r}) = \psi_0(\mathbf{r}) + G^R(\mathbf{r},0)\, T^R \psi_0(0), \qquad \psi_0(\mathbf{r}) = G^R(\mathbf{r},0) - G^A(\mathbf{r},0)$$

- In terms of scattering matrix

$$\psi(\mathbf{r}) = -G^A(\mathbf{r},0) + G^R(\mathbf{r},0)S, \qquad S = e^{2i\delta}.$$

- Solution at $r \ll \lambda_F$ is independent of energy
- Match two functions at $a \ll r \ll \lambda_F$

$$\Psi(\mathbf{r}) = \psi(\mathbf{r})(S-1)^{-1} = \frac{G^R(\mathbf{r}) + G^A(\mathbf{r})}{2} + \frac{G^R(\mathbf{r}) - G^A(\mathbf{r})}{2}\,(S+1)(S-1)^{-1}$$

- Energy dependence of scattering phase(s)

$$(S-1)(S+1)^{-1} = \tan\delta_\pm \sim v_F a^2 \nu_\pm(\epsilon)$$

- Scattering length of order a (unless resonant scattering)