BPS/CFT correspondence

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My collaborators on the theme, 1993-2019

M. Aganagic, A. Alexandrov, L. Baulieu,
H. Braden, V. Fock, A. Gorsky,
D. Gross, I. Klebanov, I. Krichever
A. Litvinov, A. Losev, S. Lukyanov,
A. Marshakov, A. Mikhailov, G. Moore,
A. Okounkov, N. Piazzalunga, V. Pestun,
A. Rosly, V. Roubtsov, K. Selivanov, A. Schwarz,
S. Sethi, C. Vafa, E. Witten, A. Zamolodchikov

My collaborators on the theme, 2003-2019

Including my students

S. Schadchin, X. Zhang, N. Prabhakar, S. Jeong, N. Lee

The **BPS/CFT correspondence**

is a principle, circa 2002-2004

Correlators of chiral observables
in four dimensional supersymmetric theories
are **holomorphic blocks (form-factors)**
of some **conformal field theory**
(or a massive integrable deformation thereof)
in two dimensions

A little bit of history

In 1994 C.Vafa and E.Witten studied twisted $\mathcal{N} = 4$ super-Yang-Mills theory on various four-manifolds X, to check the conjectured Olive-Montonen S-duality symmetry

$$\tau \longrightarrow -\frac{1}{\tau}$$

acting on the complexified gauge coupling of the theory:

$$\tau = \frac{\vartheta}{2\pi} + \frac{4\pi \mathrm{i}}{e^2}$$

Modularity

The partition function in simple cases reduces to the generating function of the Euler characteristics of instanton moduli spaces:

$$Z_X(\mathfrak{q}) = \mathfrak{q}^{-h_G \frac{\chi(X)}{24}} \sum_{k=0}^{\infty} \mathfrak{q}^k \, \chi(\mathcal{M}_{G,k})$$

$$\mathfrak{q} = \exp 2\pi \mathrm{i} \tau$$

It indeed undergoes simple transformations under

$$\tau \longrightarrow -\frac{1}{\tau}$$

More refined versions of partition function incorporate 't Hooft fluxes, distinguish between different gauge groups G with the same Lie algebra $\mathfrak{g}$ and so on. It turns out that the S-duality maps the gauge group to its Langlands (or Goddard-Nuyts-Olive) dual

$$G \longrightarrow {}^L G$$

and moreover it should be embedded into a larger group, contained in

$$SL_2(\mathbb{Z}), \qquad \tau \longrightarrow \frac{a\tau + b}{c\tau + d}$$

Nakajima algebras

Another element is the discovery of H. Nakajima,

who in 1992-1994 showed that the ground states

$$\bigoplus_{n=0}^{\infty} H^*(\mathcal{M}_{k,n})$$

of susy quantum mechanics on the moduli spaces
of $U(k)$ instantons
on the gravitational instantons,
the so-called ALE spaces $\approx \mathbb{R}^4/\Gamma,\qquad \Gamma \subset SU(2)$

Nakajima algebras

H. Nakajima showed that the ground states of SQM on the moduli spaces

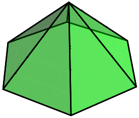
$$\bigoplus_{n=0}^{\infty} H^*(\mathcal{M}_{k,n})$$

of $U(k)$ instantons on ALE spaces $\approx \mathbb{R}^4/\Gamma$

is an irreducible level k representation
of the affine Kac-Moody algebra $\widehat{\mathfrak{g}}_\Gamma$,
where $\mathfrak{g}_\Gamma$ is McKay dual to Γ.

McKay duality

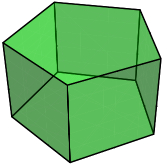
For $\Gamma = \mathbb{Z}_N$,
the binary symmetry group of



the dual is $G_\Gamma = SU(N)$,

McKay duality

For $\Gamma = \mathbb{Z}_N \star \mathbb{Z}_2$, the binary symmetry group of

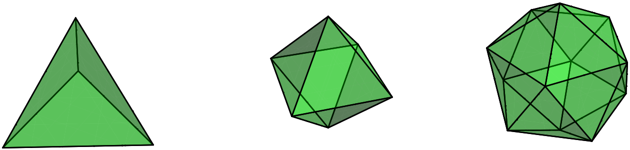


,

$$G_\Gamma = SO(2N),$$

McKay duality

For the binary groups of Platonic polyhedra:



,

The dual groups are: $G_\Gamma = E_6,\ E_7,\ E_8$, respectively.

These were the hints that the algebraic structure
of two dimensional conformal field theories
such as WZW_k models with G_Γ groups
is somehow realized in the four dimensional quantum gauge theory
with some amount of supersymmetry

Novel kind of symmetry in QFT

Novel kind of symmetry in QFT

Possibly non-local

Novel kind of symmetry in QFT

Possibly mapping one quantum field theory to another

An important tool allowing to study these questions

in the context of $d = 4$ $\mathcal{N} = 2$ theories

Z-functions

Z-function:

a refined version of Witten index

Formal definition:

Ω-deformation

In the Lagrangian of the $\mathcal{N} = 2$ theory

replace vector multiplet complex adjoint scalars σ:

$$\sigma + V^m D_m$$

$$V^m \partial_m = \epsilon_1 (x^2 \partial_1 - x^1 \partial_2) + \epsilon_2 (x^3 \partial_4 - x^4 \partial_3)$$

Also, shift the generator $\mathcal{R}_3$ of the $SU(2)$ R-symmetry group:

$$\mathcal{R}_3 \longrightarrow \mathcal{R}_3 + \mathcal{J}_3^R$$

where $\mathcal{J}_3^R$ is the generator of the $SU(2)_R$ factor of the Lorentz group

Informal definition:

View the four dimensional theory
as a limit of the five dimensional theory compactified on a circle:

$$Z^{\beta}_{5d}(\mathfrak{a}, \epsilon_1, \epsilon_2; m, \tau) =$$

$$= \mathrm{Tr}_{\mathcal{H}}\, (-1)^F\, \mathfrak{q}^{L_0}\, e^{\frac{1}{2}\beta\left((\epsilon_1-\epsilon_2)\mathcal{J}^L_3+(\epsilon_1+\epsilon_2)(\mathcal{J}^R_3+\mathcal{R}_3)\right)}\, e^{\beta\mathfrak{a}\cdot\mathcal{G}_\infty}\, e^{\beta m\cdot\mathcal{R}^F}$$

Here the charge L_0 is the topological instanton charge:

$$L_0 = -\frac{1}{8\pi^2}\int_{\mathbb{R}^4} \mathrm{tr} F\wedge F$$

(as in Nakajima's algebras) and $\mathcal{G}_\infty$, $\mathcal{R}^F$ denote the global gauge transformations and the flavor charges, respectively

Informal definition:

Four dimensional interpretation

$$Z_{4d}(\mathfrak{a}, \epsilon_1, \epsilon_2; m, \tau) = \lim_{\beta \to 0} Z^{\beta}_{5d}(\mathfrak{a}, \epsilon_1, \epsilon_2; m, \tau)$$

Here

$\mathfrak{a} = \langle \sigma \rangle \in \mathit{Cartan}(G) \otimes \mathbb{C}$ are the vev's
of the vectormultiplet complex scalars
m are the masses of matter hypermultiplets

τ are the gauge coupling(s)

The structure of Z

$$Z = Z^{tree} Z^{1-loop} Z^{inst}$$

$$Z^{tree} = \mathfrak{q}^{\frac{\mathfrak{a}^2}{2\epsilon_1\epsilon_2}}$$

$$Z^{1-loop} = \frac{\prod_{\alpha \in roots\ of\ G} \exp\gamma(\langle\alpha, \mathfrak{a}\rangle)}{\prod_{w \in weights\ of\ matter\ reps} \exp\gamma(m_f + w \cdot \mathfrak{a})}$$

Barnes double Gamma-function

$$\gamma(x) = \frac{d}{ds}\bigg|_{s=0} \frac{1}{\Gamma(s)} \int_0^\infty \frac{dt}{t} t^s \frac{e^{-tx}}{(1-e^{t\epsilon_1})(1-e^{t\epsilon_2})}$$

Additional hint for the BPS/CFT correspondence:

a related function shows up in Liouville conformal field theory

DOZZ-functions

Faddeev's quantum dilogarithm

$$e_b(x) \sim \prod_{i,j} (x - bi - b^{-1}j)$$

$$b^2 = \epsilon_2/\epsilon_1$$

Partition function Z

$$Z^{inst}(\mathfrak{a}, m, \tau, \epsilon_1, \epsilon_2)$$

for the gauge groups G

which are the products of unitary groups, such as the Standard Model

$$G = U(N_1) \times \ldots \times U(N_k)$$

can be evaluated explicitly,
as an infinite sum over special instanton configurations.

This is sometimes called the computation by localization.

Partition function Z

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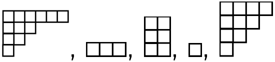
For each $U(v)$ factor one sums over the v-tuples of Young diagrams:



Partition function Z: from sums over partitions to **CFT**

The key feature of the non-perturbative Z-factor
is the combinatorics of special instanton configurations
which reproduces the structure of the Hilbert space of states

several species of free chiral fermions in two dimensions

From sums over partitions to CFT

The key feature of the non-perturbative Z-factor
is the combinatorics of special instanton configurations
which reproduces the structure of the Hilbert space of states

in the theory of several species of free chiral fermions in two dimensions

$$\int \sum_{i=1}^{N} \tilde{\psi}_i \bar{\partial} \psi^i$$

Special Ω-background: additional $SU(2)$ symmetry

$$\epsilon_1 + \epsilon_2 = 0$$

Ω-background with $SU(2) \Longrightarrow$ fermions

$$\epsilon_1 + \epsilon_2 = 0$$

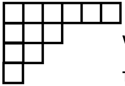
In this case Z^{inst} can be identified
with the matrix element, or a trace,
of some natural vertex operators
in the theory of ψ's

Identification of special instanton configurations

with the free fermion states

Partition $\lambda = (\lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_l)$

which is the same thing as the Young diagram



with the first row with λ_1 boxes
the second row with λ_2 boxes etc

is identified with the state

$$|\lambda\rangle = \psi_{-\lambda_1+\frac{1}{2}}\psi_{-\lambda_2+\frac{3}{2}}\ldots\psi_{-\lambda_i+i-\frac{1}{2}}\ldots =$$

$$\prod_{i=1}^{\infty}\psi_{-\lambda_i+i-\frac{1}{2}}\tilde{\psi}_{-i+\frac{1}{2}} \quad |\mathrm{vac}\rangle$$

in the free fermion Hilbert space

Bosonizations

From v free fermions to v chiral bosons

$$\psi^i =: e^{\mathrm{i}\varphi_i} : , \qquad \tilde{\psi}_i =: e^{-\mathrm{i}\varphi_i} :$$

From N free fermions to one free fermion to one boson

$$\Psi_{Nr+i-\frac{N+1}{2}} = \psi^i_r , \qquad \tilde{\Psi}_{Nr-i+\frac{N+1}{2}} = \tilde{\psi}_{i,r}$$

$$\Psi =: e^{\mathrm{i}\Phi} : , \qquad \tilde{\Psi} =: e^{-\mathrm{i}\Phi} :$$

General story leads to more general CFTs

in two dimensions, such as Liouville and Toda theories

and their q-deformations

Three classes of $\mathcal{N} = 2$ theories

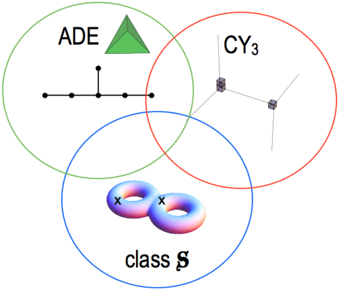
which are conformal in the ultraviolet

1) Theories which have Lagrangians.

2) Theories whose low-energy behavior is described by an auxiliary two-dimensional gauge theory (Hitchin's system)

3) Theories, for which Z can be computed using (topological) string theory.

Three ways of engineering $\mathcal{N} = 2$ theories



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- Quiver theories with Lagrangian description

The theories of class S are defined using M-theory fivebranes

The theories of class CY are defined using
string compactifications on Calabi-Yau manifolds
in the infinite CY volume limit,
where supergravity decouples

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The quiver has to be either an affine Dynkin diagram
there are no fundamentals
and the ranks of the gauge factors
are fixed up to a single integer factor

$$v_i = N a_i$$

with a_i being Dynkin labels

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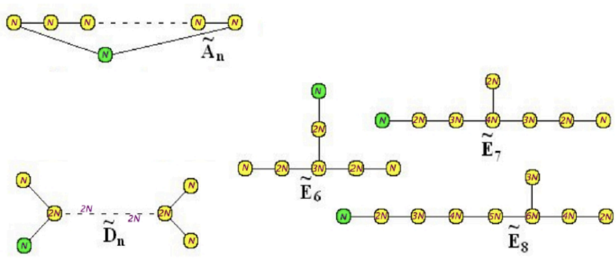
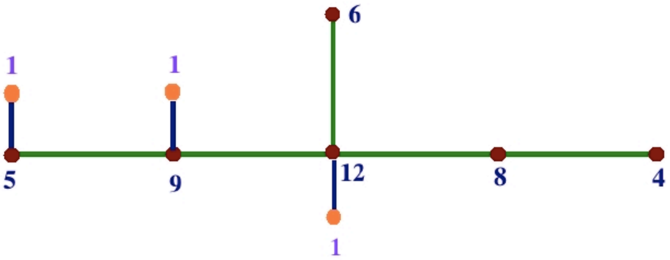
ADE quiver theories:
Aff quivers



Or the quiver is a Dynkin diagram of a finite dimensional Lie group G_Γ

In this case v_i's have more freedom

Asymptotically conformal quiver theories:
« Fin ADE quiver »
E_6 example



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These theories are solved in terms
of the auxiliary four or three dimensional
gauge theory
with the gauge group G_Γ
e.g. E_6 in the last example

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The phase space of the integrable system
describing the special geometry
of the moduli space of vacua
of the theory corresponding to Aff Dynkin diagrams
is the moduli space of charge N instantons
with the gauge group G_Γ
on $\mathbf{R}^2 \times \mathbf{T}^2$
where the geometry of $\mathbf{T}^2$ and asymptotics of the gauge fields
encode the gauge couplings and the masses

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For Fin quivers
one gets G_Γ-monopoles
on $\mathbf{R}^2 \times \mathbf{S}^1$
with Dirac singularities

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For $G_\Gamma = SU(k)$
one can employ Nahm's duality
leading to the moduli space of
solutions of $SU(N)$ Hitchin's equations
on $\mathbf{T}^2$ or $\mathbf{R}^1 \times \mathbf{S}^1$ with k singularities

For $G_\Gamma = SU(k)$
one can employ Nahm's duality
leading to the moduli space of
solutions of $SU(N)$ Hitchin's equations
on $\mathbf{T}^2$ or $\mathbf{R}^1 \times \mathbf{S}^1$ with k singularities

$$F_{z\bar{z}} + [\Phi, \bar{\Phi}] = \sum_{i=1}^{k} J_i^{\mathbb{R}} \delta^{(2)}(z - z_i)$$

$$\mathcal{D}_{\bar{z}} \Phi = \sum_{i=1}^{k} J_i^{\mathbb{C}} \delta^{(2)}(z - z_i)$$

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This picture eventually leads to
the two-dimensional conformal theory
with the Kac-Moody $\widehat{SU(N)}$ symmetry
or the corresponding W_N-algebra
of the Liouville or A_{N-1} Toda theories

$$L = \int \sum_{i=1}^{N} \partial\phi_i \bar{\partial}\phi_i + \sum_{i=1}^{N-1} e^{\phi_i - \phi_{i+1}}$$

as in the AGT conjecture

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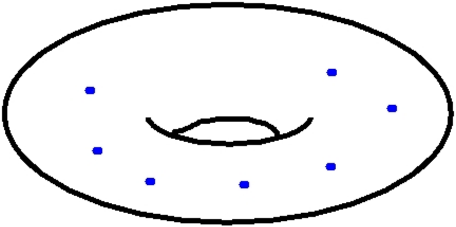
The singularities become
the vertex operator insertions



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Attempt at the theory of BPS/CFT correspondence:

NONPERTURBATIVE DYSON-SCHWINGER EQUATIONS

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DYSON-SCHWINGER EQUATIONS

INVARIANCE OF (PATH) INTEGRAL

$$\langle \mathcal{O}_1(x_1)\dots\mathcal{O}_n(x_n)\rangle = \frac{1}{Z}\int_\Gamma D\Phi\, e^{-\frac{1}{\hbar}S[\Phi]}\,\mathcal{O}_1(x_1)\dots\mathcal{O}_n(x_n)$$

UNDER "SMALL" DEFORMATIONS
OF THE INTEGRATION CONTOUR

$$\Phi \longrightarrow \Phi + \delta\Phi$$

DYSON-SCHWINGER EQUATIONS

QUANTUM EQUATIONS OF MOTION

$$\langle \mathcal{O}_1(x_1)\ldots\mathcal{O}_n(x_n)\delta S[\Phi]\rangle =$$

$$\hbar\sum_{i=1}^{n}\langle \mathcal{O}_1(x_1)\ldots\mathcal{O}_{i-1}(x_{i-1})\delta\mathcal{O}_i(x_i)\mathcal{O}_{i+1}(x_{i+1})\ldots\mathcal{O}_n(x_n)\rangle$$

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DYSON-SCHWINGER EQUATIONS

WITH SOME LUCK

$=$

GOOD CHOICE OF (POSSIBLY NON-LOCAL) OBSERVABLES

$$\mathcal{O}_i(x)$$

AND IN SOME LIMIT (CLASSICAL, PLANAR, ...)

THE DS EQUATIONS FORM A CLOSED SYSTEM

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FOR EXAMPLE

$$\hbar \longrightarrow 0$$

CLASSICAL LIMIT

$$\langle \mathcal{O}_1(x_1) \ldots \mathcal{O}_n(x_n) \delta S[\Phi] \rangle = \hbar \, (\ldots) \to 0$$

$$\Leftrightarrow \;\; \delta S[\Phi] = 0$$

GAUGE THEORY

$$\Phi \longrightarrow A = A_\mu dx^\mu \in \mathrm{Lie}\, U(N)$$

$$\frac{1}{\hbar} S[\Phi] \longrightarrow S_{YM}[A] = -\frac{1}{4g^2} \int_{\mathbf{R}^4} \mathrm{tr} F_A \wedge \star F_A$$

$$\mathcal{O}_i(x_i) \longrightarrow W_R(\gamma) = \mathrm{tr}_R\, P\exp \oint_\gamma A$$

$$\mathcal{W}(\gamma) = \frac{1}{N} \langle W_\square(\gamma) \rangle$$

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GAUGE THEORY: PLANAR LIMIT

$$N \longrightarrow \infty, \qquad g^2 \to 0,$$

$$\mathit{FINITE} \qquad \lambda = g^2 N$$

$$\Delta_\gamma \mathcal{W}(\gamma) = \frac{g^2}{N} \langle W_\Box(\gamma) \delta S_{YM}[A] \rangle =$$

$$= \lambda \delta_{\gamma=\gamma_1 \star \gamma_2} \mathcal{W}(\gamma_1) \mathcal{W}(\gamma_2) + \frac{1}{N^2} \ \mathit{correctons}$$

MAKEENKO-MIGDAL LOOP EQUATIONS

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GAUGE THEORY: MATRIX MODEL

$$\Phi \in \mathrm{Lie}\, U(N)$$

$$\frac{1}{\hbar} S[\Phi] = \frac{1}{\hbar} \mathrm{tr}\, V(\Phi)$$

$$V(X) = v_p X^p + v_{p-1} X^{p-1} + \ldots + v_1 X + v_0$$

$$\mathcal{O}(x) = \frac{1}{N} \mathrm{tr}_{\Box} \left(\frac{1}{x - \Phi} \right)$$

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MATRIX MODEL

PLANAR LIMIT: $\lambda = \hbar N$ FIXED

$$\hbar \to 0, \;\; N \to \infty$$

DS EQUATIONS $\Longrightarrow$ LOOP EQUATIONS

$$y(x)^2 = V'(x)^2 + g_{p-2}(x)$$

$$y(x) = \langle \mathcal{O}(x) \rangle + V'(x)$$

$g_{p-2}(x) =$ DEGREE $p-2$ POLYNOMIAL IN x

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QFT PATH INTEGRAL INVOLVES SUMMATION

OVER TOPOLOGICAL SECTORS

FOR EXAMPLE, IN GAUGE THEORY

$$Z = \sum_{n \in \mathbb{Z}} e^{in\vartheta} \int_{\mathcal{A}_n} \left[\frac{DA}{Vol(\mathcal{G}_n)} \right] e^{-S_{YM}[A]}$$

$$-\frac{1}{8\pi^2} \int \mathrm{tr} F_A \wedge F_A = n, \qquad A \in \mathcal{A}_n$$

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NON-PERTURBATIVE DS EQUATIONS

IDENTITIES DERIVED BY

LARGE “DEFORMATIONS” OF THE PATH INTEGRAL CONTOUR

$$A \in \mathcal{A}_n \longrightarrow A + \delta A \in \mathcal{A}_{n+1}$$

GRAFTING A POINT-LIKE INSTANTON

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COMPATIBILITY OF PERTURBATIVE

expansion in $\hbar$, g^2, ...

AND NON-PERTURBATIVE CONTRIBUTIONS

expansion in $e^{-\frac{1}{\hbar}}$, $e^{-\frac{1}{g^2}}$, ...

Resurgence, trans-series, ... A.Voros, J.Zinn-Justin, ...
Exact β-functions in SYM, Novikov-Shifman-Vainshtein, Zakharov

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TESTING GROUNDS

$\mathcal{N} = 2$ theories in $4d$

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OBSERVABLES FOR DS EQUATIONS

OBSERVABLE $Y(x)$

IN FOUR DIMENSIONAL $U(N)$ GAUGE THEORY

$$\mathbf{Y}(\mathbf{x}) \sim \mathbf{det}_{\mathbb{C}^N}(x - \sigma) \sim \prod_{\alpha=1}^{N} (x - a_\alpha)$$

NAIVELY

OBSERVABLES FOR DS EQUATIONS

$\mathbf{Y}(\mathbf{x})$ IN FOUR DIMENSIONS

MORE PRECISELY

$$\mathbf{Y}(\mathbf{x}) = x^N \exp - \sum_{k=1}^{\infty} \frac{1}{kx^k} \mathrm{Tr}\sigma^k$$

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$$\mathbf{Y}(\mathbf{x}) = x^N \exp - \sum_{k=1}^{\infty} \frac{1}{kx^k} \mathrm{Tr}\sigma^k$$

Non-perturbatively, e.g. in instanton background becomes

RATIONAL FUNCTION OF DEGREE N

UNLIKE THE NAIVE $\det_{\mathbb{C}^N}(x - \sigma)$ IT HAS POLES

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◊

FOR QUIVER GAUGE THEORY

$$G = U(N_1) \times \ldots \times U(N_r)$$

$$\mathbf{Y}(\mathbf{x}) \longrightarrow (\mathbf{Y}_1(x), \mathbf{Y}_2(x), \ldots, \mathbf{Y}_r(x))$$

Several rational functions of x

◊

MAIN CLAIM

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MAIN CLAIM

THERE EXIST

LAURENT POLYNOMIALS (SERIES FOR AFFINE γ)

$$\mathcal{X}_i(x) = Y_i(x) + \ldots$$

in $Y_j(x+$ linear combinations of masses $m_e)$ such that

$$\langle \ \mathcal{X}_i(x) \ \rangle = \text{POLYNOMIAL IN} \ \ x$$

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MAIN CLAIM

$$\mathcal{X}_i(x) = Y_i(x) + \ldots$$

COEFFICIENTS $=$ PRODUCTS OF

$$\mathfrak{q}_j, P_j(x + \text{linear combinations of } m_e), \qquad j \in \mathrm{Vert}_\gamma$$

$$P_j(x) = \mathbf{det}_{M_j}(x - \mathfrak{M}_j)$$

ENCODE FUNDAMENTAL MASSES

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WE CALL $\mathcal{X}_i(x)$

THE FUNDAMENTAL GAUGE CHARACTERS

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MORE GENERAL LOCAL OBSERVABLES $\mathcal{X}_{\mathbf{w}}(x)$

THE GAUGE CHARACTERS

$$\mathcal{X}_{\mathbf{w}}(x) = \mathcal{X}_{w_1}(x-\nu_1)\mathcal{X}_{w_2}(x-\nu_2)\ldots\mathcal{X}_{w_p}(x-\nu_p) + \qquad \text{corrections}$$

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THE MAIN CLAIM $=$ SEIBERG-WITTEN GEOMETRY

of low-energy effective theory

NN, V.Pestun, 2012

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(DOUBLE) QUANTUM SEIBERG-WITTEN GEOMETRY

when theory is subject to Ω-deformation

$$\mathcal{X}_{\mathbf{w}}(x) \longrightarrow \chi_{\mathbf{w}}(x) \text{ -- qq-characters}$$

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HIDDEN SYMMETRY OF THE SPACE OF VACUA

quantum group based on the quiver

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THE ORIGIN OF qq-CHARACTERS

$\mathcal{X}_{\mathbf{w}}(x) =$ PARTITION FUNCTION

OF A POINT-LIKE DEFECT $\mathscr{D}_{\mathbf{w}}(x)$

$\mathscr{D}_{\mathbf{w}}(x)$ CAN BE ENGINEERED

USING INTERSECTING BRANES

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Brane-world scenarios

propose that the Standard Model is confined to a brane

while gravity propagates in the bulk

Brane-world scenarios

propose that the Standard Model is confined to a brane

which could originate from the string theory D-branes

with closed strings propagating in the bulk

Brane-world scenarios

propose that the Standard Model is confined to a brane

which could originate from the string theory D-branes

spanning a nearly flat, or a nearly AdS space

What if there is more then one stack of branes?

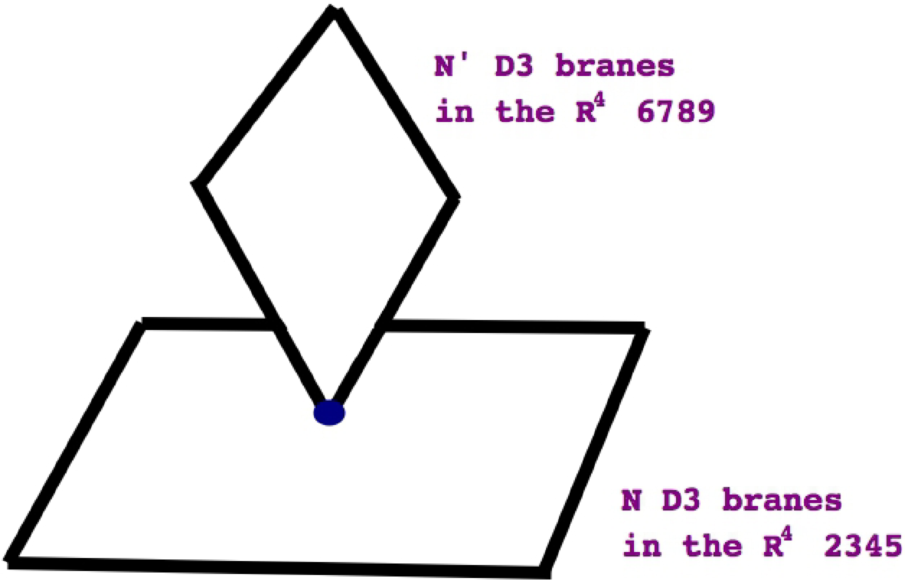
Branes that intersect?

The intersections could be either

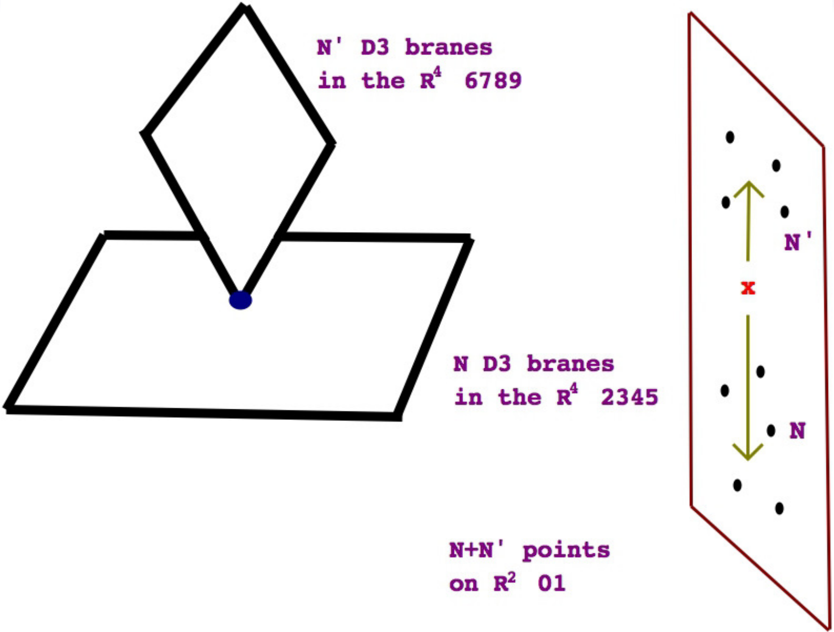
the defects in the worldvolume or

our braneworld could be an intersection

Local model: $\mathbb{R}^4 \vee \mathbb{R}^4 \subset \mathbb{R}^8$

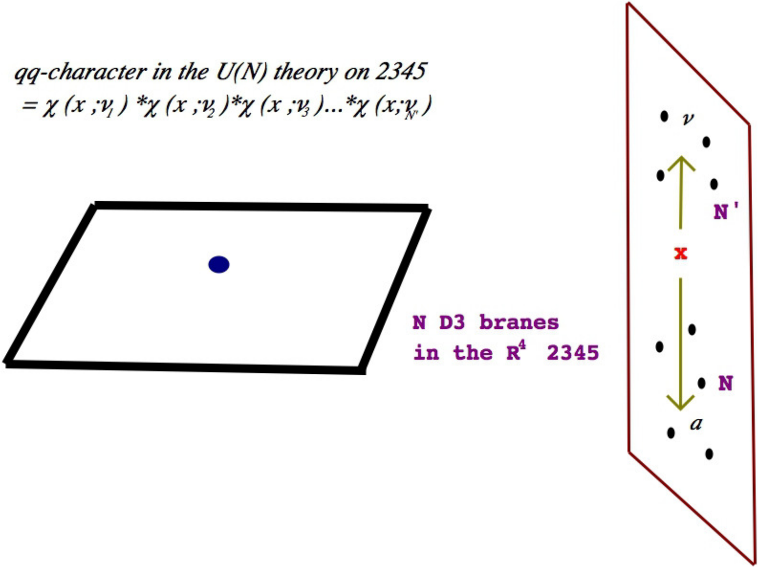


Local string model: $\mathbb{R}^4 \vee \mathbb{R}^4 \times \mathbb{R}^2 \subset \mathbb{R}^{10}$



Integrate out one of the stacks

To produce observables on the remaining stack of branes



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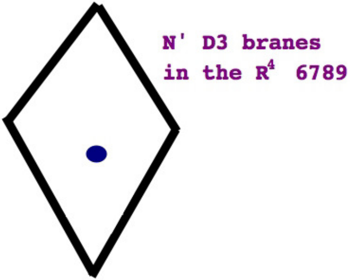
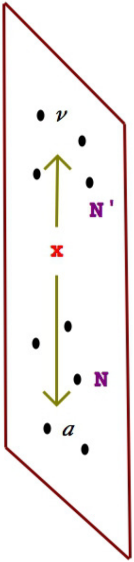
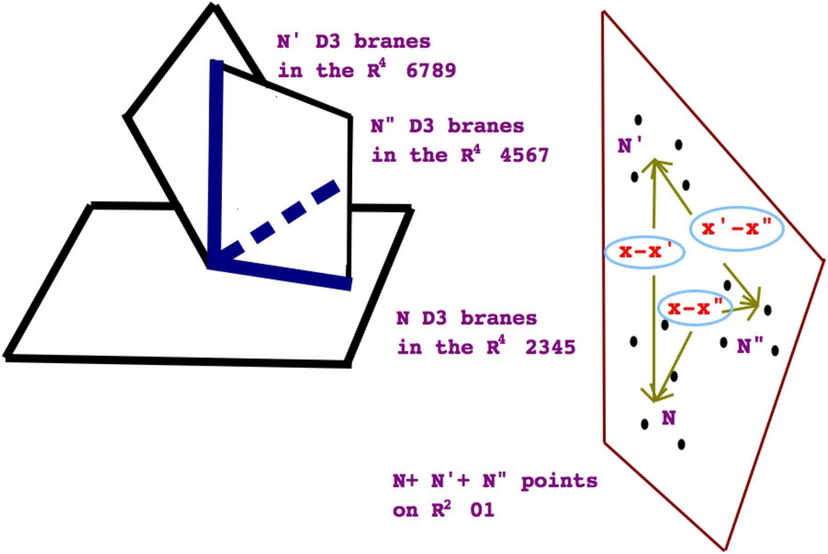


qq-character in the U(N') theory on 6789

$$= \chi(x;a_1) * \chi(x;a_2) * \chi(x;a_3) \dots * \chi(x;a_N)$$



Surface operators from intersecting braneworlds



$\diamondsuit$

EXAMPLE: $U(N)$ THEORIES

A_1 CASE: $N_c = N$, $N_f = 2N$

FUNDAMENTAL qq-CHARACTER

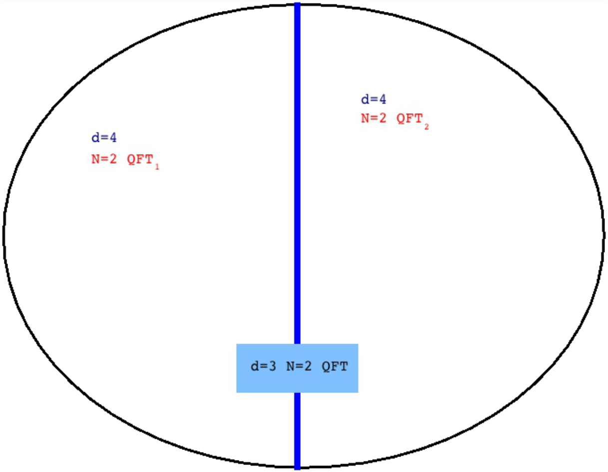
$$\mathcal{X}_1(x) = Y(x + \epsilon_1 + \epsilon_2) + \mathfrak{q} P(x) Y(x)^{-1}$$

$\diamondsuit$

For the theories with one ϵ-parameter
one finds the classical G_Γ symmetry deformed
into the Yangian symmetry $Y(\mathfrak{g}_\Gamma)$
the symmetry of the quantum spin chains

It appears that the full Yangian symmetry

is generated by the domain walls



$QFT_1 \rightarrow QFT_2$

The challenge is to extend these of observations

to the practical scheme, extending beyond the BPS-sector

The real challenge is to extend these observations

to the practical scheme, extending beyond the BPS-sector

beyond the realm of supersymmetric theories

The real challenge is to extend this sequence of observations

to the full QFT spectrum

THANK YOU

THANK YOU,

and HAPPY 100th ANNIVERSARY

ISAAK MARKOVICH