Minimal models for chaotic quantum dynamics in spatially extended many-body systems

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Joint work with Amos Chan and Andrea De Luca

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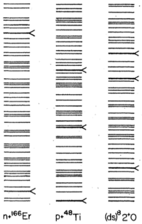
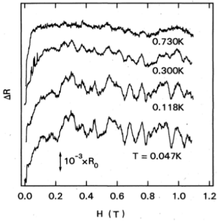
Studies of 'generic' quantum systems

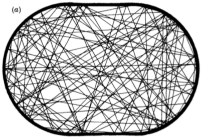
Nuclear physics



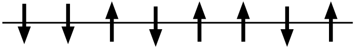
Mesoscopic conductors



Low-D systems



Spatially extended many-body systems



Characterising spectra

Evolution operator $W(t)$, eigenvalues $e^{i\theta_n t}$

Spectral form factor $K(t) = \langle \left| \sum_n e^{i\theta_n t} \right|^2 \rangle \equiv \langle |\mathrm{Tr} W(t)|^2 \rangle$

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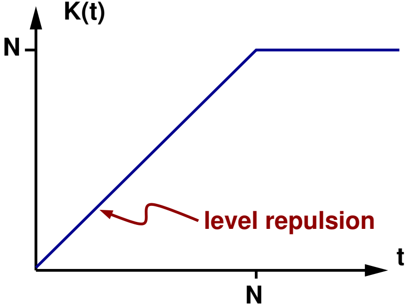
Unitary $N \times N$ random matrices



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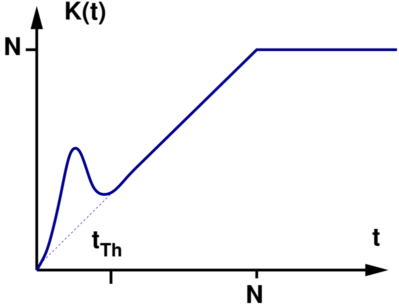
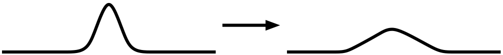
Mesoscopic conductor: Thouless time

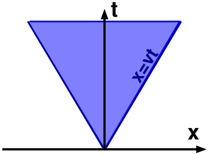


Characterising dynamics

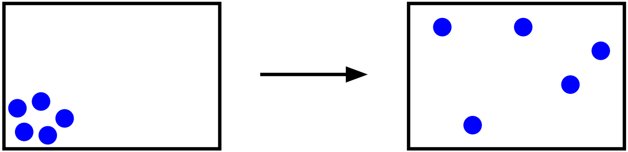
Hydrodynamics & conserved densities



Dynamics of quantum information



Equilibration under unitary dynamics



Speed limits without relativity

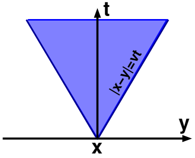
Lieb-Robinson bound (1972)

Max propagation speed v for disturbances in short-range lattice models

For local observables at x and y

$$[O(y,t), O(x)]$$

small outside lightcone

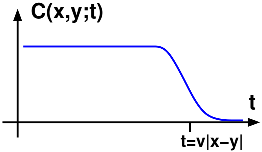


Speed limits without relativity

Out-of-time-order correlator (OTOC)

$$C(x, y; t) \equiv [O(y,t)O(x)O(y,t)O(x)]_{\mathrm{av}}$$

E.g. with $\mathrm{Tr}\,O(x) = 0$
and $O(x)^2 = \mathbb{1}$
& likewise for $O(y)$



Larkin & Ovchinnikov (1975)

Entanglement dynamics

Quantifying 'equilibration' under unitary dynamics

Density matrix $\rho(t) = |\Psi(t)\rangle\langle\Psi(t)|$ for full system

— pure state preserved under time evolution

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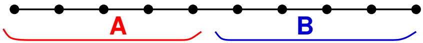
Reduced density matrix $\rho_A(t) = \mathrm{Tr}_B \rho(t)$



Entropy of sub-system may grow with time & saturate at long times

Aim: solvable models for ergodic phase

Simple physics:

Eliminate conserved densities $\Rightarrow$ time-dept evolution operator

Simple solution: Random matrices & spatial structure

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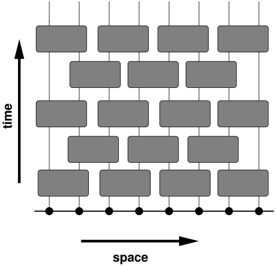
Random unitary circuits



Nahum, Ruhman, Vijay and Haah, PRX (2017)
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Fixed evolution operator w/o conserved densities $\Rightarrow$ **Floquet**

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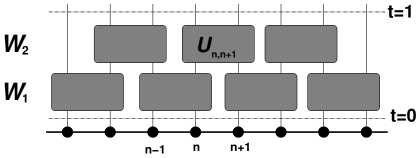
Minimal model

L-site lattice of q-state 'spins'

Floquet operator W is $q^L \times q^L$ unitary matrix



Each $q^2 \times q^2$ unitary $U_{n,n+1}$ independently Haar-distributed

Solve for $q \to \infty$

Behaviour of Floquet model

Is behaviour consistent with ergodic phase?

Relaxation of local observables

Dynamics of quantum information?

Out-of-time-order correlator

Entanglement growth

Spectral correlations?

'Thouless time' in many-body system

Relaxation of local observables

Local operator in q-state Hilbert space at site: $O(x)$

with $\mathrm{Tr}\, O(x) = 0$ and $O(x)^2 = \mathbb{1}_q$.

Want $[O(x,t)O(x)]_{\mathrm{av}}$

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Find for $q \to \infty$

$$[O(x,t)O(x)]_{\mathrm{av}} = \begin{cases} 1 & t = 0 \\ 0 & t > 0 \end{cases}$$

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Short times and finite q:

$$[O(x,t)O(x)]_{\mathrm{av}} = \begin{cases} 1 & t = 0 \\ 0 & t = 1 \\ q^{-7} & t = 2 \\ 16q^{-11} & t = 3 \end{cases}$$

Out-of-time-order correlator

Find for $q \to \infty$

$$[O(y,t)O(x)O(y,t)O(x)]_{\mathrm{av}} = \begin{cases} 1 & |t| < |x-y|/2 \\ 0 & |t| \geq |x-y|/2 \end{cases}$$

Butterfly velocity $v = 2$

Entanglement growth in Floquet model

Initial state $|\psi\rangle$ – product state in site basis

Reduced density matrix $\rho_A(t) = \mathrm{Tr}_B W(t)|\psi\rangle\langle\psi|W^\dagger(t)$

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Réyni entropies

Find with $\alpha = 2$ or 3 and q large

$$\langle \mathrm{Tr}_A[\rho_A(t)^\alpha]\rangle = \begin{cases} f_\alpha(t) q^{-2(\alpha-1)t} & t \leq L/4 \\ K_\alpha q^{-(\alpha-1)L/2} & t > L/4 \end{cases}$$

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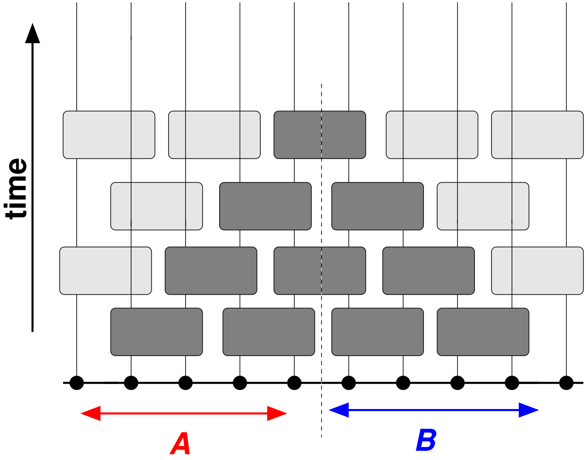
Interpretation:

$\rho_A(t)$ has q^{2t} non-zero eigenvalues, each $\mathcal{O}(q^{-2t})$

$\equiv$ Mixed (infinite temperature) state for system of $2t$ sites

Entanglement spreads at speed $v = 2$

Entanglement growth in quantum circuits



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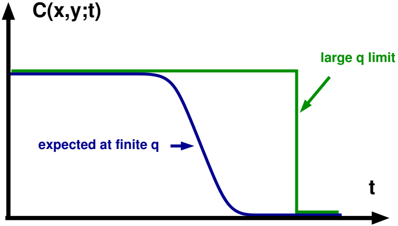
OTOC: Front is sharp, not diffuse



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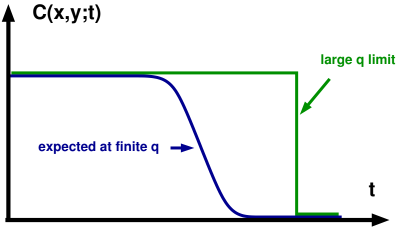

Velocities: 'Naive' value for all speeds
(butterfly, entanglement spreading ...)

Spectral form factor

Evolution operator $W(t)$ with eigenvalues $\{e^{i\theta_n}\}$

Spectral form factor $K(t) = \sum_{m,n} e^{i(\theta_m - \theta_n)t}$

Large $q \Rightarrow$ random matrix behaviour in Floquet model

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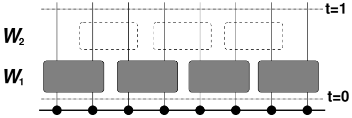
— consequence of coupling



Without W_2 find instead

$$K(t) = t^{L/2}$$

Origin of RMT behaviour?

Constructive interference of diagonal terms in double sum on paths (diagonal approximation $\sim$ diffusons)

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$$K(t) = \left\langle \sum_{nm} e^{i(\theta_n - \theta_m)t} \right\rangle = \left\langle \mathrm{Tr}[W(t)] \mathrm{Tr}[W^\dagger(t)] \right\rangle$$

$$\mathrm{Tr}[W(t)] \equiv \sum_{a_1 \ldots a_t} W_{a_1 a_2} W_{a_2 a_3} \ldots W_{a_t a_1}$$

$$\mathrm{Tr}[W^\dagger(t)] \equiv \sum_{b_1 \ldots b_t} W^\dagger_{b_1 b_2} W^\dagger_{b_2 b_3} \ldots W^\dagger_{b_t b_1}$$

Constructive interference if path $b_1 b_2 \ldots b_t$ though Fock space is reversed copy of path $a_1 a_2 \ldots a_t$

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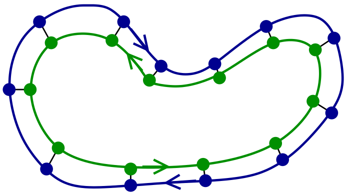
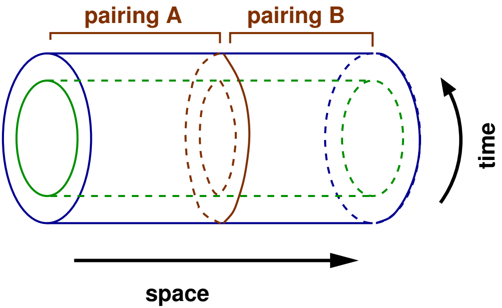
Pictorially:



t possible pairings

New possibilities in many-body systems?

Spatial domains with distinct pairings between time-reversed paths



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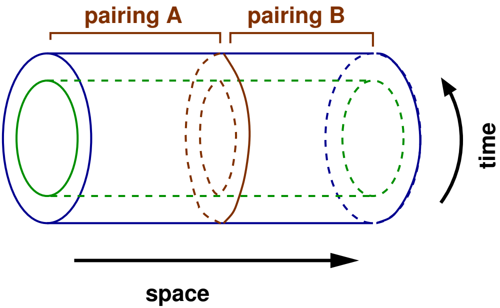

Equivalence to t-state Potts model:

t pairings in each domain

& statistical cost for domain walls

Many-body 'Thouless time'

$$\text{Small } t \quad \Rightarrow \quad L \text{ uncoupled sites} \quad \Rightarrow \quad K(t) = t^L$$

$$\text{Large } t \quad \Rightarrow \quad \text{all sites coupled} \quad \Rightarrow \quad K(t) = t$$

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Exact mapping to t-state Potts ferromagnet $K(t) = Z_{\mathrm{Potts}}$

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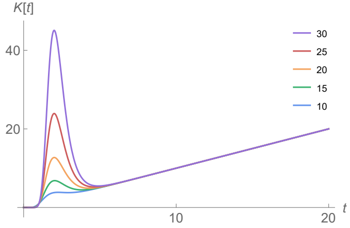

$K(t)$ vs t for $q \to \infty$

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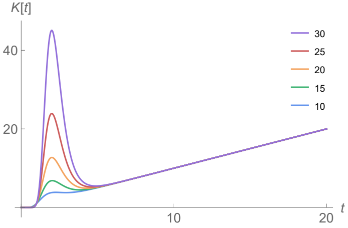

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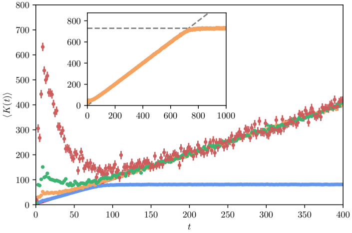

K(t) vs t for $q = 3$, $L = 4 - 10$

Summary

Floquet models at large q give solvable ergodic phase

Systematic calculations for $q \to \infty$

Rapid local relaxation

Light cone in OTOC

Ballistic growth of entanglement

Many-body Thouless time

Crossover between uncoupled sites and single RMT behaviour